Humanistic Mathematics Network Journal

Issue 25 Article 9

8-1-2001

# What Does It Mean to Understand Mathematics?

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## INTRODUCTION

There is unanimous agreement among mathematics educators (e.g., Davis 1986; Lambdin 1993; NCTM 1989) including teachers that a prime objective of mathematics teaching is to promote understanding of the subject. According to the National Council of Teachers of Mathematics [NCTM] , Teachers must help every student develop conceptual and procedural understandings... "in various aspects of mathematics (1991, 21). Unfortunately, however, there does not exist the same degree of unanimity with respect to what is meant by 'understanding mathematics.'" According to Tom Romberg (Kieran 1994, 590), "There isn't a common definition of understanding." Perceptions of mathematical understanding vary greatly and could range from a simple recall of mathematical facts to an evaluation of a mathematical masterpiece (for example, an original proof).

## DEFINITIONS

In an in-depth discussion of mathematical learning and understanding, Kieran pointed out that perceptions of mathematical understanding have changed over the years. For example, in the '70s understanding was included in learning and was equated with "knowing, applying, and analyzing" (1994, 593). She concluded that understanding is ongoing, not an "all or nothing" phenomenon, but that "some level of understanding is involved in all of mathematical learning" (1994, 598).

Bruner says, "To understand something well is to sense wherein it is simple, wherein it is an instance of a simpler, general case...In the main, however, to understand something is to sense the simpler structure that underlies a range of instances, and this is notably true in mathematics" (1995, 333). Skemp (1978) discusses "relational" and "instrumental" understanding and their differences. He describes the former as "knowing what to do and why," and the latter as "rules without reasons."

According to NCTM, understanding concepts "involves more than mere recall of definitions and recognition of common examples..." (1989, 223). It goes on to say that evidence of students' understanding of a concept is their ability to apply that concept to novel situations. NCTM agrees with Kieran (1994) that understanding is an ongoing process. It says "The development of conceptual understanding is a long-term process; understanding is developed, elaborated, deepened, and made more nearly complete over time" (1989, 69).

Cangelosi differentiates between "literal" and "interpretive" understanding. Students demonstrate literal understanding if "they can accurately translate" the "implicit meaning" of a statement. They demonstrate interpretive understanding if "they can infer implicit meaning" of a statement and can give illustrations to elucidate what is contained in the statement (1992, 98).

Cramer and Karnowski define understanding in mathematics as "the ability to represent a mathematical idea in multiple ways and to make connections among different representations" (1995, 333).

## INDICATORS OF UNDERSTANDING

In this paper I illustrate with examples some indicators of mathematical understanding and then suggest how teachers can facilitate understanding in mathematics among their students.

To understand mathematics means that the learner is able to:

1. Recognize relationships among concepts and within a concept (NCTM 1989); for example, the relationship between addition and subtraction, or between the logarithmic and exponential func-

tions; so that if the learner is given $a\text{-}c=b$, he/she must be able to conclude that $a=b+c$; and vice versa. Similarly, the learner should recognize that $\log_b a = c \Leftrightarrow a = b^c$; and that $\log(ab) \Leftrightarrow \log a + \log b$.

2. Represent a concept in different ways, identify the connections among these representations, and transform and translate easily from one representation to another (NCTM 1989; Huinker 1993). For example, the learner must be able to transform $3x + 2y = 7$ into $y = -\frac{3}{2}x + \frac{7}{2}$. The learner must also be able to translate from a concrete representation to a symbolic or other representation, and vice versa; and recognize that the slope of a line, for example, can be represented trigonometrically as the tangent of an angle, geometrically as the ratio "rise over run," and as a rate of change, all of which are related.

*"Recognize that a result is meaningful and makes sense; the learner must realize that an answer such as "eight and five-sixths buses" borders on absurdity.*

3. Recognize the underlying structure of the mathematics embedded in a situation. For example, the learner must recognize that $\log y = \log a + n \log x$ is of the same form as $Y = A + nx$; $2\left(\frac{1}{h}\right) + 3\left(\frac{r}{5}\right) = 7$ is the same form as $2x + 3y = 7$. Also, if $m \times n = 1$, then $m$ is the multiplicative inverse of $n$, and $n$ is the multiplicative inverse of $m$. More generally, the learner must recognize that $a \otimes b = e \Leftrightarrow a^{-1} = b$ and $b^{-1} = a$ where e is the identity element with respect to the operation $\otimes$.

4. Communicate mathematics orally and in writing; for example, students must be able to explain their solutions to problems to the class or to the teacher. Talking about mathematics helps students to clarify their thoughts and improve their understanding (Buschman 1995; Garofalo and Mtetwa 1990; Helton 1995; NCTM 1989; Owen 1995).

5. Apply mathematics to real-life and other situations (NCTM 1989); it does not make much sense to be able to enunciate the Pythagorean theorem, for example, but not be able to use it to answer a question in geometry.

6. Generate examples and nonexamples of concepts (NCTM 1989); for example, the learner must be able to recognize that a square is a rectangle but a rectangle is not a square; a rectangle is a parallelogram but a parallelogram is not a rectangle; and so on.

7. Monitor and control his/her thought processes so that he/she can recognize when something is not correct and take the appropriate steps.

8. Recognize that a result is meaningful and makes sense; for example, the learner must realize that an answer such as "eight and five-sixths buses" borders on absurdity. In other words, the learner must interpret the answer to a problem within its context.

**RECOMMENDATIONS**

1. Use multiple representations in teaching, including physical models and manipulatives. The learner must be provided with experiences to recognize concepts in different situations and contexts and from different perspectives. For example, a triangle should be represented in different sizes and orientations.

2. Teach relationships, for example rules, in both directions; the learner must become aware of the reversibilty of relationships; for example, given $\log a + \log b$, the learner must be able to state that this is equal to $\log(ab)$.

3. Provide opportunities for students to write and talk about mathematics. Some activities could be journal writing (students may include what they learned in a lesson), interviews, and peer tutoring.

4. Provide opportunities for students to solve a wide range of problems individually and in groups so that they can apply their knowledge, skills, and concepts to familiar and unfamiliar situations.

5. Emphasize relational understanding rather than

instrumental understanding.

6. Provide students with experiences in self-assessment to help them develop self-correcting and self-monitoring abilities.

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