10-1-2012

A Tale of Two Workshops: Two Workshops, Three Papers, New Ideas

Gizem Karaali
Pomona College

A Tale of Two Workshops

Two Workshops, Three Papers, New Ideas

In July 2009, together with my colleague and spouse Stephan Ramon Garcia (and our five-month old baby, Reyhan), I attended an AIM REUF workshop. The main motive of the workshop appealed to me, since I had been teaching at a liberal arts college since right after my post-doc. Because most of my research focused on quantum algebra, a topic not easily accessible to even the most well-prepared undergraduates, I had been struggling to find a way to initiate and sustain successful collaborations with my students.

On the first day of the workshop, the organizers introduced us to four possible topics, all intriguing. But Stephan's deep interest in number theory and my training in representation theory led us both to eventually select the project proposed by Phil Kutzko from the University of Iowa, a problem that combined classical character theory with number theory. When the workshop ended, most people went back to their regular routine, but Stephan and I, together with one of our team members, Patrick Fleming from the South Dakota School of Mines, kept at the problem. We were quite intrigued by how a simple finite group seemed to have embedded in its character theory some significant identities relating to a mathematical quantity called a Kloosterman sum. After additional focused work, we ended up with a lengthy preprint, which, when posted on the Internet arXiv of mathematics papers, generated several interesting follow-up e-mails from around the world. Our paper, "Classical Kloosterman sums: Representation theory, magic squares, and Ramanujan multigraphs," has already been published by the Journal of Number Theory.

We were pleased; none of us were experts in number theory, but this was a solid publication, and the conversations that opened up through the arXiv preprint itself were quite exciting. However, there was an element of redundancy in our work and we had a sneaking suspicion that there was something we were missing. We thought that somehow there had to be a way to clean things up so that we did not need to employ ad hoc methods to get rid of irrelevant material and reveal the desired information.

Meanwhile, back in December 2009, while we were still working on our Kloosterman sums paper, I received a most welcome invitation to attend another AIM workshop in May 2010 on supercharacters and combinatorial Hopf algebras. I had recently completed a project on combinatorial Hopf algebras and was looking to start a second one. I knew nothing about supercharacters at the time, but found the second topic of the workshop exciting. My main goal was to further connect with the active members of the combinatorial Hopf algebra research community and to possibly foster some new collaborations. However, the very first day of the workshop, when I actually understood what supercharacters were about, I had an epiphany. This was exactly what we needed!

The workshop itself brought forward some amazing results, which culminated in, with 28 authors, possibly the most co-authored paper in the history of pure mathematics. See **http://www.mathinstitutes.org/nuggets/secrete_identity.html** for more details. Later, we focused on how supercharacter theory could help us get rid of the redundancy in our previous work. We could see that the previous Kloosterman identities could be obtained through a particular supercharacter theory for the associated finite group, which then led us to wonder whether the theory could be applied in other situations. This led to fertile land for investigation. In particular, we were able to prove recently, together with Pomona undergraduate Christopher Fowler, that certain supercharacter theories for cyclic groups could be used to churn out several (known and unknown) identities related to other important quantities called Ramanujan sums. As we got more into this investigation, many new possible avenues opened up in front of us, and several undergraduate researchers joined our quest. Stephan directed a team project this summer to push this thread further with the result of a second paper already posted on the arXiv.

All in all, this has been a fantastic ride so far! And we could not have done it if it weren't for serendipity and, of course, the two distinct workshop opportunities, provided generously by the American Institute of Mathematics. ■

– Gizem Karaali