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A Linear Perspective to Art

By

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Approved by _____
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I. Early Perspective

Until the late fourteenth century the use of perspective in art was not based on mathematics. Perspective was represented in an intuitive manner, using four key techniques to imply a sense of depth and reality in the artwork. The intuitive techniques for representing perspective are classified into the categories of overlapping, vertical, diminution, and atmospheric. An example of intuitive perspective can be found in “The Battle of Alexandria” [14] (Figure 1) which was painted in the first century B.C. This picture displays the technique of overlapping to show depth, in which the partially covered men and horses are located behind other men and horses, even though the sizes are not much different.

Figure 1: An example of the overlapping technique for indicating perspective

In “St. Matthew in Ebbo’s Gospels” (816-835) [14] (Figure 2) we see an artist who uses both vertical positioning and diminution to portray depth. Vertical positioning creates a sense of depth by placing those elements that are in the distance, such as the hill, near the top of the

canvas. Those elements that are close to the viewer, such as St. Matthew are located towards the bottom of the picture. Diminution conveys perspective because the smaller elements are assumed to be farther away than the larger ones, as seen with St. Matthew and the hill, the size difference alludes to a depth difference.

Figure 2: An example of both the vertical positioning and diminution techniques for representing perspective

Worthington Whittredge's "Fight Below the Battlements" (1849) [17] (Figure 3) shows the many facets of atmospheric, or aerial perspective. It can be seen on the left side of the painting in the loss of focus, brightness, and clarity as the landscape gets farther and farther away.

Figure 3: An example of the atmospheric technique for representing perspective

II. Pioneers in Perspective

Filippo Brunelleschi (1377-1446) was a Florentine man of many trades. As an architect, sculptor and an engineer, he also worked as an “applied” mathematician, yet his work in perspective was never formally documented. Brunelleschi was the first person to carry out experiments leading to mathematical theories of perspective. His painting of the Baptistry in Florence was ground breaking work. To create the painting of the Baptistry he first set up a mirror across the square so that he could see the Baptistry in the reflection. The image in the mirror was then painted onto a wooden tablet in which a small hole had been drilled. Afterwards, observers were invited to stand facing the Baptistry and look through the hole in the painting while holding the mirror in front to reflect the artwork. The point of his experiment was to be able to move the mirror so that the observer was looking at parts of both the painting

and the Baptistery so that the observer could see that the image lined up with the actual building (e.g. was in the same ratio and perspective) [2] (Figure 5). Unfortunately, the painted panel has not survived. New rules of proportions and symmetry were created from Brunelleschi's work and also led to other explorations in art perspective.

Figure 5: A diagram of Brunelleschi's Baptistery experiment

Leon Battista Alberti was an architect and mathematician in Florence who lived from 1402-1472. He wrote the earliest account of painting using mathematical perspective in *De pictura* (1435) which was dedicated to Brunelleschi. Alberti was the first to use the term 'vanishing point' and was known for his mathematical writing. His work with floor tiling is a basis for the calculations discussed in this paper. Maple, a mathematical computer application that supports symbolic and numeric computation and graphics, was used to create a simple floor tiling in mathematically precise linear perspective. The mathematics associated with finding a vanishing point and the tiling of a floor in linear perspective is a relatively simple one. The work in Maple was accomplished with these simple steps (the computer code can be seen in Appendix A):

- Evenly spaced points are selected on the bottom of the frame. Lines are drawn from the bottom of the frame to some vanishing point along a horizon line. (Figure 4)

Figure 4: The vanishing point

- A viewing distance is determined. This is the optimal distance between the observer and the canvas. From this the viewer point is located. This is a point on the horizon line outside the frame. Its distance is the viewing distance from the frame.
- The points on the bottom of the frame are connected to the viewer point outside of the picture plane (Figure 5)

Figure 5: The viewer point is added

- Parallel lines are then drawn according to where the lines to the viewer point cross the edge (black frame) of the picture plane (Figure 6).

Figure 6: The addition of the horizontal lines

- This then creates a tiling on the ground as seen in Figure 7 and can be altered by changing the number of original lines or moving the vanishing point or viewer point.

Figure 7: The floor tiling

Piero della Francesca was an artist and a mathematician who lived from 1412-1492 and was highly influenced by Brunelleschi. He lived and wrote in places such as Florence, Rimini, Arezzo, Ferrara, and Rome. *De prospectiva pingendi* was a mathematical paper written by Francesco about perspective in art around 1465, approximately 30 years after Alberti. Piero

painted “The Flagellation of Christ” [14] (Figure 8) using the ideas of Alberti’s simple floor tiling in a more complex way. The tiling under the feet of Jesus looks like a jumble of black and white, but the geometry and mathematics utilized are quite complex and the pattern can be seen in Figure 9.

