# Mathematics Out Of Nothing: Talking About Powerful Mathematical Ideas With Children

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# With Children

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**Synopsis**

Parents and educators have powerful opportunities to introduce children to big mathematical ideas, when those ideas become necessary. Children are capable and curious. They don't need to be sheltered from big mathematical ideas. Bring out mathematical ideas when kids are ready, or when they are needed. This article describes one such instance, when I helped my six-year-old son move beyond zero in the negative direction when subtracting.

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Children are capable and curious. Parents and educators have powerful opportunities to introduce children to big mathematical ideas, when those ideas become necessary. This article describes one such instance of necessity, when I helped my six-year-old son count below zero when subtracting.

While using M&Ms as counting objects to show subtraction as removal, my son suddenly said "three subtract seven is zero". This was a pivotal moment for a father and mathematics educator. The right pedagogical move at this moment opened a new world of numbers to him.

A child's curiosity about numbers themselves can lead to powerful discussions of big mathematical ideas, when they are needed, and when the child is ready. The tendency of educators is to hide mathematical ideas until the curriculum says it is time to study them, but if we listen carefully to the children around us, we sometimes see that they are ready to go beyond where the curriculum says they should be.

A tension exists between formal programs of study of mathematics, in school contexts, and the natural curiosity of children. Curiosity is innate, and can be either developed, or muted, while curriculum is prescribed, based on various policy and political factors. Mathematics is a realm of the imagination. Children exist in a state of imagination, like Max, in Sendak's *Where The Wild Things Are* [4], who invents a new world for himself, only to return to the familiar confines of his room after.

Parents and teachers guide young children as they start to find and make sense of new mathematical ideas, and talking mathematics with young children sometimes means inventing new mathematics, right in front of our eyes. This does not mean children are discovering brand new mathematical objects, but they are discovering them as if they are new, because they are new to them. As Clements and Battista describe this active process of knowledge construction in children, "they invent new ways of thinking about the world" [2].

To live as a mathematics educator and as a father is to have a synoptic view of how kids learn mathematics. I have watched my boys grow their ideas about mathematics since birth, as they have progressed through stages of mathematical understanding, from learning the names of numbers, to counting, to learning how to add and subtract. At the same time, I have watched students of various ages grow in their own skills and understanding of various mathematical ideas. It is within this confluence of the neverending pedagogy of love, which is parenting, and my professional work teaching mathematics, that I have had the most inspiring revelations about how young children learn and understand mathematics.

Teacher-parents see children through this confluence, which we can aptly call "synopsis", or, in the Gospel scholarship sense, a "synoptic" or dual way of seeing. Nat Banting speaks to this synoptic view when he describes being a father, mathematics student, and educator at the same time: "Each arena of my life spoke to the other, and the hectic combination of the three meant that the confluence of roles was often unintentional" [1]. Other parent-pedagogues note that their teaching and parenting influence each other. For example, Simic-Mueller finds that living at the confluence of these roles has resulted in her developing a "pedagogy of caring" [5]. Steurer finds common ground between inquiry-based learning and parenting, noting that both "put us in the position of having to respond in the moment to difficult questions" [6].

Teachers need to teach the curricular topics assigned to their grade, but parents need to think about whether their child would benefit, at crucial moments, from further discussion of the mathematics. Parents are first teachers for their children, but are in a much less formal relationship with their children than are school teachers. A father, freed from formal curricular considerations, and within a relationship of love with his child, is free to explore and develop mathematics topics inspired by powerful and seemingly "off the cuff" child-like utterances. These conversations, within what we have already called a "pedagogy of love", in turn further my understanding of how children learn and understand mathematics in my classroom.

It is easy to underestimate a young child's nascent mathematical understandings. This article describes an episode in which my six-year-old son needed a new vocabulary for subtracting, when the difference of a subtraction expression ended up below zero, jarring and disrupting his sense of the possible. This led to a discussion of negative integers, and "inventing", at least in his schema for number, a new type of number.

It would have been easy to leave this problem alone, because negative integers are, some might say, beyond his current understanding. But "teachable moments" exist, and can be capitalized on, both in our home lives, and in our classrooms. Parents and teachers need to be attuned to these moments, when children spontaneously bring out big mathematical ideas, right before our eyes.

