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Some years ago there was a scandal at the Institute for Advanced Study at Princeton. It was proposed to appoint a certain social scientist to permanent membership, but the recommendations for him were ambiguous to say the least, ranging from strong approval to contemptuous dismissal. Certain leading mathematicians at the Institute led the campaign to ensure that this individual was not appointed. The story was featured in the *New York Times,* and many mathematicians, reading the account, fell to wondering if the work of a mathematician, rather than a social scientist, could have received such widely divergent judgments. Our strong belief is that this couldn't have happened.

More recently, we have witnessed the (successful) campaign of Serge Lang (see *Chronicle of Higher Education,* February 3, 1988, p. B4) against the election of a certain Professor Huntington to the National Academy of Sciences. Huntington is a social scientist who had invented certain equations relating to such quantities as 'satisfaction indices', designed to provide insight into the state of contemporary society. Lang argued that mathematics was being misused; the dispute was carried further in the columns of *The Mathematical Intelligencer* by Neal Koblitz and Herbert Simon, acting as surrogates for the main protagonists (see the Winter, Spring, and Summer issues of 1988); and, once again, mathematicians asked themselves whether there could be such utterly conflicting views about the work of a leading mathematician. Once again, too, we concluded that there could not.

Why do we distinguish in this way between mathematics and the social sciences? It is because we believe that there is an objective aspect to an assessment of the quality of a piece of mathematics which — it seems to us and evidently to others — is not necessarily present in the assessment of research in the social sciences, so that peer evaluation of mathematical research at least has the potential to be fair and reliable.[1] There may be disagreements about the relative standing of different areas of mathematics (e.g., algebra vs. analysis, hard analysis vs. soft analysis, point set topology vs. algebraic topology, algebraic topology vs. geometric topology, and so on) but, within a given branch, there is general agreement as to who are the giants and what are their major contributions. Of the Fields Medalists with whose work we are familiar — suffice it to name Atiyah, Serre, Thom, Kodaira, Thompson, Donaldson, Freedman, Novikov, Grothendieck, Smale — there is absolutely no doubt of their eminence and of the seminal significance of their work and the stimulation which it currently affords. In this respect the Fields Medals differ from Nobel Prizes, which are usually awarded long after the relevant work was done, and where there are often strong disputes over the merits of the laureates and over certain singular omissions. It is an open secret that Graham Greene has been passed over for the Literature Prize because of the prejudice of a member of the selection committee, while the award of the Peace Prize to Henry Kissinger and Le Duc Tho continues to strike most reasonable people as utterly ludicrous.

Lysenko was able to fool a lot of people, including even some biologists, into believing that he had revived Michurinism and demonstrated the inheritance of acquired characteristics. By contrast, we claim that there can be no successful chicanery in mathematics; proofs must be clear and convincing, results must be applicable. There can be no conspiracy to believe something which is ideologically acceptable or socially convenient, such as the Nazi 'theories' of racial superiority. It is true that there has been controversy over the proof of the 4-color theorem by Appel, Haken and Koch; but the question at issue is not 'Is it true?' or 'Is it important?' but 'Has it been proved?'

There has also been controversy in connection with the development of fractal geometry — the reader is again referred to *The Mathematical Intelligencer* to get the flavor of this juicy dispute (see the Fall, 1989, issue) — and, at first sight, this may appear to concern the quality of the mathematics. We claim, however, that this appearance is illusory. In reality, what is in question is not

the quality of the mathematics in the theory of fractals but whether there is a distinctive mathematical theory of fractals, distinguishable from a theory derivable from classical function theory. Inherent in the controversy, therefore, is a disagreement over who has priority for discovering the undoubtedly important Mandelbrot set. Such questions of priority, in their turn, inevitably raise ethical issues.

These examples serve, in fact, to reinforce our conviction that there is an inescapable ethical component to mathematics as a human activity. Truth and integrity play a key role in mathematical research and publication — one of us (PH) recalls Henry Whitehead's advice, which for him was a principle, never to accept in your own work a result which you could not yourself prove. Of course, this precept has a practical value, since one does not wish to act as a channel for the transmission of error; but Henry's basic point is that one must take responsibility for what one publishes. It is its relation to truth and to the integrity of its practitioners which is the humanistic aspect of mathematics which we wish to stress in this essay. We thus find ourselves in strong disagreement with the views of our friend and colleague Reuben Hersh [He] who denies that pure mathematics has an ethical component.