Figure 8: The Flagellation of Christ

Figure 9: The tiling at Christ’s feet

In his paper, Piero included a detailed drawing of how to project an octagon into perspective [5] (Figure 10), which is much like the process used to transform the elaborate two-dimensional tiling into the perspective representation of the tiles at Jesus' feet.

Figure 10: The tilting of the octagon from Piero's book

Below is a simple example of the technique that Piero used to transform the octagon. It is based on the process outlined by Alberti for projecting a tiling into linear perspective.

- An octagon is first drawn on a grid to connect the shape to a tiling (Figure 11).

Figure 11: Octagon put into a grid.

- Once the grid is transformed into a tiling in perspective, the points associated with the vertices of the octagon can be located in these transformed coordinate system. Figure 12 shows the projection of the octagon.

Figure 12: Octagon put into linear perspective.

III. Some Further Examples of the Use of Mathematical Linear Perspective

Maple can be used to show that Perugino painting “Jesus Handing the Keys to St. Peter,” (1470’s) [13] is an example of mathematical linear perspective. Figure 13 shows the painting with the perspective lines. Notice that the vanishing point is at the door of the building.

Figure 13: The painting and the associated perspective lines

Built for the Medici family around 1423 by Brunelleschi, the San Lorenzo Church in Florence is an interesting curiosity, it is a church that was erected with perspective in mind. The vanishing point is the altar at the end of the church and the perfection of perspective can be seen in the picture [16] with lines (Figure 14).

Figure 14: San Lorenzo Church in Florence

IV. Curvilinear Distortion

Curvilinear distortion is a transformation of linear perspective. This is seen in the visual distortions created by looking at wide areas. For example, if an observer stands close to a long wall and looks left, and then right any lines on the wall will not appear to be perfectly straight to the observer. This phenomenon is often seen in wide-angled photography and creates what is sometimes called the “fish-eye” effect.

To reproduce this effect on the tiling using mathematics, it must be recognized that a “wide-

angle” image takes something that is too wide to be caught in a single picture and narrows it down, creating curvature as can be seen in the simple hand drawing in Figure 15 [3].

Figure 15: A demonstration of the curvature created by “wide angle” views

To recreate this idea, this project will consider a wide tiling and transform it so that it fits into a narrower frame. Mathematically, this is accomplished using the following steps:

- A wider framed tiling is produced as before with the viewing point equidistant from the border as the vanishing point (Figure 16). This viewing point is then used to determine the viewing point of the narrower tiling which maintains the same height (Figure 17).

Figure 16: Perspective tiling with a wide base

Figure 17: Tiling with narrow base, same height, and same viewing point.

- The lengths of the horizontal lines in the wide tiling are longer than in the narrow tiling. This difference in length will create a curvature of the horizontal lines when the wide tiling is transformed into the narrow one. The exact curvature is found by using the equations associated with cords and arcs of a circle. By knowing the length of both the

cord and also the arc, we can solve the following equations for both r and θ using Maple:

$$r\theta = \text{arclength}$$

$$r \sin \frac{\theta}{2} = \text{chordlength}$$

The knowledge of r and θ allows the center of the circle to be computed and the arc to be drawn as is shown in Figure 18.

Figure 18: The first of the curves placed connecting the two points with the center above.

- This process is continued for each horizontal line to create curves as seen in Figure 19.

Figure 19: Curvature of each horizontal line.

- To find the curvature of the vertical lines involves measuring the length of the segments (horizontal edge of each tile) on the wide tiling. This length must then be measured out on the arcs shown in the narrow tiling using arc length computations. The collection of points on the arcs are connected to give the gently curving vertical lines and the associated tiling shown in Figure 20.

Figure 20: Tiling in curvilinear distortion.