The following episode occurred spontaneously, out of a verbal utterance, by Callum, age six. We play math games, and talk about mathematics at home, but in the story that follows, he simply spoke what was on his mind. This powerful and unexpected episode caused much reflection about how and when to introduce young children to negative integers.

"Three subtract seven is zero," Callum said, as we got ready for bed one night.

If this had happened in a classroom full of six-year-olds, the teacher would be at a pedagogical decision point. Negative numbers typically appear in curricula much later, after counting, whole number arithmetic, fractions, and decimals are introduced. The decision would be whether to talk about integers at this time, or to save them for later, when they are introduced in

the curriculum. Primary teachers are familiar with discussions of "readiness", which centre on whether a child or a class of children is ready to learn about a certain topic, at a certain time.

There is a vast tension that exists between letting an incomplete understanding persist in a child's mind, which seems somewhat dishonest, if sometimes prudent, given curricular and time constraints (which are often one and the same), and pushing him into a new and surprising world of mathematics. Karp, Bush, and Dougherty refer to the idea that you cannot take away a bigger number from a smaller number as a rule that expires or "breaks", when students encounter "application or word problems involving contexts that include integers, students learn that this 'rule' is not true for all problems" [3]. Teachers must decide, in the pedagogical moment, what they are going to do with a verbal expression of this idea, as I did with Callum. This instructional decision point is between letting the student hold on to their incomplete understanding, or to push them further in their thinking.

My decision was clear. Callum, like all children, is capable of handling and understanding big mathematical ideas. It was time to talk about the other half of the number line, and what happens when subtraction takes you below zero.

We took out some paper to write out our ideas, and some M&Ms to use for counters. Children are usually first taught about subtraction in its concrete form, that is, as a process of "taking away" objects from a set, so we started the conversation there.

I asked him to model eight minus four. This was no problem. He just counted backwards and removed the M&Ms (making them disappear, by eating them).

"Eight subtract four is four," Callum said, as he gobbled down exactly four candies.

"Show me ten subtract five." This question was designed as a check to see if he really knew what he was doing with subtraction, and was not just interested in eating the M&Ms.

Again, he set up ten candy counters, and removed some. "Ten subtract five is five." Five M&Ms disappeared, into his mouth.

Show me five subtract five. "Five subtract five is zero." The teacher inside of me knew very well that initial encounters with the number zero can cause puzzlement.

"Now, can you show me three subtract seven?" This was the key pedagogical question that determined whether we were going any further with this discussion.

He ate the three M&Ms we were using as counters. "Three subtract seven is zero. There's nothing left. I could only take away three."

Callum was correct: there was nothing left. If the other half of the integer number line did not exist, zero would be a correct answer. Subtraction falls apart or "breaks" at zero using this instantiation of subtraction. We reached the "zero M&M" barrier: nothing can be removed from nothing. There is seemingly nothing less than zero M&Ms. Our options in this discussion were either to stop there, or to develop a new conceptual tool for thinking about this problem.

"Can you keep counting below zero?" I asked.

"No."

Zero is itself strange: an abstract conception of nothingness that jars with the way kids are taught counting, adding, and subtracting through working with concrete objects.

I decided to draw a number line. "Difference", on a number line, is an understanding of subtraction which usually comes along slightly later in a child's schooling, after "taking away". I marked "zero, one, two, three", leaving zero as the very left hand edge of the number line.

The moment of invention had arrived. The big teaching moment had arrived. He just needed prompting to start counting: "minus one, minus two", and so on. We needed to count below zero, so we figured out how to do it.

"Can you count below zero?"

"No."

"How do they tell you the temperature in winter?"

Familiar contexts like temperature, sea level, and elevators going up and down are often used to introduce integers to students. My pedagogical intuition was that, even if he did not know exactly what 0 Celsius degrees meant, he had some idea of negative temperature. My fatherly intuition was that I had about three minutes left before this tired child gave up on talking mathematics with his father.

