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Preparing Our Students to Read and Understand Mathematics

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Abstract

This article gives techniques and tips for college mathematics instructors to increase students' ability to read and comprehend mathematics. The article also includes some relevant history of reading instruction and some motivation for incorporating these ideas into courses.

1. Introduction

We comment that our students don't use the textbook. Our students complain (on our teaching evaluations at least) that the textbook wasn't useful. Pedagogically, we have moved from pure lecture to flipped classrooms, the Moore method, and other student-centered, constructivist approaches. In addition, we are being asked more often to think about offering online courses. We worry about misleading surveys and news and wonder how we can help our students learn to navigate the large volume of technical information they come into contact with each day. What is the missing piece that weaves through all of this? The ability to read and comprehend mathematics.

Several years ago I was given the opportunity to teach a first-year symposium course at my institution. This course is taught by instructors from different disciplines, but the focus of the course is on reading and writing. Instructors for the course undergo intensive summer training. Thus began my endeavors into teaching reading and comprehension.

I soon realized that the techniques I was learning from faculty across campus would be valuable when modified for my mathematics courses. I began asking my mathematics students to read more and started investigating ways to support them in this reading.

Recently I had the chance to delve even deeper into the subject. During a sabbatical, I read research literature on literacy, on content area literacy, on disciplinary literacy, and on teaching reading in mathematics. Unfortunately, there are not a lot of resources for teaching reading in mathematics at the college level. Many times I was frustrated by being unable to locate resources or by resources being out of print or out of date. Many resources dealt only with teaching students to decode word problems.

Looking for new ideas, I began to interview my colleagues. I talked with mathematics faculty at my own and other institutions. I interviewed faculty in foreign languages, philosophy, education, English, history, and theology. I talked with K-12 educators. I asked all of these teachers how they themselves read, how they teach students to read, and how they use activities or strategies to help students make sense of written material.

It is my goal with this article to bring resources together for college instructors to incorporate more teaching of reading into college-level mathematics. This article includes information from my experiences teaching reading generally and in mathematics, my interviews with teachers and faculty at various levels and from various disciplines, research literature, and educational resources. This article is meant to give ideas and techniques that you can use in your mathematics classes, for non-majors and majors alike, right now.

1.1. Motivation

Why should you teach students to read mathematics? In our classrooms, as we have moved away from lecture as a primary teaching strategy, we often ask our students to explore mathematics concepts on their own, through a textbook or online video. We are using more class time for group work, flipped classrooms, and constructivist activities. We are asking our students to begin the learning process on their own, without necessarily equipping them with the skills they need to do so. For example, many students took high school English classes centered on reading literature.

But how many students have had the opportunity to talk about reading a mathematics text? During one of my interviews, a colleague mentioned that he didn't learn how to really read until graduate school and then he did so by seeing his professors do it. We can equip our students to read mathematics now if we take the time to do so.

If I can teach my students to read mathematics and make sense of it, then I have done them a better service than teaching them just the mathematics. For example, I can teach a student to solve a first-order linear differential equation, but will she remember it five years from now when she needs to apply this skill in her job? She is better off if I prepared her to read and understand the mathematics, and she can look up the skill and refresh herself. Preservice teachers will also find this skill invaluable. As teachers we all come across something we need to teach that we have never seen before. The ability to read about it and comprehend it is vitally important.

There are several calls for mathematicians to include more reading instruction in college-level mathematics. In *A Common Vision for Undergraduate Mathematical Sciences Programs in 2025*, Karen Saxe and Linda Braddy [25] coalesce the recommendations from seven curricular guides published by five professional associations into a set of principles to guide mathematics higher education. All of the guides recommend that, “Instructors should intentionally plan curricula to improve students’ ability to communicate quantitative ideas orally and in writing (and since a precursor to communication is understanding, improve students’ ability to interpret information, organize material, and reflect on results)” [25, page 13]. In addition, they found that pedagogy that lets students be actively involved in reading, synthesizing, and evaluating course content is recommended frequently from these organizations [25, page 19].

