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Humanistic Mathematics As Mathematics for All

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The phrase, 'Mathematics for All', has a certain ambiguity. It may be taken as an exhortation -- 'Mathematics for all!', or as an indication of a special *kind* of mathematics, 'mathematics-for-all'. These two ways of reading the phrase can be translated into two closely related questions: 1) *Should* mathematics really be for all? and 2) Are there aspects of mathematics, or parts of mathematics, truly *appropriate* for all? Since one question inevitably leads to the other, the ambiguity of 'mathematics for all' reveals only that there are two sides to this one coin. We shall begin, then, with the first question and conclude with the second.

I.

Asking whether mathematics *should* be taught to everyone presupposes that mathematics is something everyone is *able* to do. As we all too painfully know, there are plenty of people happy to confess that mathematics was something they *never* could do. There are also, at the other extreme, those sanguine pedagogues who earnestly believe that one can learn anything if only properly taught. The truth, of course, lies somewhere in between. For no one can doubt that people have limits. But, people also have potential, and they can learn more than they think and they have learned more than often they are willing to say. Thus, defining the limits of students' ability, finding ways to teach students what they can learn, and developing methods of evaluation that reveal what students have learned are, without a doubt, legitimate goals for educators, and, in fact, are the mainstay for most educational researchers. In this piece, however, I am advancing this concern: that the question of whether mathematics should be taught to everyone should not be dismissed by the argument of "impregnable barrier of inability," and that, therefore, it is at least *reasonable* to ask whether, indeed, there is some urgency to the idea that mathematics should be for all.

To begin, one should recognize that mathematics was not *always* considered an essential part of the school curriculum. In the fifteenth and sixteenth centuries, for example, manyhumanist thinkers argued against the importance of mathematics as a basic component in general education. Ironically, their reason had to do with the relevance of mathematics to human life. As Geoffery Howson says,

> "Indeed, mathematics held little appeal for the humanists. Vives was something of an exception, although even he had qualifications to as a component of general education. Erasmus considered it a subject which those sufficiently far advanced might 'taste', but, in general, the humanists were not attracted by a purely abstract, as geometry. The effect of the humanist movement on the school curriculum can be seen in the timetable of a typical Elizabethan grammar school."[1]

Even at the turn of the nineteen century, one finds in Germany, for example, that "...mathematics was taught as a marginal subject at best, given the near-monopolistic status of the classical languages."[2] Indeed, one tends to forget that elementary school was until fairly recently still commonly known as "grammar school," betraying its principal traditional function of teaching Latin and Greek grammar.

One might expect that among the Greeks, whose mathematical accomplishments were so great and so much a part of everything we think of as Greek, there would be an unequivocal recognition of the importance of mathematical education for all. Yet, when Protagoras describes to Socrates the kind of education fit for the children of the *polis*, he says that "...when the boys have learned their letters and are ready to understand the written word as formerly the spoken, they set the works of good poets before them on their desks to read and make them learn them by heart, poems containing much admonition and many stories, eulogies, and panegyrics of the good men of old, so that the child may be inspired to imitate them and long to be like them."[3] As for the other aspects of elementary education, Protagoras speaks of music[4] by which youth will "...become more civilized, more balanced, and better adjusted in themselves and so more capable in whatever they say or do...," and of gymnastics "so that a good mind may have a good body to serve it, and no one be forced by physical weakness to play the coward in war and other ordeals."[5] Protagoras' program is, more or less, Socrates' own initial proposal, in Books II and III of the *Republic*, for the education of children who are to become the guardians of the ideal state. Moreover, later in the dialogue, when Socrates reviews this initial proposal with Glaucon to find out what might be missing, Glaucon says, "...what other study is left apart from music, gymnastics, and the arts?"[6] Ian Mueller makes the obvious observation: "In neither case is there a mention of anything mathematical,"[7] and goes on to say that, despite an apparent common ability to perform calculations such as 2000/10 and 3x700,[8] "...it appears that the average Athenian citizen knew remarkable little arithmetic from our point of view and that he did not acquire his knowledge in school. But even if he did learn arithmetic at school, we have no right to assume he learned any geometry, astronomy, or music theory, despite the fact that we have plenty of evidence associating these subjects with the intellectual heights of fifth-century culture."[9]

