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Calculus

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– known to be true (but "self-referential" and not provable, as in Godel's incompleteness theorem),

– known to be false (but not disprovable),

– poorly formed (using the grammar of the model),

– well-formed but independent of the axioms (such as the Continuum Hypothesis).

LANGUAGE AND GRAMMAR

So far, we have simply regarded the brain as a processing unit, which is capable of translating (or producing) certain patterns from sequences of mental objects. These patterns may be translated and represented (not necessarily faithfully) using more universal mental objects. Call these objects "symbols". This capability of the brain might be regarded as an "abstraction device": a machine which is capable of noticing patterns from sequences of inputs. Of course, some patterns are more relevant than others. The brain is also capable of distingishing, evaluating, and selecting patterns.

The brain also has a tendency towards using universal mental objects (constructing order from chaos, if you will). Therefore, it is natural for the brain to process stimuli in terms of symbols and grammar. This leads naturally to language, which is useful for processing even more information.

The point is that the way we think about mathematics falls into one of several categories. We either

* formally manipulate symbols (such as algebraic expressions), following grammatical rules,

* formally manipulate mathematical objects (such as knots), following grammatical rules,

* experimentally determine grammatical rules using sequences of mathematical objects,

* select mathematical objects from sequences using some evaluation procedure (possibly for the purpose of manipulating them or determining grammatical rules for them).

All these involve one of the brain's capabilities discussed above.

REFERENCES

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I tell my students the story
of Newton versus Leibniz,
the war of symbols,
lasting five generations,
between The Continent
and British Isles,
involving deeply hurt
sensibilities, grievous
blows to national pride;
on such weighty issues
as publication priority
and working systems

of logical notation:
whether the derivative
must be denoted by a "prime",
an apostrophe atop
the right hand corner
of a function, evaluated
by Newton's fluxions method
or by a formal quotient
of differentials dy/dx,
intimating future possibilities,
terminology that guides
the mind.

The genius of both men
lies in grasping
simplicity
out of the swirl of ideas
guarded by Chaos,
becoming channels,
through which
her light
poured
clarity on the relation
binding slope of tangent line
to area of planar region
lying below a curve,
The Fundamental Theorem of Calculus,
basis of modem mathematics,
claims nothing more.

While Leibniz -
suave, debonair,
philosopher and politician,
published his proof
to jubilant cheers
of continental followers,
the Isles seethed
unnerved,
they knew of Newton's
secret files, locked in
deep secret drawers -
for fear of theft
and stranger paranoid delusions,
hiding
an earlier version
of the same result.
The battle escalated
to public accusation,
charges of blatant plagiarism,
excommunication
from The Royal Math. Society,
a few blackened eyes,
(no duels);
and raged for long
after both men were buried,
splitting Isles from Continent,
barring unified progress,
till black bile drained
and turbulent spirits becalmed.

Calculus -

Latin for small stones,
primitive means of calculation;
evolving to abaci;
later to principles of enumeration
advanced by widespread use of
the arabic numeral system
employed to this day,
as practiced by algebristas
barbers and bone setters
in Medieval Spain;
before Calculus came
the $\sum$ (sigma) notion,
sums of infinite
yet countable series;
and culminating in
addition of uncountable many
dimensionless line segments,
the integral $\int$
snake,
first to thirst
for knowledge, at any price.

That abstract concepts,
applicable—at start,
merely to the unseen
unsensed objects:
orbits of distant stars,
could generate
intense earthly
passions,
is inconceivable today;
when Mathematics
is considered
a dry discipline,
depleted of life sap,
devoid of emotion,
alive only
in convoluted brain cells
of weird
scientific minds.