Between Childhood and Mathematics: Word Problems in Mathematical Education

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How should we educate our children to help them develop into competent, productive members of the modern society? The purpose of these notes is to address this question. We shall concentrate on that period of life, which comes roughly to middle and high school, when childhood is over, but professional use of mathematics is not yet possible. This age seems to be critical for success vs. failure in rigorous abstract thinking: some students get prizes at olympiads, some are confused and frustrated. Our main point is that good teaching of word problems is essential at this period.

Since this article is about word problems, we need first of all to define the subject. To keep as close as possible to the exact meaning of the words, I suggest that a non-word problem is a problem, which is formulated using only mathematical symbols and technical words like "Solve the equation..." Correspondingly, a word problem is a problem which uses non-mathematical words to convey mathematical meaning. At the K-12 level there is not much room for sophisticated formalisms of professional mathematics, so non-word problems, which deal with formalisms, are necessary, but not exciting, exercises. No wonder that most interesting problems available at this level are word problems.

There is an important similarity between children's play and all aspects of modern culture: in both cases creative imagination is essential. On one hand, all life of children is a continuous play of imagination. On the other hand, all phenomena of modern civilization involve imagination. When we go to a theater or cinema or art gallery or read a book, we imagine certain events, but at the same time we know that they are not real. Modern mathematics is not an exception: imagination is essential not only to work in it, but even to understand it. It is only natural that school should not interrupt what is common for childhood and culture, namely creative play of imagination. When a teacher of geography tells her students "Today we shall travel across Africa," all normal children understand that this should not be taken literally: this will be an *imaginary* travel. Similar understanding takes place when a teacher of literature says "Today we shall spend in the company of Hamlet" or a teacher of biology says "Let us look inside a living cell". The function of school is to enlarge children's outlook, to teach them facts, images, ideas, laws, phenomena that go beyond their personal experience and everyday life. At school, as much as elsewhere. students are expected to have imagination and use it. Mathematics is not an exception from this rule. When a teacher says "Peter had ten apples and gave Mary three of them," all children understand that these are abstract Peter and Mary and abstract apples. This understanding is essential for children to study mathematics, which is a science about abstractions. Now look at the following problem:

A plane takes off and goes east at the rate of 350 mph. At the same time, a second plane takes off and goes west at the rate of 400 mph. When will they be 2000 miles apart?

I see nothing wrong with this easy problem. To my mind it is usable and even has some merits. For example, it may be used to demonstrate the idea of relative movement, which helps to solve it without algebra: in the coordinate system associated with one plane, the other moves at a speed $350 + 400 = 750$ mph, so the time needed to increase the distance by 2000 miles is $2000/750$ hour = 2 hours 40 min. However, a few years ago it was mentioned in *Mathematics Teacher* with the following pejorative comment: "Any normal student ought to ask, 'Who cares?' No one cares except the algebra teacher who assigns these problems and the student who wants a grade. Our curriculum is too crowded to allow us the luxury of such frills" [11, p. 159]. I am very worried by this comment, and it is a matter of principle. We can manage without this or any other particular problem, sometimes we need to exclude something from curricula, but we should not *approve* asking "who cares?" instead of applying an intellectual effort, especially on the pages of an educational journal.

According to my experience, only a few students ask "Who cares?" instead of solving a simple problem, and these few students already are in trouble: mentally deficient or delinquent. Clearly, that "normal" student who asks back "Who cares?" does it because he *can not* solve it. This is really scary, especially if we remember that this is a college-bound student. I certainly don't want my children, indeed any children to be educated under such guidance. But perhaps instead of this problem we proposed some new, better one. Look at the following problem, which was presented as an example telling us why algebra is important to learn [12, p. 34]:

A batter hit a baseball when it was 3 feet off the ground. It passed 4 feet above the 6-foot-tall pitcher 60 feet away. It was caught by an outfielder 300 feet away, 5 feet off the ground. How far from the batter did the baseball reach its maximum height, and what was that height?

