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Review: On symplectic self-adjointness of Hamiltonian operator matrices

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On symplectic self-adjointness of Hamiltonian operator matrices. (English summary)

Sci. China Math. **58** (2015), *no. 4,* 821–828.

Let X be a complex Hilbert space. A *Hamiltonian operator matrix* is a block operator matrix of the form

$$H=\begin{bmatrix} A & B \\ C & -A^* \end{bmatrix},$$

acting on $X\times X$ with A,B,C closed and densely defined, B and C self-adjoint, and H densely defined. A Hamiltonian operator matrix H satisfies $JH\subseteq(JH)^*$, in which

$$J=\begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}.$$

If $JH=(JH)^*$, then H is a *symplectic self-adjoint Hamiltonian operator matrix*. The main theorems of the paper (Theorems 3.2 and 3.6) give conditions on the entries A,B,C that are equivalent to the Hamiltonian operator matrix H being symplectic self-adjoint. Several corollaries provide equivalent conditions in terms of the relative boundedness of the matrix entries. The paper concludes with an application to symplectic elasticity.

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