He thought about it for a while, and then said, "minus five". I interpreted this as an example of a temperature he had heard recently.

As seen in Figure 1, we sketched jumps backwards from three on the number line: two, one, zero.
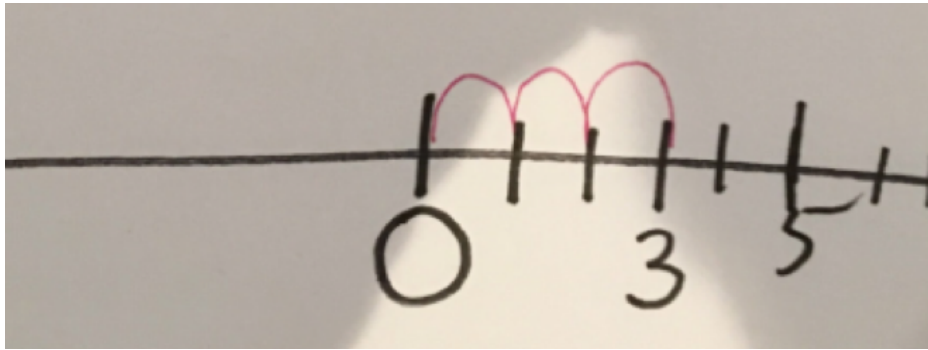


Figure 1: We sketched jumps backwards from three on the number line: two, one, zero.

"Is minus one next?"

"Yes."

Most adults, after their formal schooling, have internalized the idea that every positive integer has its opposite, an equal distance away from zero, on the negative side of the number line. For young children, the other half of the number line does not exist yet. As Figure 2 shows, we jumped from zero to minus one, to minus two, to minus three, and to minus four. The other half of the number line suddenly came into being.

We practiced counting in the negative direction. "Minus one, minus two, minus three, minus four." Then we counted up from negative four, all the way to three.
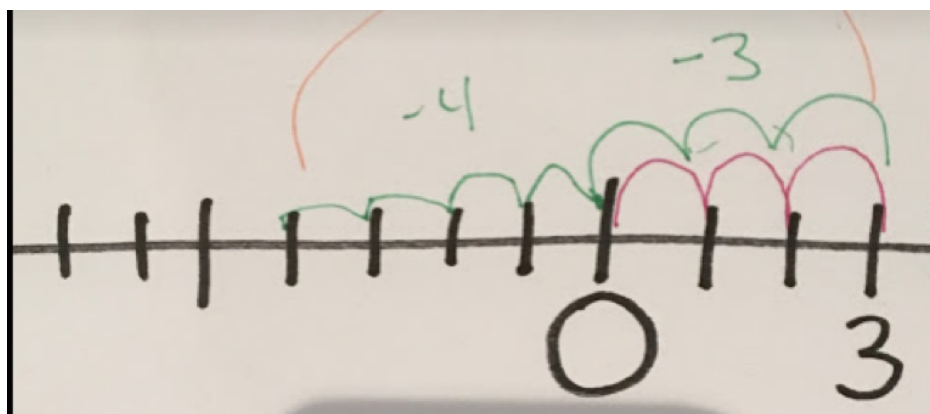
Figure 2: We jumped from zero to minus one, to minus two, to minus three, and to minus four.

I helped Callum trace with his finger to prove to him that he had made seven jumps.

"What is three subtract seven?"

"Minus four!"

We opened up an entire new world of counting, where you can count forwards and backwards from zero, whenever you want. My son used his newfound mathematical power to start counting, as kids often do.

"Minus one, minus two, minus three, minus four..." He got as far as negative ten and then stopped, satisfied.

Children want to know about big and important mathematical ideas. There is a tension that exists between the demands of formal schooling in mathematics, and the natural curiosity and interest of children. It is through my dual role as father and mathematics educator that I came to see that talking about powerful mathematical with young children is putting an often lacking human face on mathematics.

Educators and parents alike can look for seemingly spontaneous mathematical utterances that show a desire or curiosity to go places the curriculum, bound by policy and politics as it is, cannot or will not go. If we are attuned to the wonderings of the children around us, we can help them to bring new and powerful mathematics into being.

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