To solve this problem, we have to assume that trajectory of the ball is a parabola (that is, ignore air resistance), introduce some coordinate system and describe the trajectory by an equation, say

$$y(x) = k(x - b)^2 + m,$$

where y is the height and x is the distance from the batter along the earth (which is supposed to be flat). Then $y(0) = 3$, $y(60) = 10$ and $y(300) = 5$, whence we can find k, b and m. This problem is more difficult than the previous one, but I do not deem that it is better. In any case it is not more "real-world." As any school problem, it creates an imaginary situation, provides certain data about it and requires one to deduce the answer from these data. As usual, this imaginary situation is not real in the literal sense. How were all the heights and distances measured in the heat of the game? Why do we need to know the maximal height and how far from the batter it was reached? No answer is provided to these questions. This is normal and usual for traditional word problems, but in another article Zalman Usiskin called the traditional word problems *phony* and said that they are not needed because there are many "real applications" [11, pp. 158,159]. He should not use such a pejorative term even if he were right, but in fact he is not. Idealization of reality, reducing it to a definite formal system with a finite, strictly defined set of parameters and relations between them and asking all kinds of questions about this system, is the essence of scientific modeling, and there is nothing phony about it. What about "real applications?" We have just seen an example.

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> ***...a mathematical problem is interesting and educationally useful because of its intrinsic mathematical structure.***

Why was such an ordinary, even somewhat cumbersome problem chosen for such an imposing purpose? Wait a little...notice that this problem involves *baseball*...many children love to play baseball...this suggests a guess: probably, the author hopes to convince them that algebra is important to learn because they will use it when they play baseball! The surrounding problems confirm my hypothesis: they are about such attractive topics as an around-the-world trip, a marching band and rock music. Clearly, they are intended to be interesting because of this. At this point I sharply disagree. To me a mathematical problem is interesting and educationally useful because of its intrinsic mathematical structure. I strongly disagree with the idea to attract students to mathematics pretending that it helps to play baseball, organize marching bands or enjoy rock music. This is a false promise.

I have invented many problems and always made no secret that they were *mathematical* problems. First of all I cared about their mathematical meaning. To this I might add some fun. For example, I invented the following problem for the Russian School by Correspondence:

A mathematician who had a hat and a stick in his hands was walking home upstream a river with a speed which was one and a half times greater than the speed of the current. While walking he threw his hat into the river because he mixed it up with the stick. Soon he noticed his mistake, threw the

stick into the river and ran back with a speed twice that with which he had walked ahead. As soon as he caught up with the hat, he immediately got it out of the water and went home with the former speed. 40 seconds after he got his hat, he met his stick carried by the stream against him. How much earlier would he come home if he did not mix up his hat with his stick? [4, p. 8]

This problem was liked by some students and their teachers, although it is clearly not "real-world." One way to solve it is to denote v the speed of the current and t the time the mathematician spent running back, where time is measured in minutes. Then the distance he run back is $3vt$, the distance he went forward from getting the hat to meeting the stick is $3/2v \cdot 2/3 = v$, and the distance the stick moved back till he met it is $v(t + 2/3)$. So we can write the equation

$$3vt + v = v(t + 2/3),$$

where v cancels and we get $t = 1/4$ min. The time the guy lost consists of two parts: the time he ran back, i.e. 1/4 min, and the time he went forward the same distance, which is two times greater, that is 1/2 min. So the total time he lost is $1/2 + 1/4 = 3/4$ min. This problem has one interesting feature which the previous one does not: in solving it, we had to introduce an extra variable, in this case v, which cannot be found, but cancels. Alternatively, we could introduce a special unit of distance to make the speed of the current equal 1. Another class of problems that have this useful feature are often called work problems. This is an example:

Three workmen can do a piece of work in certain times, viz. *A* once in 3 weeks, *B* thrice in 8 weeks, and *C* five times in 12 weeks. It is desired to know in what time they can finish it jointly.

This problem was included by Newton into his textbook and cited by Polya [6, p. 47]. The solution is based on the well-known (unrealistic) assumption that each workman has a constant rate. We can take the "piece of work" mentioned in the problem as a unit of work and call it "job." Then *A*'s rate is 1/3 job/week, *B*'s rate is 3/8 job/week, and *C*'s rate is 5/12 job/week. When they work together, their rates add, and the total rate is

$$\frac{1}{3} + \frac{3}{8} + \frac{5}{12} = \frac{9}{8}$$

Then, the time they need is 1 job divided by 9/8 job/week, that is, 8/9 of a week. Why did Newton and Polya consider such problems valuable? This is an answer [6, p. 59]:

> *Why word problems?* I hope that I shall shock a few people in asserting that the most important single task of mathematical instruction in the secondary schools is to teach the setting up of equations to solve word problems. Yet there is a strong argument in favor of this opinion. In solving a word problem by setting up equations, the student translates a real situation into mathematical terms; he has an opportunity to experience that mathematical concepts may be related to realities, but such relations must be carefully worked out.