Other sources echo these calls. Fang and Coatoam [10] state that, “developing disciplinary literacy is a long-term process that begins in upper-elementary grades and continues through college.” The NCTM Principals and Standards for School Mathematics [22, page 60] states that, “students who have opportunities, encouragement, and support for writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically.”

Finally, in *Literacy Strategies for Improving Mathematics Instruction*, Joan Kenney [17, page 11] states that, “If we intend for students to understand mathematical concepts rather than to produce specific performances, we must teach them to engage meaningfully with mathematics texts.” While Kenney and the NCTM may have been speaking to K-12 educators, higher education needs to attend to reading mathematics as well.

1.2. History

There are many books and articles that provide a history of literacy education, of constructivist learning theories, and of other modern movements in education. One recommendation for a general overview is Chapter 1 of the 2013 book *Theoretical Models and Processes of Reading* by Reynold *et al.*, where Alexander and Fox summarize theories of reading over the last century, including historical and political perspectives [4]. For this section, I am mainly concerned with current theories and understanding of reading, so I note the relevant, recent ideas. My goal is to provide a brief recent historical account to motivate and support some of the strategies and activities.

The importance of motivating students to read emerged in the early 2000s [14]. The importance of motivation coincides with the idea that readers don’t just passively absorb the material; there is more of a transaction between the reading and the reader, as well as a requirement that the reader participate actively when reading. The transactional and motivational theories of reading are a lot like the constructivist theory of learning. In addition, during the early 2000s, educators and researchers began to pay more attention to the fact that growing as a reader is important for people of all ages and abilities [2]. Educators and researchers also began to study the role of alternative forms of text in reading comprehension as technology changes the way we encounter written material [27].

In the next decade, researchers began to explore what it means for students to develop a deeper understanding of a text more closely tied to critical thinking and taxonomies of thinking. Kulikowich and Alexander [18] extended the idea of being an engaged reader to being engaged with higher-order thinking goals related to critical analysis and evaluation. Murphy *et al.* [21, page 741] in their meta-analysis defined critical literacy as “higher order thinking and critical reflection on text and discourse”.

By this definition, the authors suggest that students should be able to go beyond a basic reading and comprehension of texts and instead should be taught to think critically and reflectively about what they are reading. To achieve this goal, Murphy *et al.* [21] encourage teachers to have students do more than offer opinions on a text; rather they suggest students be taught and challenged to defend and support what they say about a text or how they interpret a text. Furthermore, the authors believe this behavior needs to be modeled by a competent person.

Other researchers studied the importance of disciplinary differences between texts and what it means to teach students to read with comprehension in different disciplines, such as science [19]. Alexander [3] introduced a Model of Domain Learning (MDL), which is applicable to many traditional school subjects and to mathematics, in particular. In this model, development of expertise in a particular domain is broken down into three categories: knowledge, interest, and strategic processing. Knowledge is further broken down into breadth of knowledge across the domain and depth of knowledge on particular topics in the domain. Interest is measured in terms of interest in a particular situation, such as a student's interest in learning about limits at infinity while reading a Calculus book on the topic, and in terms of the overall individual level of interest in a domain. Finally, strategic processing centers on strategies the individual employs while reading in the particular domain, such as summarizing and self-evaluation. Furthermore, Alexander [3] describes surface-level reading strategies, such as rereading, that help a student gain basic understanding of a text. Deep-level processing strategies, like relating a topic to prior knowledge, are employed by individuals who can convert what they have read into their own message.

Recently there have been more studies into reading mathematics at the college-level. Weinberg *et al.* [29] completed a study of 1156 undergraduates in introductory mathematics classes in an effort to understand how students use their math textbooks. The researchers found that students often use examples, rather than the written explanations, to help them understand a concept and noted that this tendency could be problematic. They suggest that mathematics faculty should ask students to read and should provide instruction on how to do so. Weber [28] researched how successful math students read mathematical proofs. From videotaped sessions with four undergraduates, Weber found four typically used strategies:

Try to do it yourself, identify the proof framework, break the proof into smaller subproofs, and use examples to understand difficult parts. Carducci [8] has used technical instructions, such as instructions on how to do a complex card trick, to help students reflect on how they read the instructions. Students are then asked to translate these same skills into reading mathematics.