The expectation one has that the Greeks should promote universal mathematics education is, of course, not *completely* unfounded. In the passage quoted above from the *Republic*, Plato, in the voice of Socrates, points out to Glaucon that the guardians' education lacks training in the mathematical sciences, arithmetic, geometry, and astronomy. In the case of geometry, Glaucon recognizes the practical importance of geometry, especially in the conduct of war: "For in dealing with encampments and the occupation of strong places and the bringing of troops into column and line and all the other formations of an army in actual battle and on the march, an officer who had studied geometry would be a very different person from what he would be if he had not."[10] Socrates does not deny this practical importance, but says that these applications only require "...a slight modicum of geometry and calculation..."; Socrates thinks that the study of geometry (as he does the other mathematical sciences) turns one's soul to the true being of things and eventually to the idea of the good itself.[11] One must keep in mind, however, that even here the dialogue is referring to the education of an *elite*, the education of the *guardians*, the future rulers of the ideal state, not the education of all. Plato himself perhaps recognizes the desirability of all citizens of a republic to have some true understanding of mathematics, but he has profound reservations about the ability to achieve such a goal. This is clear from a well known passage in the *Laws*, in which the Athenian says to Clinias, the Cretan, that "...there are, of course, three subjects for the freeborn still to study. Ciphering and arithmetic make one subject; mensuration, linear, superficial,, and solid, taken as one single study, forms a second; the third is the true relations of the planetary orbits to one another. The elaborate prosecution of all these studies into their minute details is not for the masses but for a select few...For the multitude it will be proper to learn so much of the matter as is indispensable, and as it may truly be said to be a disgrace to the common man not to know, though it would be hard, or actually impossible, to pursue the research into minute detail."[12]

II.

Plato's discussion of the importance of mathematics education is, as we have just said, **was** directed towards the future leaders of Athens or the select few who become guardians of the ideal state. Democratic traditions have developed since then, and we now tend to see *all* children as future leaders, or, at least, this is our rhetoric in speaking about such things.[13] This is why, perhaps, the justifications given for mathematics education *for an elite* within the text of the Platonic dialogues, and two in particular,[14] are still familiar *mutatis mutandis* as justifications of mathematics *for all*. Thus, most arguments supporting mathematics for all are elaborations of two basic claims: 1) that mathematics is useful, and 2) that mathematics leads to a way of thinking that allows one to understand other things beyond mathematics itself. Not much needs to be said about the first. As for the second, Plato thought that mathematics could lead to thinking about things in themselves, for mathematical objects were, for Plato, intermediaries between the ever-changing things of this world and the world of completely unchanging and intelligible forms where one can find the true source of human knowledge and, more importantly, of human justice.[15] One usually hears the second claim in a form in which Plato's emphasis on the *objects* of mathematical thinking has been redirected to the character of mathematical *thinking* itself, its precision and its rigor.[16] The stress on the importance of mathematics as a problem-solving activity, for example, is one version of this claim. Indeed, the interest in problem-solving would not be nearly so extensive if one thought that problem-solving in mathematics had nothing to do with problem-solving in other aspects of our lives.

