Humanistic Mathematics Network Journal

Issue 26 Article 5

6-1-2002

History of Mathematics, an Intuitive Approach

Alejandro R. Garciadiego
National Autonomous University of Mexico

History of Mathematics, an Intuitive Approach

Dr. Alejandro R. Garciadiego
Dept. of Mathematics
Faculty of Sciences
Universidad Nacional Autonoma de Mexico

ABSTRACT
The main goal of this essay is to discuss, informally, an intuitive approach to the history of mathematics as an academic discipline. The initial point of departure includes the analysis of some traditional definitions of the concept of 'history' taken from standard dictionaries. This concise dissection attempts to suggest the complexity of the discipline.

KEY WORDS: HISTORY, HISTORY OF MATHEMATICS, HISTORIOGRAPHY, METHODOLOGY, CHRONOLOGY.

The term 'history' is familiar to almost everyone, and most people believe they know its meaning intuitively. The lay person sometimes thinks of history in terms of dates, names, places and of colorful anecdotes on interesting characters. History provides a record of where someone lived and what he did. In short, history is the repository of the past. But what exactly is the history of mathematics as an academic discipline?

The events that took place yesterday are now part of history. Photographs are history: they reflect the way we once were. History is studied in elementary school, especially that of student's native countries. Teachers, who often seem old enough to have taken part in some of the historical events, teach -over and over again- anecdotes, names, places, dates, and so on. Historical movies, TV programs and books are popular. The media affects the way people understand history as a discipline. Unfortunately, sometimes the public's knowledge of important historical issues is derived from the popular media (especially movies), and not from professional sources. Thus, their understanding of these events can be distorted.

Unlike the word mathematics, the term history is used on a daily basis by the news media. Some reporters may believe that history is actually being made when they type or read the news. On occasions, reporters and anchormen have trouble attempting to disassociate themselves from the event they are reporting on.

Of course, in most cases, the term history is misused by professional reporters. Often, for example, TV sports commentators discuss player's individual records while narrating a baseball game. These statistics include the player's batting average, number of times at bat in the game, number of stolen bases, etc. Sometimes, the commentators also narrates the player's background. They mention the college the player attended; where he played in the minor leagues, his previous professional teams and so on. The commentator may also discuss some of the player's qualities as a human being (e.g., generosity, sportsmanship). Then, the sportscaster may try to explain why the player was motivated to become a professional. After the commentator has finished with these statistics and anecdotes, he often says: '... and, ladies and gentlemen, ... the rest is history'.

This popular expression 'and, ladies and gentlemen, ... the rest is history'- suggests that once the information or anecdote has been revealed to the audience, the rest of the account is common knowledge and requires no further explanation. Or, perhaps, the phrase '... the rest is history' is synonymous with '... the rest is unimportant' or '... the rest is well-known'. In fact, the term 'history' may suggest that something is not of current concern or lacks importance (e.g., My youth is history) [2, p. 614].

History, however, is much more than long lists of data. Think back to your student days. Did your teacher demand knowledge that was more substantial and profound that mere factual information contained in a list of data, especially on a test? Sometimes they asked essay questions which required some explanation of historical events. Consider an often asked question on why XV century navigators attempted to find a new route to India. Students, apparently, fail to distinguish between the factual aspects of historical data and the interpretation of historical evidence. In gen-

eral, young students do not understand that the words 'history' and 'chronology' are not synonymous.

The word history may be defined as "[...] a narrative of events; [...]" [2, p. 614]. This definition seems to indicate that there is indeed a subtle difference between the terms chronology and history. History develops a narrative and, therefore, it is not simply a list of dates and names arranged sequentially. History, viewed from this perspective, is familiar to us. Historical works (e.g., a movie, a book, a play) contain more than just a simple list of names and dates. In many cases, the presentation (or form) of the material is as important as the raw material (or historical data) itself. For example, the movie industry attempts to produce and sell good films to entertain an audience, not necessarily to reproduce good history. For this, producers, directors and writers pay special attention to the presentation of the narrative. In most occasions, they even add external elements to produce a more interesting movie or a more attractive story. In fact, almost any historical movie or TV show contains a disclaimer asserting that some characters, situations and dialogues were added for dramatic purposes. Similarly, a scientist -being not a professional historian- may judge a historical textbook by its literary and entertainment qualities and not necessarily by its historical accuracy and objectivity. Some classical texts used for many years by the mathematical community may provide excellent examples [in particular, items 3 and 7 of the references].