Pay attention that what Polya calls "real situation" is not real in the literal sense. Clearly, Polya took for granted that everybody has imagination and highly valued traditional word problems. He would be very astonished if somebody called them phony in his presence, and I completely agree with him: I believe that traditional word problems are very useful.

Another strange, but widespread, idea is that word problems are more uniform than non-word ones. For example, the influential "standards" [8, Summary of changes in content and emphases in 9-12 mathematics] recommend to decrease attention to "word problems by type" and never mention non-word problems by type or word problems not by type. This recommendation shows that the authors feel that there is something wrong with teaching word problems, but fail to analyze what exactly is wrong. They say nothing about how to teach them. Or, perhaps, the phrase "by type" means some bad manner of teaching? What about types of problems? They are everywhere. Give me a problem which you think is not by type, and I shall invent ten similar problems which will put it into a type. In fact, I often have to do this when I teach: first I solve a problem at the board, then I give a similar problem for all to solve in class, then I give a similar problem as a homework, then I give a similar problem on a test. All these stages (often more) are necessary, otherwise many students will not grasp the method.

In fact, at the K-12 level there are *many more* different word problems than non-word ones. Word problems enormously *increase* the variability of problems solved in the classroom. In addition to those limited formalisms of pure mathematics, which are available at the K-12 level, word problems bring a plethora of images, such as coins, buttons, matches and nuts, time and age, work and rate, distance and speed, length, width, perimeter and area, fields, boxes, barrels, balls and planets, price, percentage, interest and discount, volume, mass and mixture, ships and current, planes and wind, pumps and pools, etc., etc. It is an invaluable experience for children to discern those formal characteristics of these images, which should be taken into account to solve the problem. What is at least equally important, in my opinion, is that in solving word problems, children have to comprehend and translate into mathematics a multitude of verbs, adverbs and syntactic words indicating actions and *relations* between objects, such as put, give, take, bring, fill, drain, move, meet, overtake, more, less, later, earlier, before, after, from, to, between, against, away, etc. Although I say "children," I actually mean a wide range of ages, including college undergraduates, for whom all this may be quite a challenge [10]. How did that strange idea of uniformity of word problems come into existence? I think that some teachers and educators, too incompetent to cope with the richness of word problems, reduced them to a few types, and this secondary phenomenon, which went *against* the grain of word problems and came from incompetence rather that from potential of word problems, was mistaken more than once for an inalienable feature of word problems. For example, the influential "Agenda for Action" recommends [1, p. 3]:

> The definition of problem solving should not be limited to the conventional "word problem" mode.

What did the authors mean by the "conventional 'word problem' mode?" Perhaps, that uninspiring manner of teaching which still plagues classrooms, and which is caused by poor preparation of teachers? Who knows? In any case, they expressed their ideas in such an obscure manner that no meaningful action could be undertaken based on this recommendation. It is not a secret any more that some teachers of mathematics don't know enough mathematics. In this light the "who cares?" recommendation is especially dangerous because it may be used as a pretext by some teachers. Regretfully, [11] is not the only occasion when word problems are referred in a pejorative way. For example, the second chapter of an otherwise sound book [2] is filled with deteriorative jokes about word problems. Clearly, Morris Kline would not indulge in such frivolous mockery if he were not sure in advance that it would please some readers. Lately a member of an e-mail list proposed to *define* word problems as those given with the intention to evoke a knee-jerk reflex from students. When I objected that it is better to use the term "word problems" according to the meaning of the words, that is, apply it to problems that use words besides mathematical terms, this professional educator was very astonished and admitted that this idea was new to him. It seems that word problems were almost always taught so badly that most students could not separate word problems themselves from the dreadful manner of teaching. Ralph Raimi is one of those who made this distinction [7]:

> I was a tractable student and did what I was told, and they told me what boxes to put certain numbers into, for a limited range of problems, few enough to memorize. It was hard going, and I later realized how easy the problems were, but since I was told how to do them, and since I was rewarded with praise, that's what I did, totally without insight. Nor did the insight emerge as it does in language learning, when one puts words into sentences and inflections on verbs in a sort of continuous process of accretion. In my case algebra did not come to me that way, and what I learned later, that caused me to see how idiotic my high school exercises were, was not rooted in the

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> ***...in solving word problems, children have to comprehend and translate into mathematics a multitude of verbs, adverbs and syntactic words indicating actions and*** **relations** ***between objects***

> boxes I had learned earlier. The fault was not in the problems, nor in the "type" idea. The fault was in the teaching.