The two claims listed above, that mathematics is useful and that mathematics aids thinking in general, certainly make mathematics a worthy subject, but are they truly arguments showing that mathematics *should* be taught to everyone? They are, of course, to the extent one considers schools as *democratic* institutions, that is, as institutions that *serve* democracy. In general, the centrality of education in determining a successful democracy is common to almost all thinking about democracy. In Thomas Jefferson's thought, for example, one cannot separate the concern for the health of the republic and the education of its citizens.[17] Thus, in the preamble to his "Bill for the More General Diffusion of Knowledge" (1779), Jefferson writes, "Whereas it appeareth that however certain forms of government are better calculated than others to protect individuals in the free exercise of their natural rights, and are at the same time themselves better guarded against degeneracy, yet experience hath shewn, that even under the best forms, those entrusted with power have, in time, and by slow operations, perverted it into tyranny; and it is believed that the most effectual means of preventing this would be, to illuminate, as far as practicable, the minds of the people at large, and more especially to give them knowledge of those facts, which history exhibiteth, that, possessed thereby of the experience of other ages and countries, they may be enabled to know ambition under all its shapes, and prompt to exert their natural powers to defeat its purposes."[18] In Jefferson's vision of education, there was, moreover, no question as to the importance of mathematics and the sciences in general for this end. Accordingly, he says that in the grammar schools (roughly equivalent to our high-school) "...shall be taught the Latin and Greek languages, English grammar, geography, *and the higher part of numerical arithmetick, to wit, vulgar and decimal fractions, and the extraction of the square and cube roots* [emphasis added]."[19] Jefferson's sense that an enlightened polity means, among other things, a polity that is no stranger to scientific thought is clear also from his proposed program of studies for the University of Virginia, which included under the heading "Mathematics, pure," "Algebra; Fluxions [calculus]; Geometry, Elementary, Transcendental; Architecture [!], Military, Naval," and under the heading "Physico-Mathematics," "Mechanics; Statics; Dynamics; Pneumatics; Acoustics; Optics; Astronomy; Geography."[20] Indeed, these make up almost a third of the total subjects to be taught at the university.

That mathematics is both useful and develops clear analytical thinking, needless to say, fit well into the Jeffersonian scheme. Obviously, clear analytic thinking, the ability to form arguments and counter-arguments, the power to discern sham reasoning, and the habit of being precise about terms form a firm defense against the perversion of a good government into a tyranny. The democratic implication of mathematics' utility, on the other hand, is evident not only in its allowing a wider range of people to make a positive contribution to society, but also in its removing the barriers which might prevent people taking up more sophisticated and lucrative professions (particularly, one might add, professions to which political power is attached). Jefferson makes the last point plain when he writes that "[At the district schools] might be taught English grammar, the higher branches of numerical arithmetic, the geometry of straight lines and of the circle, the elements of navigation, and geography to a sufficient degree, and thus afford the greater numbers the means of being qualified for the various vocations of life, needing more instruction than merely menial or praedial labor..."[21] In one of its "modern" incarnations, the argument from utility is the "technology argument" – we live in a technological society, mathematics is indispensable in technology, therefore, the mathematics is essential to success in society. For many, the "technology argument" is *the* most potent argument implying the endorsement of mathematics education for all.

Yet, despite the plausibility of the two arguments given so far for mathematics for all, something about them, nevertheless, does not ring true. To begin, consider the arguments claiming that it is the *usefulness* of mathematics that gives mathematics for all its urgency. These have a wide range. At the crudest level -- though, I must sadly confess, the argument here is a very common one – one makes the case that acceptance into universities or other institutions or organizations is barred for anyone who cannot demonstrate some proficiency in mathematics, proficiency that is usually determined by an examination or school grades. The kindest comment one can make regarding the "institutional utility" of mathematics is that it begs the question. Anyone who genuinely cares about mathematics, however, should further object that the argument to "institutional utility," in fact, discounts any deeper use of mathematics beyond its being a mere device for selection, a capacity for which it would be an arbitrary choice except for the fact that most people find mathematics hard. Though this is, I admit, *a* use, and one *does* use such words as "device" or "tool" to describe mathematics in this very instrumental role; it is not really a use of mathematics *per se* since it is related, at best, only tangentially to its content.

As for the "technology argument," there is no doubt that mathematics has had a decisive role in the development of the machines and systems that we have come so much to depend on, and there is no sign that the importance of mathematics here will, in the future, in any way diminish. Thus, it is *not* an arbitrary policy that future computer engineers be required to have a relatively deep understanding of, say, combinatorics, probability, and number theory. But what of the *users* of technology? After all, while the number of people who actually devise some new technology is relatively small, there is hardly a person, in the developed world at least, who does not use fairly advanced technology almost every day. For users of technology, however, the importance of knowing what *makers* of technology know is not at all clear. Indeed, it is almost in the very nature of technology that knowledge of the inner workings of machines becomes progressively irrelevant for those who only use machines. Thus, as the writer Robert O'Brien says,