Before developing our theme, we should stress that we are not speaking of the ethical or humanistic aspects of *teaching* mathematics.[2] Our concern is with the humanistic aspects of mathematics itself. On the other hand, neither we nor Hersh would deny that all teaching of mathematics provides the opportunity — indeed, we would say, the obligation — to bring to our students' attention the ethical commitment which the proper practice of mathematics requires. This obligation, deriving as it does from the nature of mathematics itself, does fall within our purview. We regard it as especially urgent to emphasize it in view of the fact that, for easily comprehensible reasons, it is so often neglected. Let it therefore receive our immediate attention.

COMMUNICATING ETHICAL VALUES TO STUDENTS

We believe that most of the difficulty encountered by our students in trying to learn mathematics at the university level stems from the fact that they have never seen any *real* mathematics before. They have been 'taught mathematics' in such a way that they don't recognize its relationship to the real world and don't understand that it is much more than merely a game. They don't realize that the symbols they write must mean something and that that something should always make sense; they don't understand that there is an unbridgeable gap between truth and falsehood in mathematics, not a mere continuum of meaningless statements. They don't realize that each statement they write down should follow logically from its predecessor; and they don't appreciate why an argument is not complete unless every step does in fact follow from the previous one. In a word, they do not appreciate the integrity of the subject — but this is scarcely their fault. Their experience has left them blissfully unaware of the fact that mathematics involves any question of integrity at all!

Crucial to any attempt to repair this situation is the understanding that the students are not to blame for it. When they reach the university they find themselves in the position of desperately trying to learn material for which they have not been suitably prepared. The response of students to their pre-college mathematics education[3] is, we believe, perfectly natural and should have been expected. All too often, that education has consisted of being given, each day, *the rule of the day*, followed by a set of exercises for which this rule produces *an answer* (that may, for odd numbered problems, be looked up in the back of their textbook!). We claim that students who have been taught mathematics in this catechistic way have been doubly cheated. They have not been given the opportunity to learn what mathematics really is, and they are not able to use what they have supposedly learnt.

For obvious reasons — at least to anyone who either appreciates or uses mathematics — we believe that it is absolutely essential that the teaching of mathematics, at all levels, should embody Henry Whitehead's emphasis on understanding what you use. This has a very important long-term practical aspect in that what students understand they will continue to have at their disposal. Even though some details of a mathematical result may fade over time, if one really understands the underlying principles then it is very likely that one will be able to reconstruct the desired result when it is needed.[4] We believe an equally important, and more immediate, consequence is that students who are taught mathematics for understanding (even at the expense of speed) will have a much better opportunity to learn — and so to appreciate — the real nature of mathematical thinking and hence, as we have said, to make the ethical commitment which the proper practice of mathematics requires. A devotion to the pursuit of truth, in all its aspects, would bring students, and teachers, closer to an understanding of the essential content of a mathematical statement. Thus, for example, they would understand that not all wrong answers are equally wrong, and that being able to recognize whether or not an answer is plausible is much more important than memorizing

meaningless formulas long enough to pass a test. However, herein lies a severe practical problem.

We believe that mathematics, when practiced properly by students, should incorporate the ethical commitment inherent in mathematics itself. But to achieve this is difficult. Students naturally want to make good grades. They have been systematically programmed to become successful grade-grubbers. We cannot change their need for good grades — and even if we could it might not be desirable — but we can change the way we test and the way we grade.

We can give credit to the student who recognizes an answer is wrong, says so, and explains why the answer is not a reasonable one. Moreover, recognizing that an answer is unreasonable is itself a sign of a maturing awareness of an important feature of mathematics itself. It is a sad fact that most people do not realize that it is perfectly possible to know something is wrong without knowing what the correct answer is. If they had learned, and understood, the technique of casting out nines (see [HP]) to check an arithmetic calculation, they would thereafter fully appreciate the fact that one can, in some situations, know for certain that a calculation cannot be correct. They would also realize that if the check "works", that doesn't guarantee that the calculation *is correct*.

We can also give credit to the student who begins a proof, knows how it should end and *admits* that the intervening steps are missing; so much the better, of course, if the student also states the nature of what should be filled in. But we would not give credit to the student who puts in a few steps at the beginning and just before the end, hoping the instructor won't notice that there is a gap in the middle. We believe that this kind of behavior, which we would call fundamentally dishonorable, should be strongly discouraged, and that students should be made aware of the fact that there is an internal structure to mathematics that should not be violated. By giving the student credit for the correct thinking he or she does, we encourage both honesty and effective mathematical thinking.

