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Mathematics, the Liberal Arts, and Slavish Devotions

J.D. Phillips
July 2, 2002
Chersonissos, Crete

Thank you for the opportunity to join you today.

As I began preparing my talk it occurred to me that the sorts of institutions and departments that we all come from are decidedly different, in fact, maybe even pair wise orthogonal. Those of you at, say, mid-sized regional universities are up to something markedly different with your undergraduates than we who are at liberal arts colleges are up to with our students. To say nothing of those of you at community colleges, big research universities, polytechnic institutions, and so on.

So I should warn you that my remarks are grounded in a particular understanding of undergraduate education that is part of a very old liberal arts tradition, a tradition that, alas, has fallen on hard times recently, especially, ironically enough, in mathematics departments. And by “liberal arts” education, I should point out that I do not mean simple fealty to a hodge-podge of introductory “general education” courses in a random assemblage of discipline-based departments (which in mathematics departments generally means something like “math for poets” courses), or by the incantation of tired slogans like “critical thinking”, both of which are industry standards in American higher education. I mean something rather different, which will hopefully begin to emerge, slowly, in the body of my talk.

The ancient Greeks (I can mention them here in Crete, can't I?) understood the Liberal Arts *to be* Mathematics, more or less, *viz*. what came to be known as their *Quadrivium*: arithmetic, applied arithmetic (harmony), geometry, and applied geometry (astronomy). These arts are characterized by the signal fact that, unlike, say politics, their content can be *learned*—recall that the Greek root of the word 'mathematics' translates roughly as "that which is learnable". And so to the ancients, mathematics was paradigmatic of all learning, and hence, central to any liberal education.

But this sort of education—an education in the liberal arts—is (and I will argue *necessarily* is) burdened with a number of conflicting tensions. For instance, there is the tension between the inexorable press of politics on *all* education (from Meno asking Socrates "can virtue be taught" to "education for citizenship" in our own time—to say nothing of programs, from the NSF to the DOD, that reward achievement in mathematics based exclusively on mathematics' ability to serve political aims) against the freedom a *liberal arts* education both requires and aims for. By the way, understanding the tension like this—teaching virtue versus a liberal arts education's required (and aimed for) freedom—helps explain why conservative moralists, politically correct professors, and religious fundamentalists are up to the same thing, more or less, in education: straightening us out morally; teaching virtue. And so we see that, at least as professors, the dogmatic right and the tolerant left are simply obverse images of the same worn coin. Their self-delusion about the deep-seated desire to remain morally unchallenged is coextensive. It is a self-delusion which *itself* challenges the freedom which is the necessary backdrop for a liberal arts education; the possibility of genuine inquiry is simply passed over in silence.

And this presages another tension, specific to mathematics. You see, unlike many of the humanities disciplines, and even some of the science disciplines, what we do as *mathematicians* bears little resemblance to what we do as mathematics *teachers*.

Mathematicians pursue proof. But when we *teach* mathematics, we do not pursue proof. Nor, generally, do we direct our students toward the pursuit of proof. As mathematics teachers, our energies are directed elsewhere, toward what a friend of mine calls the assertive pursuit of scientifically intentioned problem sets. More on this later.

Compare this with the situation in other disciplines. One of my philosopher friends, for instance, is a Plato scholar. So not only is he engaged, at least in part, in scholarly research as he prepares for class each day by simply reading The Republic, his very act of teaching is a philosophical act—I'm thinking here of Socrates the seducer and midwife.

I know of no area of current mathematics research that is regularly a part of the undergraduate mathematics curriculum. Still, in spite of this, I suppose that, strictly speaking, the pursuit of proof is still *possible* in undergraduate mathematics. But, alas, this possibility actually realizes itself only in those rare activities that are entirely marginal in the landscape of undergraduate mathematics education. More on this—the ultimate goal of this talk—shortly. First, though, I should remind you of what actually goes on in undergraduate mathematics.

Mostly, what goes on is conditioning, as above, via the pursuit of problem sets. Jacob Klein's colleague, Eva Brann, describes this activity as the asking and answering of sham questions. Of course, a genuine question is one that the questioner doesn't know the answer to; in fact, *a genuine question is nothing more than the desire for an answer*. It leaves uncertainty in its wake, and generates genuine conversation, in which none of the participants knows the answer, but all desire it. A sham question—a problem set, for instance—on the other hand, is one that the questioner already knows the answer to, and hence, cares very little about; for instance, from a typical problem set in the calculus, "What is the derivative of the hyperbolic sine function?" Or this from an algebra class, "Show that all groups of exponent 2 are abelian." And this—namely using problem sets (sham questions) to condition—is what mathematicians do when we teach. It bears no resemblance to what we do as mathematicians. So there is a tension, then, between our two primary activities—doing mathematics, and teaching mathematics.