> "Increasingly, engineers are designing people out of the machine process. The purposes are practical: to eliminate human error, fatigue, boredom; to introduce faster, safer, more economical and consistent methods of production; to fulfill the incessant human striving for abundance. And even as their designers toil, the machines themselves seem to be groping more and more toward self-sufficiency, impelled by the same blind will with which a vine climbs toward the sun. The telephone system that began with hand cranks and 'Hello' girls is today a marvel of automatic control. In place of the philosophic elevator man who joked about his 'ups and downs' is a panel of plastic push buttons."[22]

As a technology progresses, it seems, one needs to be less and less of a specialist to use it; the development of a technology is marked by increasing "user-friendliness." This means that mere *functioning* in a technological world does not depend on knowing its technological foundations. However, functioning *intelligently* in the technological world does, I think, involve an understanding of its conceptual foundations; one should know, for example, the difference between the "precision" of 3.1415926 and the "precision" of π. This implies an approach which is more reflective and less attached to immediate applications; one might conceive such a reflective approach as still directed towards mathematics' usefulness but, it must be granted, only in an equivocal sense. Similar things could be said regarding mathematics' usefulness in the exact sciences.

Mathematics as a path to general types of thinking is, in a sense, also an assertion of the usefulness of mathematics, and it is a very strong one at that. With that, however, it is also open to the same criticism regarding the "institutional utility" of mathematics, namely, that thinking of mathematics in this light makes the content of mathematics of only secondary importance, if important at all. But, beyond that criticism, one wonders about the truth of the claim itself. Consider the most common version of the claim: namely, that "A strong emphasis on mathematical concepts and understandings ... supports the development of problem solving."[23] The first question one should ask is, What kind of problems are meant here? As I mentioned above, the very strength of the claim comes from its referring to problems *other* than purely mathematical ones. Certainly, Pólya, to whom we owe this concentration on problem solving in mathematics education probably more than anyone else, saw his work on *heuristics* as applicable to areas other than mathematics, for he saw problem solving as, in some way, central to all thought. In fact, "The tenet underlying all of Pólya's writing and teaching on problem solving was that if taught and learned appropriately, mathematics improves the mind and implants good habits of thought."[24] But Pólya began with mathematical problems, his books concentrate on mathematical problems, and one suspects that his many beautiful insights relate most properly only to mathematical problems or problems that can be framed as mathematical problems. The sad fact, though, is that, despite the optimism born in the 16th and 17th centuries that *all* problems would eventually be open to a mathematical-like treatment,[25] most real-life problems of real significance still remain outside the realm of mathematics. This is because such real-life problems involve, more often than not, a judgment of ends rather than a choice of means. Thus, Eva T. H. Brann writes:

> "The presumption in coming to school to learn to solve human problems is that human affairs are amenable to something like an algebraic treatment, that they can be clearly formulated in terms of knowns and unknowns, and that they maybe foretold as well as resolved by an application of the proper technique. It is a faith encouraged by certain academics who, though no less mere theoreticians than any of their colleagues, want to invest their subject with irresistible urgency. Be that as it may, the mere formulation of such a problem represents...so enormous a rational determination that students need a thorough intellectual foundation to give them some critical independence with respect to the whole genre of theories constructed for practical applications."[26]

The "thorough intellectual foundation" Brann speaks about clearly contains some thoughtful exposure to mathematics, but it is far from clear that mathematics has any *dominant* part here; even Jefferson, whom we cited as a supporter of scientific education, still saw *history*, and not mathematics, as probably the most important of one's studies.[27]

III.

There is no denying that, while most problems of deep concern to citizens and policy makers alike are *not* amenable to the type of thinking one applies in mathematical problems, still the range of questions for which mathematical reasoning is relevant is very great. This is certainly true in the exact sciences where attempts to *remove* the mathematical presentation usually result in an oversimplification at best and a complete distortion of the facts at worst.[28] Therefore, I do not disagree that for a true understanding of the exact sciences, at least, one needs a firm grasp of its mathematical underpinning. The question is, however, as it was in my remarks about technology, whether this is an appropriate argument for mathematics *for all*. What shall we say to the student who has no interest whatsoever in physics or computer science? Here, I would like to return to the passage from Plato's *Laws* quoted at the end of the first section. For besides the two arguments we derived from that and the passage from the *Republic*, there is, I think, another more subtle argument in the former.

