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Review: Crystal bases of q-deformed Kac modules over the quantum superalgebras Uq(gl(mln))

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Crystal bases of q-deformed Kac modules over the quantum superalgebras $U\_q(\mathfrak{gl}(m|n))$.

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Introduced by M. Kashiwara in 1990 [Comm. Math. Phys. 133 (1990), no. 2, 249–260; MR1090425], the crystal base construction has proved to be a versatile tool in understanding the representation theory of quantum groups. In 2000, G. M. Benkart, S.-J. Kang and M. Kashiwara [J. Amer. Math. Soc. 13 (2000), no. 2, 295–331; MR1694051] generalized this construction to all irreducible integrable modules of the quantum superalgebra $U_q(\mathfrak{gl}(m|n))$, the quantized enveloping algebra of the general linear Lie superalgebra $\mathfrak{gl}(m|n)$. In the paper under review, the author further extends the notion of a crystal base to apply to the q-deformed Kac modules over $U_q(\mathfrak{gl}(m|n))$. Furthermore, it is shown that the crystal base of a q-deformed Kac module is compatible with that of its irreducible quotient $V(\lambda)$ given by Benkart, Kang, and Kashiwara when $V(\lambda)$ is an irreducible polynomial representation.

The paper is well written and straightforward to follow. The construction and the uniqueness theorem are developed in a manner that loosely follows earlier work. The main connectedness result, Theorem 4.8, is analogous to the corresponding statement in the Benkart-Kang-Kashiwara paper mentioned earlier. However, the author notes explicitly where the difficulties lie and where any natural analogies need to be abandoned.

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