I spent the first forty years of my life in Russia, where presence, even abundance of word problems in mathematical education was always taken for granted. Larichev's textbook for 6-8 grades (13-16 years old) [3], which was used when I was there, contains a lot of word problems. At that time I thought that [3] was just an ordinary textbook. Now, after several years of teaching American college freshmen, many of whom get confused even by simple word problems, I am astonished by the high level and quality of Larichev's work. If graduates from American high schools could solve all the problems from his textbook, they would be prepared better than many of those who are actually sent to calculus. In particular, Larichev's book includes many historical problems, e.g. the following:

A flying goose met a flock of geese in the air and said to them: "Hello, 100 geese." The leader of the flock answered: "We are not 100 geese. If there were as many of us as there are and as many more and half as many more and quarter as many more and you flew with us, then there would be 100 of us." How many geese there were in the flock? [3, p. 27]

Do I need to mention that we solved this problem without calculators and in two ways—without algebra and with it? Russian textbooks for elementary school also contain plenty of word problems. These books are written with an eye to the future, and the problems prepare children to solve more difficult problems in the next years. Example:

A house is to be repaired and 150 window frames need to be painted. One painter can do it in 15 days and another can do it in 10 days. In how many days can the two painters do this job if they work together? [5, p. 190]

This problem may be considered as a preparation for Newton's problem. It follows straight from the data that the first painter makes 10 frames per day and the second one makes 15 frames per day. So together they paint $10 + 15 = 25$ frames per day. Therefore they need $150/25 = 6$ days. The crucial point here, as in all work problems, is to understand which quantity is additive. This is one reason why work problems are useful. Some students naïvely add 15 days and 10 days and come out with 25 days as the answer. It is essential how the teacher reacts to such wrong suggestions. She should quietly observe that this answer contradicts common sense: when two persons work together, they finish sooner that if only one of them worked. Here we get into the realm of teacher's competence, which is measured first of all by her reactions to wrong or partially wrong solutions. A skillful teacher encourages students to use common sense, which in this way gradually ripens into mathematical competence. In the subsequent grades students are proposed similar, but more difficult problems, so that their skills build one on another. In the 6-8 grades students are already proposed to solve problems with data denoted by letters, for example:

Two workers, working together, can fulfill a task in *t* days. The first worker can do this job in *a* days. How many days does the second worker need to do this job alone? [3, p. 166]

If word problems are so useful in Russia, why can't they be equally useful in America? There is a certain theory behind this. I shall call it no-transfer theory. This theory claims that children can not transfer ideas, methods and skills from one task, problem, situation to another and therefore it makes sense to teach them to solve only those problems which they will meet in everyday life. This fantastic theory is often taken for granted by American educators and is almost unknown to all other people. It was developed by Edward Thorndike about a century ago. In 1926 Thorndike published his influential book [9], where he claimed:

> Solving problems in school is for the sake of problem solving in life. Other things being equal, problems where the situation is real are better than problems where it is described in words. Other things being equal, problems which might really occur in a sane and reasonable life are better than bogus problems and mere puzzles.

Thorndike gives no examples of "bogus problems," but based on his argumentation one may conclude that this pejorative term refers to all problems which have no literal counterpart in everyday life. But then all modern mathematics is bogus! Based on his ideas, Thorndike included in his book a chapter called "Unreal and Useless Problems", which starts as follows (p. 258):

> In a previous chapter it was shown that about half of the verbal problems given in standard courses were not genuine, since in real life the answer would not be needed. Obviously we should not, except for reasons of weight, thus connect algebraic work with futility.

Pay attention: Thorndike thought that whenever children are given a problem which they cannot meet in everyday life, they feel a sense of futility. All my experience as a teacher tells me that children's interest in mathematical problems is not determined by straightforward relevance to everyday life. It has much more complicated causes. A lot of my students were excited by various problems, whose wording was fantastic or jocular. In this connection let us consider the following problems:

Mary has forty coins in her piggy bank, all pennies and nickels, which total a dollar. How many pennies and how many nickels are there?

There are rabbits and pheasants in a cage. Altogether they have 100 legs and 36 heads. How many pheasants and how many rabbits are there?

