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Review: The classical Hom-Yang-Baxter equation and Hom-Lie bialgebras

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A *Hom-Lie algebra* $(L, [\cdot,\cdot], \alpha)$, defined first by J. T. Hartwig, D. Larsson and S. D. Silvestrov [J. Algebra 295 (2006), no. 2, 314–361; MR2194957], is a nonassociative algebra generalizing a Lie algebra. It consists of a vector space L, an anti-symmetric bilinear operation $[\cdot,\cdot]\colon L \otimes L \to L$, and a linear map $\alpha\colon L \to L$, such that the *Hom-Jacobi identity*

$$[[x, y], \alpha(z)] + [[z, x], \alpha(y)] + [[y, z], \alpha(x)] = 0$$

holds for all $x, y, z \in L$.

The author of the article under review introduced the *Classical Hom-Yang Baxter Equation*, adapting the Classical Yang-Baxter Equation (CYBE) to the Hom-Lie setting, in [J. Phys. A 42 (2009), no. 16, 165202; MR2539278]. The current article develops this theory further, by first showing that solutions of the CYBE give rise to infinite families of solutions to the new equation. The analogues of Lie bialgebras, so-called Hom-Lie bialgebras, are defined and studied. In short, the article answers some natural questions, extending well-known results to the Hom-twisted case.