1.3. Article Organization

The next two sections are divided into strategies and activities for planning a course that incorporates instruction on reading mathematics (Section 2) and strategies and activities to prepare students before an assigned reading (Section 3). Section 4 includes a short conclusion. A follow-up article is planned on strategies and activities during reading and post reading. In some cases, a strategy may be used at more than one stage. In addition, some strategies may have parts that extend into multiple stages. Each of these stages is important for comprehension. Strategies are described as for an individual, for a small group, or for the whole class; however, many can be modified to be used in one of the other ways.

The strategies for pre-reading help prepare and motivate students for the reading assignment. In both the research literature I have explored and in interviews I had with faculty, it was apparent to me that reading with a purpose that students understand is very important. If we think of learning as being goal-directed, then we have to think about what our students interpret as the goals of our reading assignments.

Finally, you might consider using some of these strategies for helping students make sense of other types of materials, such as online videos. Unfortunately, research on the topic is limited, so careful reflection is needed on what would be appropriate and helpful to the students. Modifications may need to be made to make the activities appropriate for other types of resources.

2. Designing Your Course

There are many strategies and activities you can incorporate into your course while it is in progress. However, if you have the luxury of planning ahead, there are some things to consider and some things you may want to plan in your syllabus and course.

2.1. Textbooks

If you plan to assign readings from the textbook, pick your text very carefully. Many textbooks are written in a way that is difficult to understand for students that are first learning a subject. Since motivation is often important in getting students to read, a textbook that is too hard can be very discouraging. Look for texts that are meant for students to read. Bullock and Millman [6] studied how mathematicians write versus how students read. They found that mathematicians value brevity and conciseness in mathematics. However, students who are reading mathematics are not best-served by these traits. Instead of following the conventions of mathematics, Bullock and Millman [6] argue that texts meant for students to read should meet the needs of the student. In particular, if you plan to ask your students to read from their textbook, look for textbooks that incorporate more expository writing, more examples, and mathematics that is not always what we think of as the “best” (e.g., shortest possible proof of a theorem or calculus example worked out with all the algebra steps skipped), but will instead give students the information they need to process the text.

Some faculty that assign readings from the textbook suggest using a textbook with short sections or chapters. These faculty note that students find the shorter sections less intimidating. Other faculty who I interviewed emphasized the need for students to write in texts as they read; so they discourage their students from renting books or using ebooks. Students may be able to use software or buy ebooks where they can take notes in the ebook.

We’ve all encountered bad texts. Even in texts that we like, there are often elements that we don’t like. Sometimes we disagree about what is good or bad in a text. The point here is that the authors of these texts are human, they make mistakes, and they have to make decisions about how to present the ideas in their texts; we may not always agree with these decisions. Help your students to see that if they can’t understand something in a text, the author may have done a bad job explaining it!

2.2. Other Readings

Farmer and Schielak [11] argue that other types of readings, besides textbooks, are very important to include in mathematics classrooms. They cite the need to change attitudes toward mathematics as a primary reason.

They also suggest that readings on recreational mathematics can help students see the beauty and fun of math, which can lead to an increased interest in the subject. Thus, even if you are assigning reading from the textbook, consider incorporating other types of readings. These other readings might include history, expository writing, newspaper articles, research articles, articles for a wider audience, and fiction. For ideas, consider the Mathematical Reading List [30] made available online in 2015 by the University of Cambridge Mathematics Faculty for their students. The list is broken into categories like history, recreational, and readable mathematics. Each suggestion includes a description. Farmer and Schielak [11] also provide a reading list, broken down by topic, as well as sample study guides for readings. In addition, Karaali [15] suggests using philosophical readings in mathematical courses. Specifically she provides a list of philosophical readings to be used in a linear algebra course, with the goal of helping students to see why the mathematics they are learning is relevant to the world around us.