Before attempting to refine the definition of history, it is important to understand that a historical scholarship does not necessarily arise from a description of past events -analogously, doing mathematics involves much more than dealing with numbers. Some people, other than historians, are constantly interested in the past. Take, for instance, a private investigator. He may be trying to solve a homicide case. His task is to reconstruct how a murder took place. Depending on the case, the private eye will have to answer certain questions. Some may deal with 'historical' factual inquiries: for example, attempting to answer when, where and how the event took place. Other questions may involve non-factual issues; for example, the state of mind of the killer at the moment of the crime. Is the private eye an historian?

He is trying to reconstruct the past, presumably the goal of the historian. No doubt there are some similarities between the two professions. Every private detective must be acquainted with some of the methodological techniques used by historians. He may have visited a newspaper archive and read old items. Perhaps, he may have studied autobiographical sources (e.g., diaries, address books, old photographs, unpublished correspondence, etc.) to determine the activities of the person he is investigating. On the other hand, the historian may enjoy the detective work of his profession. A wonderful example is remarkably illustrated by Reid's attempts to determine where Eric T. Bell spent his childhood [12]. Furthermore, it is always an intellectual challenge to find a difficult source, or to devise new ways of understanding or interpreting historical data, or to prove a point in a more effective way.

Nevertheless, private investigators do not always attempt to reconstruct the past. They are usually investigating events that occurred in the immediate past, and have not yet formed a conclusion. His investigation will lead to a conclusion (e.g., a court verdict), and will likely be influenced by the actions of the detective. Furthermore, his goal is not to reconstruct the past in itself, but to use some information about the event for other purposes. For example, some detectives/reporters have written books (or reports) on police investigations attempting to reconstruct the events associated with crimes committed in the past. One of these books [18], more than three hundred pages long, attempts to reconstruct the last day in the life of Marilyn Monroe. Some authors, on occasions, criticize the original investigation, presenting new evidence that may reveal the real reason that somebody died (e.g., Marilyn Monroe and JFK, among many others [see, for example: 15, 16 and 17]).

Historians may not only find certain aspects of a detective's methodology questionable (including the private investigator's use of sources, inferences, conjectures, goals and extrapolations), but may also criticize his/her lack of objectivity. It is extremely difficult for detectives not to become personally involved in the events surrounding the case. Detectives usually receive an economic reward for their activities (except perhaps for the fictional character Mike Hammer who seems to get always involved for friendship and personal reasons) and have an obligation to get results for their clients, not necessarily to find the his-

torical truth. On the contrary, history, mathematics and most other academic disciplines, demand the highest possible level of rigor and objectivity. For example, Frege criticized Cantor's definitions of the concept of set [menge] and cardinal number because Cantor relied on each individual's mental capacity to abstract certain properties. That sort of definition was too subjective, depending on the personal perspective of each particular mathematician. In the same way, the closer the historian is to the person, event or idea that he/she is studying, the more difficult it is for him/her to present an objective reconstruction of the event. But let us attempt, once again, to define the term 'history'.

Some biographical treatises include, as a guide, a chronology listing major events and corresponding dates in a person's live. As discussed earlier, a chronological treatment of events is not necessarily the same as a historical investigation. Furthermore, some researchers may begin an investigation intending to conduct a historical study, but may not produce results that meet contemporary professional standards. For example, consider the term 'class'. A historian may ask who first conceived this term, and when and where the concept was first used. He might examine an old reference to find out whether the term class was already in use. If it was, then he could look up an even earlier reference and eventually find out who first used the term. It is very possible that the researcher's final report will simply be a chronology itself, naming the person who initially used the term, and how its meaning may have changed over time. But this chronological narrative may not explain why the concept was formulated in the first place or modified over time.

A historian may argue that a chronological description of events does not necessarily fulfill the criteria of rigor established by the professional community of historians. In fact, it is quite easy to attempt to explain facts retrospectively. A mathematician may argue along similar lines. To the general public, any text using numbers may be thought to involve mathematics. Mathematicians, however, would regard this as simplistic; in fact, most professional books or articles in mathematics contain no numbers at all, except for the page numbers.

