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Review: Nevanlinna-Pick spaces with hyponormal multiplication operators

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The main result of this paper (Theorem 1.1) shows that the classical Hardy space H^2 is essentially the only complete Nevanlinna-Pick space whose multiplication operators are all hyponormal. To be more specific, if $\mathcal{H}$ is an irreducible complete Nevanlinna-Pick space on a set X with kernel K such that all multiplication operators on $\mathcal{H}$ are hyponormal, then either:

1. X is a singleton and $\mathcal{H}=\mathbb{C}$.
2. There is a set of uniqueness $A\subset\mathbb{D}$ for H^2, a bijection $j\colon X\to A$, and a nowhere vanishing function $\delta\colon X\to\mathbb{C}$ such that

$$K(\lambda,\mu)=\delta(\lambda)\overline{\delta(\mu)}k(j(\lambda),j(\mu)).$$

   Here $k(z,w)$ denotes the Szegő kernel on H^2.

In particular, the map $f\mapsto\delta(f\circ j)$ is a unitary operator. Continuity and analyticity are also respected: if X is endowed with a topology such that K is separately continuous on $X\times X$, then j is also continuous. If $X\subset\mathbb{C}^n$ and K is holomorphic in the first variable, then j is holomorphic.

An interesting consequence of the main theorem is Corollary 1.3, which asserts that if $n\geq 3$ and $U\subset\mathbb{R}^n$ is an open set, then there is no irreducible complete Nevanlinna-Pick space on U which consists of continuous functions and whose multiplication operators are all hyponormal. The proof appeals to Brouwer's invariance of domain theorem.

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References

1. Jim Agler and John E. McCarthy, *Complete Nevanlinna-Pick kernels,* J. Funct. Anal. **175** (2000), no. 1, 111–124, DOI 10.1006/jfan.2000.3599. MR1774853 (2001h:47019) MR1774853
2. Jim Agler and John E. McCarthy, *Pick interpolation and Hilbert function spaces,* Graduate Studies in Mathematics, vol. 44, American Mathematical Society, Providence, RI, 2002. MR1882259
3. Călin-Grigore Ambrozie and Dan Timotin, *On an intertwining lifting theorem for certain reproducing kernel Hilbert spaces,* Integral Equations Operator Theory **42** (2002), no. 4, 373–384, DOI 10.1007/BF01270919. MR1885440 (2002m:47012) MR1885440
4. William Arveson, *Subalgebras of C^*-algebras. III. Multivariable operator theory,* Acta Math. **181** (1998), no. 2, 159–228, DOI 10.1007/BF02392585. MR1668582 (2000e:47013) MR1668582
5. Joseph A. Ball, Tavan T. Trent, and Victor Vinnikov, *Interpolation and commutant lifting for multipliers on reproducing kernel Hilbert spaces,* Operator theory and analysis (Amsterdam, 1997), Oper. Theory Adv. Appl., vol. 122, Birkhäuser, Basel, 2001, pp. 89–138. MR1846055 (2002f:47028) MR1846055
6. L. E. J. Brouwer, *Beweis der invarianz des n-dimensionalen gebiets* (German), Math. Ann. **71** (1911), no. 3, 305–313, DOI 10.1007/BF01456846. MR1511658 MR1511658

7. John B. Garnett, *Bounded analytic functions,* 1st ed., Graduate Texts in Mathematics, vol. 236, Springer, New York, 2007. MR2261424 (2007e:30049) MR2261424
8. Allen L. Shields, *Weighted shift operators and analytic function theory,* Topics in operator theory, Amer. Math. Soc., Providence, R.I., 1974, pp. 49–128. Math. Surveys, No. 13. MR0361899 (50 #14341) MR0361899

Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.