What especially adds to the educational value of these problems is that they can be solved in various ways, even without algebra. For example, we can solve the piggy bank problem as follows. First we assume that all the coins are pennies. Then they total forty cents. This is sixty cents less than we need. Now observe that every time we substitute a penny with a nickel, the amount of money increases by four cents. So, to increase it by sixty cents, we need to perform this substitution 60/4 = 15 times. Thus we get 15 nickels and 40-15 = 25 pennies. We can check this answer by calculating 25 + 15 x 5 = 100 cents = 1 dollar. The rabbits-pheasants problem is included in [3, p. 90] as an "ancient Chinese problem." Polya included a similar problem in [6]. From one teacher I heard a charming way to explain its solution to children: Imagine that all the rabbits stand on their back legs. Then the number of legs standing on the ground is twice the number of heads, that is 72. The remaining number of legs is 28, and these are front legs of rabbits. So the number of rabbits is half of it, i.e. 14, whence the number of pheasants is 36 - 14 = 22. According to all my experience, normal children like these problems and don't ask "who cares?" or "when shall we apply this to everyday life?" Also, all normal children notice that in spite of their different imagery, the piggy bank problem and the rabbit-pheasants problem are similar, and having solved one of them helps to solve the other.

To Thorndike's credit he admits that some of those problems, which he calls unreal and useless, may be interesting for "pupils of great mathematical interest and ability" (p. 259). Many gems might be mentioned as examples of this, e.g. irrationality of $\sqrt{2}$ or the fact that there are infinitely many prime numbers. However, Thorndike mentions only a few historical problems, including the following:

The square root of half the number of a swarm of bees is gone to a shrub of jasmine; and so are eight-ninths of the swarm; a female is buzzing to one remaining male that is humming within a lotus, in which he is confined, having been allured to it by its fragrance at night. Say, lovely woman, the number of bees.

I can testify that in this Thorndike is right: there are students interested in such problems. When I asked Yuly Ilyashenko, who is a professor of mathematics now, how he became a mathematician, he remembered this problem. (It is included in [3, p. 167].)

Thorndike's ideas were criticized by several thinkers, including Vygotsky, who wrote [13, p. 233]:

> ...to refute the Herbartian conception, Thorndike experimented with very narrow, specialized and most elementary functions. He exercised his subjects in distinguishing of lengths of linear segments and then studied how this learning influenced their ability to discriminate magnitudes of angles. Of course, no influence could be found here.

Vygotsky conducted his own experiments, which showed that when dealing with higher mental activities, such as learning of arithmetic and native language, transfer takes place. In this connection Vygotsky spoke about another imporant notion: *mental discipline* (which he called "formal discipline"). The notions of transfer and mental discipline are so closely connected that it is practically impossible to accept one and reject the other. It is well-known that the greater part of mathematics taught in high school has no straightforward application to everyday life. For this reason, when discussing the importance of mathematical education, we cannot avoid speaking about mental discipline embracing all the non-literal, non-direct and far-reaching results of schooling. Criticizing Thorndike, Vygotsky wrote [13, p. 233]:

> Partially the underdevelopment of the theory of formal discipline and mainly the inadequacy of its practical implementation for the tasks of the modern burgeois pedagogics led to demolition of the whole doctrine of formal discipline in theory and practice. Thorndike was the ideologist; in several works he tried to show that formal discipline is a myth, legend, that teaching has no remote influences, no remote consequences for development. As a result of his studies Thorndike came to a complete denial of existence of those interrelations between learning and development, which the theory of formal discipline correctly anticipated, but presented in a very ludicrous form. However, Thorndike's statements are convincing only insofar as they concern the ludicrous exaggerations and distortions of this theory. They do not concern and certainly do not destroy its kernel.

Thorndike's and Vygotsky's conceptions have quite different consequences for mathematical education and the usage of word problems in it. Let us list some of them.

1) If Thorndike is right and mental discipline does not exist, then problems solved in school should be identical with those which students have a chance to face in their present or future life. However, professions are very specialized now, and it is impossible to tell in advance who will follow which profession. More than that, even if we knew somehow that a certain student would become, say, a computer programmer, we still could not teach him exactly that computer language which he will use, because it is a safe bet that this language is not yet invented. Those who had been taught Basic had to program in Pascal or Fortran, and those who were taught Pascal, now program in C++. If mental discipline were just a myth, all their school time would be lost, but many people think that it was not. In fact, some people think that solving mathematical problems, say on geometrical constructions, also helps future programmers. This can be understood only if we accept the notion of mental discipline: if it exists, then transfer is possible and productive. Facing a new problem, children may exclaim: "This is analogous to the problem which we have solved, only with different words and numbers." In other words, they can notice intrinsic similarity between problems and transfer skills and ideas developed in dealing with some of them to solve other, more difficult ones. This is what I always try to achieve as a teacher.