Sometimes, the nature of a historical discipline may be partially understood by asking why some people study it. Some people want to clarify who discovered a particular idea. Hubbert Kennedy ingeniously proposed 'Boyer's law', in part because of the many occasions in which credit for a concept has been given to the wrong person. Boyer's law states that "mathematical formulas and theorems are usually not named after their original discoverers" [8, p. 67]. Boyer mentions [4], between the XVII and the XIX centuries, at least thirty cases, of mathematical results that have not been credited to the appropriate person.

Sometimes, influenced by political ideology, historians are affected by nationalist sentiment, attributing (or attempting to attribute) discoveries (previously attributed to western mathematicians) to their own countrymen [13]. There is another reason why people study history: To understand the present by studying the past. Some professional mathematicians, sooner or later, decide to investigate the origins of the concepts they use today.

> ***...history lacks the absolute character of mathematical reasoning. Within mathematics, once the basic premises are settled, there is no space for alternative results.***

At present, less emphasis is given to describing events in the lives of the great men of mathematics than to listing discoveries in the various branches of mathematics. Historians of mathematics place greater emphasis on the development of ideas. They attempt to find the key concepts that influenced the development and evolution of their discipline. In particular, they try to find the key questions that played the role of Ariadne's thread conducting mathematical research. So, some historians attempt to reconstruct how mathematical ideas originated, evolved and learn how they influenced other ideas. Historians might also focus on the development of a particular school of thought and its 'philosophical' program. Others might attend the social influences that affected the thought of mathematicians or the relationship between the mathematician's work and the society in which he lived. Others might be interested on the social conditions surrounding the academic community or, on the contrary, on the effects of mathematical ideas on society. Some historians analyze the way that institutions and governments implement scientific policy. Others study the effects of economic factors such as produc-

tion and consumption on science. Indeed, historians claim that ideas do not occur in a vacuum. Knowledge is a human creation, not a preexisting entity, closely associated with specific individuals, institutions and schools of thought.

No doubt, some people are tempted to write embellished accounts of events for posterity, including sports, knowledge (in this case, mathematics), politics, and, most importantly, war. Most likely, historians belonging to opposite ideological camps would present radically different accounts of the same event. For example, a Spanish soldier and an Aztec Indian would describe the conquest of Mexico in the XVI century from very different perspectives. The same situation occurs in the history of the American Civil War, as in the description of almost any human event. The triumphant narrative of a Yankee might differ from the pessimistic account of the Southerner. They may differ on the origins of the conflict, and the importance and consequences of some events. Here rests one of the major methodological differences between mathematics and history. History provides room for alternative interpretations. In fact, if different persons have witnessed an event, there may well be as many different accounts, as beautifully illustrated by the late Akira Kurozawa's Rashomon [Film. B/W. 1950], in which four different characters, all apparently equally trustful and credible, narrate four conflicting interpretations of the same event. The nature of history provides room for diverse reconstructions of the past, as reflected by the diversity of published historical material. These reconstructions will vary in their degrees of plausibility and consistency in terms of both chronological and technical characteristics of the discipline. Thus, history lacks the absolute character of mathematical reasoning. Within mathematics, once the basic premises are settled, there is no space for alternative results. For example, if within Euclidean geometry we have shown that the sum of the inner angles of any triangle is equal to the sum of two right angles [Euclid, 1-32], then there is no room for any possible alternative, thanks to the principle of excluded middle.

The possible existence of alternate interpretations may explain why, on historical disciplines, many historical monographs are published on the same topic. For example, Drake [5] and Koyré & [10] have presented different historical accounts on whether Galileo conducted empirical experiments. Later, other historians commented on the works of Drake and Koyré, discussing each author's viewpoints [19]. Still later, some other historians, using additional sources, may discuss the works of the earlier critics. And so on. But historians are not satisfied by merely presenting an alternative view of the events in question; more importantly, they want to convince their readers that their own conceptualization is better than earlier or alternative accounts. This last point is extremely important.