In research literature, education literature, and interviews, I heard many bring up the importance of including history in the teaching of mathematics. In interviews, faculty mentioned that mathematics is a human enterprise, but that it is not often seen this way by students because of the dry way it can be presented in textbooks or in lectures. Sharing readings on the history of mathematics with students will help them to see the human side of mathematics and to see that throughout history humans have struggled with mathematical ideas. In addition, during interviews, faculty pointed out that history can help provide a context for the mathematics content and the vocabulary. An English teacher introducing a Victorian-era novel to her students would provide context in terms of the history of the time and life of the author; such steps could also be useful in mathematics.

Grabiner [13] gives three reasons for including history in the teaching of mathematics. She notes that an historical perspective can help students to see the inherent difficulty of some mathematical concepts. Secondly, she suggests that seeing the historical development of mathematics can motivate students to study mathematics. Finally, Grabiner [13] believes history can help tie mathematics into the greater tradition of human thinking and advancements. Byers [7] gives a history of including mathematics history in mathematics classes.

In addition to different types of readings, you might have students read on the same topic from several sources. Think of each text as a teacher — some are better than others. No one resource is going to give you everything you need to understand something. By reading multiple sources on the same topic, the reader has to consider the same idea from multiple perspectives and can develop a deeper understanding. As instructors we might do this when we are preparing a class on a topic (we might look at the same topic in multiple textbooks), but our students can benefit from this repetition in a similar way.

2.3. Class Time

As educators adjust to new pedagogical methods, they occasionally struggle to find class time to “give up” to devote to group work, hands-on activities, or flipped classrooms. However, when used well, these activities enhance student learning, rather than detract from it. In the same way, devoting class time to comprehension of mathematics texts has benefits that outweigh the costs of “giving up” more class time. Many of the comprehension strategies involve rich, student-centered discussion, which allow students to make meaning of the written mathematics. Often the teacher does not lead the class, but rather is a part of the classroom conversation.

By devoting class time to an activity and emphasizing it through course policies and assignments, we show students what we think is important. By devoting class time to the comprehension of written mathematics, we communicate to students that we think it is important. Consider using class time to read mathematics. There are strategies for getting students to read in pairs, but students could also read out loud or quietly on their own. Letting students read in class with other students and you there for support is like a flipped classroom for reading. Letting students read out loud helps them to slow down, focus, and hear things in a different way. In addition, having students read out loud means that all students have heard the same thing, so can have a democratizing effect on the class.

If you plan to assign readings from a text, take the time in class to orient students to the text. Let them explore the text and ask questions. Talk about special features the text might have. Have the students get the text out, locate things in the text, and get used to it as a classroom resource.

Let the students see you use the text and reference it. During interviews, several faculty mentioned having the students access the text during class and frequently have the text in their hands.

2.4. Other Considerations

By helping students to feel comfortable reading mathematics, you also help them to see that, as their instructor, you are not the only source of knowledge on the topic. Students should be given the ability to find answers, additional information, and perspectives from other sources.

Consider assigning students to read the same thing from the same source more than once, leaving time between the two readings. Many faculty across disciplines read material more than once, but students may not understand how important this is. Even if we suggest to students that they read something more than once, they may not do it because they do not understand why it is important. If you can incorporate two assignments of the same reading with clear goals, students will begin to see the value in rereading.

We often assess comprehension, but, in a very important study, Dolores Durkin [9] found that we are not teaching students how to comprehend. Strategies in this article can help make actively engaging with written material part of learning how to comprehend mathematics.

In one interview, a foreign language professor mentioned that when teaching reading in a foreign language, there is non-native empathy: the idea that, for example, a Spanish teacher that is a non-native Spanish speaker has gone through the same thing that the students are going through. In some sense, we are all non-native to the vocabulary and notation that we use in mathematics. Let the students know that you have struggled and can still struggle with reading mathematics.

3. Pre Reading

3.1. Vocabulary and Notation Instruction

Before being assigned a reading students often need exposure to new vocabulary and notation. McKeown, Beck, and Blake [20] declare that for vocabulary instruction to be helpful, it needs to be meaningful;

students need to find ways to make sense of the new words that they will be able to remember. We want students to connect new vocabulary with prior knowledge and with associated known words. Here I describe some techniques that can help do just that. In each of these cases, students may also benefit from sharing their work in groups or as a class.