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...we want children to be able to solve problems with* arbitrary data *and answer* arbitrary *questions rather than only those which they meet in everyday life.

2) If mental discipline does not exist, then word problems should be taken literally, at face value. For example, a coin problem makes sense only as related to dealing with real coins, a rabbit-pheasant problem is related only to rabbits and pheasants, etc. Data which are given should be the same as actually available in practice, and questions which are asked should be the same as those which we usually need to answer in everyday life. Problems in this case are grouped into types according to their paraphernalia, such as coin, rabbit or work. If, on the other hand, mental discipline exists, then intrinsic mathematical structure of the problems is most important, while coins, rabbits etc are just superficial external features. In this case we want children to be able to solve problems with *arbitrary data* and answer *arbitrary* questions rather than only those which they meet in everyday life. Without this, children will not be able to make the next step: from numerical data to data denoted by letters. This also means that numerical data do not need to be cumbersome, because they do not need to look like data taken from a concrete real situation.

3) If transfer is impossible, then interaction between

> mathematics and other school subjects, e.g. physics, is impossible, and it is not worthwhile to care about it. This is what we usually can observe in the public schools of America, where subjects are isolated from each other. If, on the other hand, Vygotsky is right and different school subjects interact, then it makes sense to coordinate curricula in mathematics and physics, so that mathematical notions will be applied in physics and vice versa. This is done systematically in Russian schools.

When I read Thorndike's "The psychology of algebra" [9], I got a strange impression. On one hand, it was clear that Thorndike was a hard, persistent worker. On the other hand, he seems to have had no idea of the essence of mathematics. All my classmates, all my students, all children whom I ever met, knew that problem apples were not real apples, but Thorndike did not know this! It is my impression that having spent much time experimenting with animals, Thorndike became convinced that living beings make efforts only in view of some material reward and uncritically transferred this idea to human beings. Every parent knows that children are spontaneously curious and have fantasy and love fairy tales and fantastic stories, but Thorndike seems not to know this. Every parent also knows that children enjoy *contrafactual* statements and images, often (inexactly) called "absurd" or "nonsense." Lewis Carroll and many other authors elaborated this idea with much success, but Thorndike argues as if he has never heard of it!

Today those ludicrous exaggerations associated with the notion of mental discipline, which Vygotsky mentioned, are almost forgotten. Their place is taken by equally ludicrous exaggerations of the opposite kind. An example: lately a new phrase was introduced into educational literature: "real-world problems." Nobody knows exactly what this phrase means, and different authors use it in different, sometimes contradictory, ways. In any case, it is clear that "real-world problems" are very different from that important and well-known part of mathematics, which is traditionally called applied mathematics. Applied mathematics needs precision, because it deals with hard reality, while problems presented as real-world are often vague and loose and have been said to have many answers (nobody ever said how many). To do applied mathematics one needs a lot of mental discipline, while usage of the phrase "real-world problems" often comes with the assumption that mental discipline is but a myth.

It is obvious that future mathematicians need to study mathematics. Let us ask another question: Why is mathematics important to learn for those who will not become mathematicians? Why is mathematical literacy important? One reason is that we read and write and calculate for our everyday practical needs. There is, however, another reason: a literate person is another kind of person than an illiterate one. Literacy and its analogs, such as mathematical literacy, are not just gadgets. They add new dimensions to personality. A literate person, in particular a mathematically literate person, not only answers old questions better, she asks new questions also. Mathematical literacy includes ability and habit to produce abstract *closures* which go beyond immediate necessity. Incompetent people often think that mathematical abstractions are difficult, and they are right in their own way, but abstractions would not be needed if they were not easy in some other sense. Indeed, to solve an abstract problem is easier (if you can do it) than to tamper with every particular case. This contrast is well visible in the case with word problems. For mathematicians they are so easy that some (like Morris Kline [2]) fail to recognize their importance. On the other hand, for people with undeveloped abstract thinking (some of whom regretfully are teachers of mathematics) word problems are enormously difficult. This is because every type of word problems is a small closure: as soon as you grasp the general idea, you can apply it to many particular cases. In this way word problems give some taste of abstract work to everyone who can cope with them. Let us teach all children to solve them.