Indeed, in most cases, historians are not satisfied by merely narrating past events. For example, Ferreirés [6] convincingly argues that some of Dedekind's ideas underlie Cantor's theory of transfinite numbers. Likewise, Rodriguez-Consuegra consistently disputes [14] that there is a general philosophical method inspired by Russell's philosophy of mathematics (which he developed at the turn of the century), in spite of Russell's apparently endless intellectual evolution. In current thinking, an ordered (or unordered) collection of information on a particular topic does not constitute a historical study. As May has already argued: "history arises when chronology is selected, organized, related and explained." [11, p. 28 (my emphasis)].

History is also defined as "a branch of knowledge that records and analyzes past events" [2, 614]. The key word here is 'analyzes'. Contemporary historians do not record past events in a passive manner, but evaluate the past from a critical perspective. "If error and ignorance," as Adler and van Doren say, "did not circumscribe truth and knowledge, we should not have to be critical" [1, p. 166]. By critical, at this level, one simply means a position that evaluates and judges. Certainly, every historical treatise is based on a biased framework affected by the author's ideology and background. Historians cannot disassociate themselves from previous knowledge and be completely impartial. When an author has a critical attitude, however, he/she does not intentionally supports a political or ideological position.

The reader should not misunderstand the meaning of the word judge. The historian does not make ethical or moral assertions, nor is he/she concerned about the empirical truth of a concept. His goal is to determine why a particular novel idea, theory or interpretation was accepted over other conceptualizations current at the time, or immediately beforehand. In order to do so, the historian is required to ask appro-

priate questions: What problem was the mathematician attempting to solve? What conceptual tools were available? What comprises a 'rigorous' solution to the problem according to the standards of the time? In the case of the history of mathematics, Wilder has asserted that: "we don't possess, and probably will never possess, any standard of proof that is independent of the time, the thing to be proved, or the person or school of thought using it" [20, p. 319].

Historical understanding does not exist independently of other kinds of knowledge. Firstly, it is highly dependent on the intellectual background and historical assumptions of the practitioner. Secondly, it is abstracted from a totality surrounding it. The historian cannot encompass this totality and necessarily omits possible associations with other intellectual disciplines; very frequently he/she omits sociological factors. The historian understands that scientific thought is profoundly influenced by science, arts, technology, philosophy and theology, among other areas. However, the relationship of mathematics to these other disciplines may be obscured by current thinking which emphasizes the independence of mathematical thought. The task of the historian is to transcend the constraints of the present and reveal these influences.

There is no single approach to studying historical questions. Historical research is strongly affected by personal values, background, interests, the surrounding environment and the characteristics of the time in question. The only way to formulate interesting historical questions is to expand the knowledge of the past. The more one knows, the more one would like to know. It is obviously necessary to read the classics and the works of great historians. While doing this, keep in mind that, there was less of a rigid distinction between mathematics, the sciences, philosophy and others branches of knowledge up to the turn of the present century than there is today. Most of the intellectuals of the past were competent in all these disciplines. In order to understand the scientific ideas of an important historical figure, it is necessary to understand the person's theological and/or philosophical thought. Descartes, for example, made important contributions to philosophy, mathematics, physics, music and medicine, and, perhaps, to other disciplines. If the historian of mathematics wishes to understand Descartes' contributions to geometry, he/she needs to study his writings in other subject areas.

To conclude, consider the following analogy to historical research. Suppose that you are attempting to put together a one thousand piece puzzle. Most people begin with the following strategy: they separate out border pieces with a straightedge to form the frame of the puzzle. Then, if possible, they try to separate out pieces with the same color, to build small sections of the puzzle. Finally, with the help of the picture on the top of the box, they attempt to assemble all of the smaller sections together, to produce the whole picture. Similarly, historians are trying to reconstruct the past by presenting a picture, story or narrative to their audience. Unlike a puzzle, however, historians do not know the shape, size or the number of pieces of the puzzle! To make matters more complicated, the historian does not even know the overall 'image' of the events he/she is trying to reconstruct, and therefore has no framework (like the pieces that form the border) to begin developing the story. Moreover, the historian knows that he will never possess all pieces of the puzzle (except in trivial cases). So, at some point, he will have to conjecture (or imagine) the potential shape, size and design of some of his pieces. But to make the situation even more difficult, others may visualize the image(s) he wishes to present in a different way. Furthermore, even if they have the same general perspective, another colleague may visualize an individual piece or sections of the puzzle in a different way. The reconstruction of the past is never final, unique or totally accurate; but a plausible and logical synthesis of the available evidence.