THE ETHICS OF MATHEMATICS IN EVERYDAY LIFE

If we are right in asserting that the pursuit of mathematics has an inescapable ethical content, should not that moral component transfer itself to other aspects of our lives, professional and personal? Should we not be more consciously aware of this moral component? Should we not extend our respect for truth and our concern for the probity of our research to some of our other activities? We claim that we often do — to our serious disadvantage as advocates and opinion-formers! Let us elaborate.

- It is a feature of our professional work to distinguish sharply between what we know well and the rest of mathematics. Thus, in particular, we are very well aware of our areas of ignorance; this awareness is, indeed, as an important criterion of the educated person. Unfortunately, such awareness is all too often absent among those who exert an influence on public opinion; unfortunately, too, it is no advantage if one wishes to popularize one's cause, since a confession of ignorance is usually taken — quite erroneously — to be an admission of incompetence. How many politicians today admit their ignorance of European history?

- We are naturally liberal at a time when extreme views tend to command more support. We live in an age of single-issue fanatics (to borrow Bernard Levin's vivid phrase); mathematicians should find such fanaticism very distasteful, if not impossible.

- Mathematicians are opposed to the use of force, as it is not possible to establish a theorem by intimidation of the sceptics, or by the demonstration of superior strength. It is thus offensive to their sense of proper order in the universe that disputes should be resolved by means which pay no heed to the worthiness of the cause.

- Since rational thought, and hence reasonableness, are our research method, we tend to see the other person's point of view. Such reasonableness is scarcely conducive to the evocation of fanatical support — one does not persuade people to man (or woman) the barricades by arguing that one's point of view is in certain clearly defined respects superior to that of the enemy.

- We tend to believe in the reasonableness of others, especially of those with whom we are in dispute or whose opinions we wish to influence. This belief is, unfortunately, naive and often mistaken. In such cases we are at a serious disadvantage and are likely to be completely outmaneuvered.

However, we do have some conspicuous successes to our credit. As we have said, our awareness of the existence, within our store of knowledge, of significant areas of ignorance often causes us to be unduly reluctant to participate in deliberations which range over a broad front (university mathematicians are all too rarely active

on university-wide committees); but some of us, while retaining our intellectual integrity and honesty — indeed, largely because we bring those qualities to bear — are outstandingly effective in public office. Let us cite two enormously successful university presidents, John Kemeny and Paul Olum, and the man who constitutes for us the supreme vindication of our argument, the Polish mathematician Janusz Onyckiewicz, sometime spokesman for Solidarity and later Deputy Minister of Defence in the Solidarity government of Mazowiecki.

Of course we do not deny that there are counterexamples to our claim that the discipline of mathematics imposes standards of integrity and truthfulness on its practitioners which should inform their activities outside mathematics — the name of Ludwig Bieberbach comes all too readily to mind. We must emphasize that we are only asserting that everyday life offers scope for the exercise of virtues which should have been developed by activities devoted to the understanding of mathematics and research in mathematics. But human frailty is a factor whose strength and ubiquity we recognize.

COLOPHON

We believe that we do a disservice both to mathematics and to education by failing to insist as teachers, explicitly but, of course, not constantly, on the potential role of mathematics in the development of character and morals. For the proper — and, hence, the successful — pursuit of mathematics requires a dedication to truth and integrity. We should always be modest in our claims for ourselves as mathematicians — but there is every reason for us *not* to be modest in our claims for the vast ambit of mathematics itself.

ENDNOTES

[1] One must express oneself cautiously. The impression is widespread that peer evaluation of NSF research proposals exemplifies the dictum, 'You scratch my back, I'll scratch yours.'

[2] We were surprised that the great majority of papers contributed to the *Symposium on Mathematics and Humanism* at the Winter meeting of the AMS in Louisville in January, 1990, were concerned with teaching and not with mathematics itself.

[3] It would be more accurate to describe it as *training*, rather than education. Unfortunately it is usually not even good training.

[4] In the very short term understanding can bring disadvantages because the time required to reconstruct an argument is almost certain to be greater than that required simply to regurgitate a formula learned by rote.

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