V. Bibliography

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Appendix A

I: Maple code plotting animation of perspective

```

> with(plots) :
> n:=6;
                                 $n := 6$ 

> a:=1/2;
                                 $a := \frac{1}{2}$ 

> t:=1;
                                 $t := 1$ 

> c:=t->1+(t/10) ;
                                 $c := t \rightarrow 1 + \frac{1}{10}t$ 

> b:=1;
                                 $b := 1$ 

> Perspective:=seq(plot([
  (1/(a))*x,
  ((b/((a)-(1/(n-1))))*x)-((b/((a)-(1/(n-1))))*(1/(n-1))),
  ((b/((a)-(2/(n-1))))*x)-((b/((a)-(2/(n-1))))*(2/(n-1))),
  ((b/((a)-(3/(n-1))))*x)-((b/((a)-(3/(n-1))))*(3/(n-1))),
  ((b/((a)-(4/(n-1))))*x)-((b/((a)-(4/(n-1))))*(4/(n-1))),
  ((b/((a)-(5/(n-1))))*x)-((b/((a)-(5/(n-1))))*(5/(n-1))),
  (b/(c(t)))*x,
  ((b/((c(t))-(1/(n-1))))*x)-((b/((c(t))-(1/(n-1))))*(1/(n-1))),
  ((b/((c(t))-(2/(n-1))))*x)-((b/((c(t))-(2/(n-1))))*(2/(n-1))),
  ((b/((c(t))-(3/(n-1))))*x)-((b/((c(t))-(3/(n-1))))*(3/(n-1))),
  ((b/((c(t))-(4/(n-1))))*x)-((b/((c(t))-(4/(n-1))))*(4/(n-1))),
  ((b/((c(t))-(5/(n-1))))*x)-((b/((c(t))-(5/(n-1))))*(5/(n-1))),
  (b/(c(t)))*1,
  ((b/((c(t))-(1/(n-1))))*1)-((b/((c(t))-(1/(n-1))))*(1/(n-1))),
  ((b/((c(t))-(2/(n-1))))*1)-((b/((c(t))-(2/(n-1))))*(2/(n-1))),
  ((b/((c(t))-(3/(n-1))))*1)-((b/((c(t))-(3/(n-1))))*(3/(n-1))),
  ((b/((c(t))-(4/(n-1))))*1)-((b/((c(t))-(4/(n-1))))*(4/(n-1))),
  ((b/((c(t))-(5/(n-1))))*1)-((b/((c(t))-(5/(n-1))))*(5/(n-1)))
], x=0..2.0, y=0..2.2,
color=[red, red, red, red, red, red, blue, blue, blue, blue, blue, blue, bla
ck, black, black, black, black, black, black]), t=1..9) :
> border1:=pointplot([ [1,0], [1,2] ], connect=true, thickness=3) :
> Border1:=seq(pointplot([ [1,0], [1,2] ], connect=true, thickness=3), t
=1..9) :
```

```

> border2:=pointplot ({ [1, 2], [0, 2] }, connect=true, thickness=3) :
> Border2:=seq(pointplot ({ [1, 2], [0, 2] }, connect=true, thickness=3), t
=1..9) :
> BB1:=display (Border1, insequence=true) :
> BB2:=display (Border2, insequence=true) :
> PP:=display (Perspective, insequence=true) :
> display (PP, BB1, BB2) ;

```

II: Maple code for the projection of an octagon into perspective

```

> with (plots) :
(Normal octagon in a flat 2-dimensional plane)
> aa:=pointplot ({ [0, 18], [18, 0] }, connect=true, axes=BOXED) :
> bb:=pointplot ({ [18, 0], [42, 0] }, connect=true, axes=BOXED) :
> cc:=pointplot ({ [42, 0], [60, 18] }, connect=true, axes=BOXED) :
> dd:=pointplot ({ [60, 18], [60, 42] }, connect=true, axes=BOXED) :
> ee:=pointplot ({ [60, 42], [42, 60] }, connect=true, axes=BOXED) :
> ff:=pointplot ({ [42, 60], [18, 60] }, connect=true, axes=BOXED) :
> gg:=pointplot ({ [18, 60], [0, 42] }, connect=true, axes=BOXED) :
> hh:=pointplot ({ [0, 42], [0, 18] }, connect=true, axes=BOXED) :
> display (aa, bb, cc, dd, ee, ff, gg, hh) ;
(Octagon projected into perspective)
> A:=pointplot ({ [8, 22], [18, 0] }, connect=true, axes=BOXED) :
> B:=pointplot ({ [18, 0], [42, 0] }, connect=true, axes=BOXED) :
> C:=pointplot ({ [42, 0], [52, 22] }, connect=true, axes=BOXED) :
> E:=pointplot ({ [52, 22], [45.5, 38] }, connect=true, axes=BOXED) :
> F:=pointplot ({ [45.5, 38], [39, 45] }, connect=true, axes=BOXED) :
> G:=pointplot ({ [39, 45], [21, 45] }, connect=true, axes=BOXED) :
> H:=pointplot ({ [21, 45], [14.5, 38] }, connect=true, axes=BOXED) :
> J:=pointplot ({ [14.5, 38], [8, 22] }, connect=true, axes=BOXED) :
> display (A, B, C, E, F, G, H, J) ;