Most importantly, after a historian has developed a plausible interpretation of the evidence, he has the difficult task of proving the soundness of his formulation, just as a mathematician must prove a new result. It is not acceptable just to assert a new point of view. Knorr [9], in his monograph discussing the history of some pre- Euclidean concepts, provides a convincing elegant argument for the specific date for the discovery of incommensurable quantities and a general goal for the entire corpus of Euclid's Elements. Then, he goes on to argue why this is a plausible case.

We have considered several important and profound differences in the methodologies used in mathematics and historical research. One of the most important differences is apparent when the scholar begins a new

project. The mathematician is usually aware of the future outcome of his/her research (e.g., a proof of the well-ordering theorem, a proof of Fermat's last theorem, etc.). It is usually extremely difficult to reach the goal (sometimes it has taken more than 300 years!), but, at least, the goal is known. On the contrary, the historian does not usually know what conclusions he/she will make. He does not yet know the key questions, concepts and results. Most importantly, the historian is unaware of the driving factors that influenced a mathematician's career. To some extent, it may be very simple to list and describe the publications of a certain individual. But it might be extremely difficult to learn what influences affected the mathematician's ideas and contributions. On some occasions, after reading dozens of articles or hundreds of manuscript unpublished folios, which may or may not be related to each other, the historian might find himself in complete darkness, without understanding the pivotal ideas behind the mathematicians's concepts and publications. As May pointed out, despite much hard work and effort in conducting his/her research (searching, finding, organizing and selecting), the historian may not be able to explain what underlay a mathematician's thinking.

REFERENCES

1. Mortimer Adler and Charles van Doren. 1972. *How to read a book.* New York: Simon & Schuster.
2. *American Heritage Dictionary, The*. 1985. Boston: Houghton Mifflin Company. (2nd college ed).
3. Eric T. Bell. 1937. *Men of Mathematics*. New York: Simon and Schuster.
4. Carl B. Boyer. 1968. *A history of mathematics.* New York: John Wiley & Sons, Inc.
5. Stillman Drake. 1978. *Galileo at work. His scientific biography.* Chicago: Chicago University Press.
6. José Ferreirés. 1999. *Labyrinth of thought. A history of set theory and its role in modern mathematics*. Basel: Birkhäuser.
7. Leopold Infeld. 1948. *Whom the Gods Love.* New York: Whittlesey House.
8. Hubbert Kennedy. 1972. *"Who discovered Boyer's law?"* American Mathematical Monthly 79: 66-77.
9. Wilbur Knorr. 1975. *The evolution of the Euclidean elements.* Dordrecht.
10. Alexandre Koyré. 1966. *Etudes galileennes*. Paris: Hermann. 1966.
11. Kenneth O. May. 1974. *Bibliography and research manual of the history of mathematics.* Toronto: University of Toronto Press.
12. Constance Reid. 1993. *The search for E T Bell, also known as John Taine.* Washington, DC: Mathematical Association of America.
13. K. Ribnikov. 1987. *Historia de las Matematicas.* Moscow: Mir. [Translated into Spanish by Concepción Valdés from the original Russian edition 1974].
14. Francisco Rodriguez Consuegra. 1991. *The mathematical philosophy of Bertrand Russell.* Basel: Birkhäuser.
15. Anthony Scaduto. 1976. *Who Killed Marilyn?* New York: Woodhill.
16. Milo A. Speriglio. 1983. *Marilyn Monroe: Murder Cover-Up*. New York: Seville.
17. Anthony Summers. 1980. *Conspiracy.* London: Victor Gollancz.
18. ——. 1985. *Goddess: the Secret Lives of Marilyn Monroe*. New York: Macmillan.
19. Pierre Thuillier. 1983. "*Galileo et l'expérimentation.*" La Recherche 14 (# 143, Avril). Pp. 584-597.
20. Raymond L. Wilder. 1944. "*The nature of mathematical proof*'. The American Mathematical Monthly 51: 309-323.