There are different models for introducing new vocabulary that help achieve the goals McKeown, Beck and Blake [20] set forth. One strategy is commonly called the Frayer Model [12]. Here students think of four categories: definition, interesting facts, examples, and non-examples. Sometimes the categories may have different names such as essential characteristics, non-essential characteristics, examples, and non-examples. Here is a completed example of using a Frayer Model to define a subgroup in abstract algebra.

Subgroup

Definition: If $(G, *)$ is a group and H is a subset of G , then H is a subgroup of G if $(H, *)$ is also a group.	Interesting Facts: G is a subgroup of itself. The set consisting of just the identity element with the operation forms a subgroup.
Examples: $(2\mathbb{Z}, +)$ is a subgroup of $(\mathbb{Z}, +)$. $\{0, 4\}$ is a subgroup of Z_8 .	Non-examples: $\{0, 2, 3\}$ is not a subgroup of Z_8 since $2 + 3 = 5$, which is not an element of the subset.

Another model emphasizes a visual representation, where students think of the word, the definition, something they associate with the word, and draw a picture. Here is a completed example of this model to define an open set in topology.

Word Open	Definition: A set is open if it is a neighborhood of every point.
Associated Word: neighborhood	Picture:

Another model, the feature analysis strategy [16] uses connections between vocabulary words. Students make notes or highlight important examples in each box to illustrate whether the words down the left-hand side have the characteristics along the top. Here is an example from calculus that has not been completed.

	Is always continuous	Always has domain all real numbers	Always has range all real numbers	May have removable discontinuities	May have irremovable discontinuities
constant function					
linear function					
quadratic function					
polynomial function					
rational function					
trig function					

Many words in mathematics have a meaning that is different from their meaning in everyday language. The Typical to Technical Approach [24] helps students with these common areas of confusion. In this approach, start by discussing the usual meaning of the word in everyday language. Then contrast this with the technical definition of the word. Reinforce the meanings with exercises that require that students differentiate between the two meanings. As an example, consider the word “solution”. Students sometimes have trouble understanding what we are asking when we ask students to find a solution to an equation, i.e., a value of the variable that makes the equation true. In everyday language we think of a solution as a way to deal with a difficult situation. We can contrast these typical and technical definitions with sentences like the following.

- A *solution* to $2x + 3 = 5$ is $x = 1$.

- A first step in a *solution* to finding a line that is parallel to $y = 2x + 3$ is to find the slope of the line.

Some other mathematical words that also could be confusing to students because of a difference between the typical and technical meanings are open, limit, union, ratio, acute, complementary, congruent, group, and function.

Use history and other context to help make sense of new vocabulary. If there is a storyline, students may have an easier time remembering the word. Many mathematical words are chosen for a reason; the words themselves are meant to be a clue into the meaning. One idea from an interview is that before introducing new words, you might deliberately design sentences that use math jargon and ask students to guess at the meaning. By doing so, we are getting students to reflect on why someone might choose the word and also to activate prior knowledge related to the word. Once you have discussed the technical definition of the word, you might then engage the class in a discussion of what word they would choose if they were the mathematician who got to name it. This technique can help to humanize mathematics. In some cases, it is necessary to discuss how the word is related to another language or why a particular language is used.

Although there are less techniques and research into teaching notation, mathematical notation is its own language, and learning this notation is similar to learning new vocabulary. In this way, instruction on new notation should be deliberate and meaningful. Emphasize to students why we are using notation: to be more compact, to allow us to write more quickly, or to help us remember something. Once again history can be a helpful tool in helping students to see that someone selected this notation for some reason. Why? What could we do differently? Is there a different notation that is better in some ways? Notation may make things easier sometimes and harder other times. The Frayer Model and other graphic organizers could also be used to help students with notation. Model reading sentences containing the new notation out loud while students follow along. Have students practice reading the sentences out loud.

The following graphic organizer can help students with new notation and, by forcing the students to write the meaning out in words, can help students to see why the notation is useful. Here is a completed example of this model to practice the notation for the complement of a set from basic set theory.