At the end of the passage from the *Laws*, the Athenian says that it is a *disgrace* for the citizens not to know the details of mathematical researches. The sort of disgrace I think Plato has in mind can be illustrated by the apocryphal story told about Plato and the doubling of the cube: "Eratosthenes in his work entitled *Platonicus* relates that, when the god proclaimed to the Delians by the oracle that, if they would get rid of a plague, they should construct an altar double of the existing one, their craftsmen fell into great perplexity in their efforts to discover how a solid could be made double of a (similar) solid; they therefore went to ask Plato about it, and he replied that the oracle meant, not that the god wanted an altar of double the size, but that he wished, in setting them the task, *to shame the Greeks* [emphasis added] for their neglect of mathematics and their contempt for geometry."[29] The twist in the story, of course, is that the shame the Delians were to feel was *not* that from a sense of ineptness before a practical task, although they were led to believe that it *was* a practical task they were to carry out, and thought, accordingly, that it was a job for their *craftsmen*.[30] The Delians' shame was in not knowing the deeper foundations *of their own altar*; their contempt for geometry meant that they were not, in some way, in full possession of their own world. The lesson of the story, then, is that knowing mathematics and not scorning geometry is important at bottom not because it allows one to solve practical problems, though it may do this, but because it allows one more fully to belong in one's own world, and, therefore, the neglect of mathematics is a disgrace not for the craftsman alone among the Delians, but *for all* the Delians.

The need to belong, I think, provides a sound motivation for *mathematics for all*. The sense of belonging that one aims for, however, applies to the greater world, the natural world, the world that science contemplates, as well as the more provincial world of one's own culture.

I want to end this piece with some remarks about the latter; these remarks also touch on the kind of mathematics appropriate for all, the second side of the coin with which this essay began. To start, I do not want to consider the presence of mathematical thinking in *different* cultures, though this has in recent years become a lively area of research with many impressive findings. I only want to stress that mathematics has a central place in *western* culture, and, therefore, mathematics is as much a part of cultural education as the study of literature is. In the English speaking world, one reads Shakespeare not because one aims to be a Shakespearean scholar but, to borrow Plato's phrase, because it is a *disgrace* for an English speaker never once to have savored a line of Shakespeare's English. It should be as natural to expect someone in the western world to have savored Euclid's proof of Pythagoras' theorem. Yet, this is not the case. One tends to see the reading of works by mathematicians and scientists as a somewhat esoteric activity reserved for historians, biographers and archivists. There is, in this way, a kind of double standard in education. For at any level of education it is taken for granted that the study of literature is the study of the *works* of literature; hardly anyone doubts the inadequacy of a mere textbook approach to literature in which the stories of Shakespeare's tragedies are merely told or the themes in Frost's poetry merely listed. In *mathematics* education, however, the textbook approach is the *only* approach. Accordingly, our students end up, more often than not, only with a textbook understanding of the subject; they are not left with the sense of having faced great ideas of great thinkers, a sense which can truly enrich and inspire.[31] Emerson said that "Activity is contagious. Looking where others look, and conversing with the same things, we catch the charm which lured them...Talk much with any man of vigorous mind, and we acquire very fast the habit of looking at things in the same light, and on each occurrence we anticipate his thought."[32] One seems painfully aware of this in literature education but oblivious to it in mathematics education.