REFERENCES

[1] *An Agenda for Action. Recommendations for School Mathematics of the 1980s*. NCTM, Reston, VA, 1980.

[2] Morris Kline. *Why Johnny can't add: the failure of the new math*. New York, St. Martin Press, 1973.

[3] P. A. Larichev. *Collection of problems in algebra. Part I for 6-8 grades*. "Uchpedgiz," Moscow, 1961 (in Russian).

[4] *Mathematical Olympiads by Correspondence*. N. B. Vasilyev, V. L. Gutenmakher, Zh. M. Rabbot and A. L. Toom. "Nauka," Moscow, 1986. (in Russian.)

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as if it were a transparent medium and writing skills as if they were generalizable across all contexts) with a belief that language can never be completely "clear," can never be completely rid of analogy, and, even if it could, it shouldn't. As scientists and humanists work together to better understand the languages and conventions that do characterize our disciplines, we may also better understand each other.

REFERENCES

Bizzell, Patricia and Bruce Herzberg, eds. *The Rhetorical Tradition: Readings from Classical Times to the Present.* Boston: St. Martin's P, 1990.

Dawkins, Richard. *The Blind Watchmaker: Why the Evidence of Evolution Reveals a Universe Without Design.* New York, Norton: 1987.

Hesse, Mary. *Models and Metaphors in Science*. Notre Dame, IN: U of Notre Dame P, 1966.

Kuhn, Thomas S. "Metaphor in Science." in *Metaphor and Thought*, Ed. Andrew Ortony. Cambridge: Cambridge UP, 1993.

Leatherdale, W. H. *The Role of Analogy, Model, and Metaphor in Science*. New York: American Elsevier Publishing Co., 1974.

Lewis, Ricki. *Human Genetics: Concepts and Applications*. Dubuque, IA: W. C. Brown, Publishers, 1994.

Locke, David. *Science as Writing*. New Haven: Yale UP, 1992.

Mayer, Richard E. "The Instructive Metaphor: Metaphoric Aids to Students' Understanding of Science." in *Metaphor and Thought*, Ed. Andrew Ortony. Cambridge: Cambridge UP, 1993.

Ortony, Andrew. *Metaphor and Thought.* 2nd ed. Cambridge: Cambridge UP, 1993.

Richards, I. A. *The Philosophy of Rhetoric*. Oxford: Oxford UP, 1964 (1936).

Russell, David. *Writing in the Academic Disciplines, 1870-1990: A Curricular History*. Carbondale, IL: Southern Illinois P, 1991.

Sentilles, Dennis. *Applying Calculus in the Economic and Life Sciences*. Belmont, CA: Wadsworth, 1989.

Snow, C. P. *The Two Cultures: And a Second Look* (includes original 1956 essay). Cambridge: Cambridge UP, 1964.

Steen, Gerard. *Understanding Metaphor in Literature: An Empirical Approach*. New York: Longman, 1994.

Turbayne, Colin Murray. *The Myth of Metaphor*. New Haven: Yale UP, 1962.

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[5] *Mathematics 4. Textbook for the fourth grade of elementary school*. Ed. by Yu. M. Kolyagin. "Prosveshenie," Moscow, 1992 (in Russian).
[6] George Polya. *Mathematical Discovery. On understanding, learning, and teaching problem solving*. Combined edition. John Wiley & Sons, 1981.
[7] Ralph A. Raimi. E-mail communication.
[8] *Curriculum and evaluation standards for school mathematics*. NCTM, March 1989.
[9] Edward L. Thorndike a.o. *The psychology of algebra*. The Macmillan Company, New York, 1926.
[10] A. Toom. "How I Teach Word Problems." *Primus*, v. VII, n. 3, September 1997, pp. 264-270.
[11] Zalman Usiskin. "What Should Not Be in the Algebra and Geometry Curricula of Average College-Bound Students?" *Mathematics Teacher*, v. 88, n. 2, February 1995, pp. 156-164.
[12] Zalman Usiskin. "Why Is Algebra Important To Learn?" *American Educator*, v. 19, no. 1, Spring 1995, pp. 30-37.
[13] L. S. Vygotsky. *Thought and Speech. Collected Works in 6 volumes*, Moscow, "Pedagogics," V. 2, 1982 (in Russian).