Symbol A'	Read out loud as (may be more than one way) A complement the complement of the set A
Example in symbols $U = \{1, 2, 3, 4, 5\}$ $A = \{1, 2, 3\}$ $A' = \{4, 5\}$	Example in words The universal set U contains the elements 1, 2, 3, 4, and 5. If A is the set consisting of 1, 2, and 3, then A complement is the set consisting of 4 and 5.
Alternative notations A^C	Other ways the same symbol is used single tic mark is also used for first derivative

3.2. Background Knowledge

Before students complete an assigned reading, instructors can aid comprehension of the reading by helping students activate their background knowledge on the topic. In addition, taking time to think about background knowledge helps motivate students to complete the reading and gives more of a purpose to the assignment. Students might use the chapter or section titles, the vocabulary or other notation, or a prompt provided by the instructor. Just giving students time to brainstorm about background knowledge could be helpful, but the following are some more formal strategies.

Give students time to answer the following questions and then discuss in small groups or as a class.

1. Have you ever seen these words before? Was it in another class? Was it in real life?
2. Do you know any words or concepts that are related to these words? How might they be related?
3. Did you ever wonder about these words or ideas in the past? Were you ever confused by something related to these words or ideas?

Have students brainstorm questions they might have on a topic. After reading, have the class come back to this list of questions and decide if the reading:

1. Answered the question explicitly.
2. Did not answer the question. In this case, help the students brainstorm about where they might find the answer to the question. Or maybe the students can consider if the question is really related to the topic or related to something else.
3. Assumed the students already knew the answer to the question. If so, do the students still have the question? Where they able to infer an answer to the question by information given in the reading?

The K-W-L strategy, developed by Donna Ogle [23], has the students make a chart with three columns: Know, Want to Know, and Learned. The first two columns are filled out as pre reading. The final column is filled out after completing the reading.

3.3. Structure Analysis

In my interviews with faculty from other disciplines, instructors often talked about the need to look for structure in a reading, such as looking for words that divide or transition words. These same ideas can apply to looking for structure in mathematics texts. As mathematicians, we are used to a certain structure, which may be foreign to our students. Explicit instruction can help students learn about the particulars of mathematics structure and syntax. Students may need more explicit instruction in the meaning of the words theorem, corollary, and lemma, for example. Faculty in other disciplines also suggested rewriting each line of a reading in your own words or in shorthand. This method leaves you with an outline of the reading, which can help you look at the bigger picture and the overall structure.

3.4. Paced Reading

In many disciplines, mathematics included, students may not understand that reading takes a long time. Other disciplines have developed techniques to help students slow down when they are reading and to identify areas of confusion. You might have students read the first paragraph and underline words they don't know. Various techniques, such as writing all of the unknown words on the board and going over them or having small group discussions, can be used to help clarify points of confusion.

Another idea is to have students read out loud. Faculty that I interviewed mentioned that this helps slow down thinking. In addition, faculty mentioned that this technique has an equalizing effect on the class because everyone has then had the same experience with a reading. Also, words have a different impact when heard out loud. In mathematics, reading out loud can also help students learn new notation as they are forced to make connections between the notation and the meaning of the notation.

3.5. Anticipation Guides

Several sources suggest giving students Anticipation Guides before a reading (for example, [1] and [26]). When using this technique, you start by giving students a set of true/false statements about the topic in the reading. Researchers suggest using sources of common confusion to write statements. In addition, you may use Bloom's taxonomy [5], or another taxonomy, to incorporate true/false statements at different levels of reasoning.

Before the reading students decide if they think the statements are true, false, or sometimes true. Students should record their predictions. After completing the reading, students go back to the same set of statements and use the reading to justify if the statements are true, false, or sometimes true. Whenever possible, students should use the text for justification by writing down page numbers, theorem numbers, etc. Students should also note the differences between their pre reading assessment and post reading assessment. What changed? Why? It may be the case that not every answer can be found explicitly in the reading. In this case, the instructor has an opportunity to model how you reason out an answer from the information provided in the reading.

4. Conclusion

The goal of this article is to motivate college math instructors to include more instruction for reading and comprehending mathematics. In addition, the article details activities for instructors to include in their classes to prepare students to read on their own. A follow-up article is planned to detail activities for students to help during reading and after reading.

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