By the way, as an aside, I note that this tension is present even in very sophisticated classes at the graduate level. For instance, consider John Thompson's celebrated theorem: "A finite group with a fixed point free automorphism of prime order is nilpotent." Now, it's certainly true that most of us couldn't prove this theorem on the spot. But it *is* true that we could all simply look up the proof and follow it. More importantly, we all know it to be true; it is simply *not* the object of our desires—you don't desire to know what you already know. And in this sense, the only way you can ask about something as sophisticated as even Thompson's theorem, is as a problem set, not as a genuine question.

I don't know how to resolve this tension. In fact, it's probably not resolvable, I think, because sham questions are a necessary part of mathematics education, and they necessarily interfere with genuine questions. I do, however, think we can respond constructively to it.

Of course, we all know one way to respond to this tension—student research. But since we all know this, I won't spend any time commenting on it here.

Another way to respond constructively to this tension though, is with so-called core or seminal texts. And this response is so uncommon as to be almost entirely marginal. So it seems like a good place to remind you of Abel's famous dictum:

> "It appears to me that if one wants to make progress
> in mathematics one should study the masters."

I should qualify this. Recall from above that mathematics is paradigmatic of all learning. And so built in to every mathematics class is the possibility that bracing and accompanying all of the other efforts in the class, will be the purposeful attendance to nothing less than learning itself, (re)learning how to learn. *This is hard to do if you're just pursuing problem sets.* But it's not hard to do if you're reading, say, The Elements.

Reading The Elements demands the student's careful attention to the details of actually working through the demonstrations. And this, in turn, opens up the conversation to the sorts of questions we're all familiar with. Why will those two circles intersect? Why is there only one triangle with this property? And so on. Of course, most mathematics textbooks and problem sets allow for these sorts of questions, but unfortunately most textbooks and problem sets answer these questions before students have a chance to actually struggle with them for themselves.

But The Elements, as you know, inevitably generates another sort of genuine question—the question about the deductive apparatus and the axiomatic system itself. What is the difference between a postulate and a common notion? What *is* a point? Do points *exist*? Are the propositions *true*? Can you tinker with the axioms and still prove some of the propositions? If so, what recommends one set of axioms over another? I suppose good textbooks and problem sets can generate these sorts of questions, but it is rare.

Ultimately, though, if you read The Elements attentively, you might get the feeling that Euclid is up to something more than just presenting "results" (as in a typical mathematics textbook), that he is actually working *toward* something—The prime number theorem? A commentary on the Platonic solids? An insight into the nature of logic and deduction? These are genuine questions. They demand genuine conversation. And this, in turn, stokes passion. [As an aside, I should note that it is this passion, by the way—the passion to find the truth about things, to satisfy the simple desire to know—that drives us as mathematicians, not the assigning and grading of problem sets.] And finally, all of this invites reflection on the nature of human learning. To say nothing of giving the student an authentic taste—not contrived—of what we actually *do* as mathematicians. Textbooks and problem sets—even the seemingly "sophisticated" activity of working through Thompson's nilpotence theorem—don't do this. Textbooks and problem sets *can't* do this. And this seems like a good place to note that the root of the word "sophisticated" is the same as the root of the word "sophistry".

So seminal and core mathematics texts generate precisely the sort of erotic uncertainty necessary for a genuine conversation that stokes the passion for mathematical discovery that drives us as mathematicians. This conversation, by the way, is part of a tradition that fosters habituation toward truth and beauty by liberating students from slavish devotion to their own parochial opinions (opinions formed against the backdrop of nothing more than tribalism and bloodlines). Seminal mathematics texts, not just the pursuit of problem sets, are particularly well suited to this. This approach recognizes the "skills" and "competencies"—developed via the pursuit of problem sets—as means, subordinate to other ends, for instance, wisdom and freedom. And as above, the slavish devotion to technique and training that is part of the pursuit of problem sets mistakes these skills and competencies as ends, and thereby expresses contempt for wisdom and freedom. It is self-undermining insofar as the wisdom required to see through all this is grounded in mathematics. And, by the way, it encourages us to flatter ourselves by obsessively “assessing student learning outcomes”, that is, by teaching (and valuing) primarily what is easy to assess.

I don’t mean to say that the pursuit of problem sets shouldn’t be part of a mathematics education. In fact, it must be; remember, I claimed that the tension in mathematics is irresolvable. I *am* suggesting, though, that this pursuit can be tempered—and again, only in an entirely marginal way.

Thank you for your kind attention