```

III: Maple code to find the radius and angle for curves to create curvilinear distortion.

```

> f:=(r, theta)->r*theta-52;

$$f := (r, \theta) \rightarrow r\theta - 52$$

> g:=(r, theta)->r*sin(theta/2)-19;

$$g := (r, \theta) \rightarrow r \sin\left(\frac{1}{2}\theta\right) - 19$$

> fsolve ({f(r, theta), g(r, theta)}, {r, theta});

$$\{r = -19.57206587, \theta = -2.656847793\}$$

> restart:
> f:=(r, theta)->r*theta-58;

$$f := (r, \theta) \rightarrow r\theta - 58$$


```

```

> g:=(r,theta)->r*sin(theta/2)-22;

$$g := (r, \theta) \rightarrow r \sin\left(\frac{1}{2} \theta\right) - 22$$

> fsolve({f(r,theta), g(r,theta)}, {r,theta});
{ theta = -2.503192199, r = -23.17041417 }

>
> restart:
> f:=(r,theta)->r*theta-67;

$$f := (r, \theta) \rightarrow r \theta - 67$$

> g:=(r,theta)->r*sin(theta/2)-25.5;

$$g := (r, \theta) \rightarrow r \sin\left(\frac{1}{2} \theta\right) - 25.5$$

> fsolve({f(r,theta), g(r,theta)}, {r,theta});
{ r = -26.92187661, theta = -2.488682382 }

> restart:
> f:=(r,theta)->r*theta-79;

$$f := (r, \theta) \rightarrow r \theta - 79$$

> g:=(r,theta)->r*sin(theta/2)-30.5;

$$g := (r, \theta) \rightarrow r \sin\left(\frac{1}{2} \theta\right) - 30.5$$

> fsolve({f(r,theta), g(r,theta)}, {r,theta});
{ theta = 2.426239787, r = 32.56067286 }

> restart:
> f:=(r,theta)->r*theta-99.5;

$$f := (r, \theta) \rightarrow r \theta - 99.5$$

> g:=(r,theta)->r*sin(theta/2)-38;

$$g := (r, \theta) \rightarrow r \sin\left(\frac{1}{2} \theta\right) - 38$$

> fsolve({f(r,theta), g(r,theta)}, {r,theta});
{ theta = 2.473821779, r = 40.22116744 }

> restart:
> f:=(r,theta)->r*theta-125;

$$f := (r, \theta) \rightarrow r \theta - 125$$

> g:=(r,theta)->r*sin(theta/2)-50;

$$g := (r, \theta) \rightarrow r \sin\left(\frac{1}{2} \theta\right) - 50$$

> fsolve({f(r,theta), g(r,theta)}, {r,theta});
{ r = 55.25581923, theta = 2.262205171 }

```

IV: Maple code for the partial angles to determine the placement of the vertical lines.

```

> restart ;
> alpha1 := (6 + (2/3)) / (25.42) ;
      α1 := .2622606871

> a := convert(alpha1, degrees) ;
      a := 47.20692368  $\frac{\text{degrees}}{\pi}$ 

> alpha2 := (7 + (1/3)) / (29.37) ;
      α2 := .2496878901

> convert(alpha2, degrees) ;
      44.94382022  $\frac{\text{degrees}}{\pi}$ 

> alpha3 := (8 + (2/3)) / (30.69) ;
      α3 := .2823938308

> convert(alpha3, degrees) ;
      50.83088954  $\frac{\text{degrees}}{\pi}$ 

> alpha4 := (10) / (32.85) ;
      α4 := .3044140030

> convert(alpha4, degrees) ;
      54.79452054  $\frac{\text{degrees}}{\pi}$ 

> alpha5 := (12) / (38.59) ;
      α5 := .3109613890

> convert(alpha5, degrees) ;
      55.97305002  $\frac{\text{degrees}}{\pi}$ 

> alpha6 := (15) / (45.45) ;
      α6 := .3300330033

> convert(alpha6, degrees) ;
      59.40594059  $\frac{\text{degrees}}{\pi}$ 

> alpha7 := (16) / (53.05) ;
      α7 := .3016022620

> convert(alpha7, degrees) ;
      54.28840716  $\frac{\text{degrees}}{\pi}$ 

```