The approach to learning mathematics by reading *works* of mathematics[33] would be misconstrued if it were taken as a purely *historical* approach. It involves, rather, the recognition that living mathematics is a product of the imagination of real people.[34] I mean this quite seriously: mathematics is a fundamentally human endeavor, as much as literature, and must be studied in this way; it is not so much a matter of knowing Euclid did this or Gauss did that, but that this *thought* is in Euclid this and this in Gauss. Tymoczko makes a distinction similar to the one I am trying to make when he says, "It is mathematics with a human face because there is no mathematical discipline without a human face. Stories of mathematicians are 'color'. It is interesting that Tartaglia was a stammerer who extracted a promise from Cardano. But stories about what historical individuals say when they looked on the mathematical universe at historical points of time are not color. They are mathematics. No one can learn mathematics without being inculcated into this tradition."[35] One might say that this is *precisely* the historical approach, and perhaps I would say so too if it were not so common to think of the historical approach in the pejorative sense of being about dead ideas. I am not proposing 'mathematics for all' consists in learning dead ideas. One thinks one can ignore great works such as Euclid's *Elements* or Descartes' *La Géométrie* as being no longer relevant, or Shakespeare's plays as no longer current, but this is like thinking that one can come into the middle of a conversation and still really know what is going on. Indeed, being engaged in the reading of such great works is, as Robert Hutchins put it, to be engaged in a "great conversation."[36] Thus, while it does not seem to me an injustice not to provide one with technical training for a scientific profession that one does not really want, it *does* seem to me an injustice to keep one out of this "great conversation," for this is something truly belonging to *everyone* in society and is behind any sense of belonging in one's culture. What allows one to participate fully in this "conversation," therefore, should be our guide in thinking about 'mathematics for all', both in its justification and its content. And it is the humanizing participation in the 'great conversation' that makes 'mathematics for all' deeply humanistic mathematics.

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[1]Geoffrey Howson, *A History of Mathematical Education in England*, Cambridge: Cambridge University Press, 1982, p.9. One hears, I think, an echoes of the humanists' complaint regarding mathematics whenever our students challenge us with the question, "What *good* is all this mathematics?," by which they do not mean what are its applications in science and technology (*that* they have been told, *ad nauseam*), but what is its relevance to their everyday troubles and pleasures, to passions, and to genuinely human dilemmas.

[2] Gert Schubring, "Germany to 1933" in the *Companion Encyclopedia of the History and Philosophy of the Mathematical Sciences,* I. Grattan-Guinness ed., Vol. II, p.1443.

[3] Plato, *Protagoras*, 325e-326a.

[4] Music must be understood here in the wider sense of music *and* poetry. Later, of course, the study music, that is, of music theory became a strictly mathematical science associated, in particular, with certain aspects of the theory of proportion. Indeed, Euclid wrote a book about music theory, as did Ptolemy, and, by the Middle Ages, music became one of the four mathematical sciences included in the quadrivium (see Theodore C. Karp "Music," in *The Seven Liberal Arts in the Middle Ages*, David L. Wagner, ed. (Bloomington: Indian University Press, 1983)).

[5] *Ibid*., 326b-c.

[6] *Republic*, 522b.

[7] Ian Mueller, "Mathematics and Education: Some Notes on the Platonic Program," in *ΠΕΡΙ ΤΩΝ ΜΑΘΗΜΑΤΩΝ*, special edition of *Apeiron*, vol.XXIV, number 4, December 1991, p.87. The absence of mathematics in the Greek curriculum is evident also in Aristotle's account of this in the *Politics*: "The customary branches of education are in number four; they are—(1) reading and writing, (2) gymnastic exercises, (3) music, to which is sometimes added (4) drawing. Of these, reading and writing and drawing are regarded as useful for the purposes of life in a variety of ways, and gymnastic exercises are thought to infuse courage. Concerning music a doubt may be raised—in our own day most men cultivate it for the sake of pleasure, but originally it was included in education, because nature

herself, as has been often said, requires that we should be able, not only to work well, but to use leisure (*scholē*) well..." (VIII, 1337b). Ernest Barker attributes the absence of scientific subjects in Aristotle's scheme to the purely political goal of education of making good citizens (a point we shall consider soon in our discussion of Jefferson's views). Thus he writes: "Not only does [Aristotle] regard the State, rather than the individual, as the primary object of attention; but he also regards character rather than knowledge as the end to be sought, and will rather than intelligence as the subject to be trained and developed. This being the aim of education in Aristotle's conception, there will result certain differences between the means of education which he prefers to use, and those which we employ. Working on the intelligence, we use the means that influence the development of intelligence, the subtleties of grammar, the abstractions of mathematics: working on the will, he lays stress upon those influences which are calculated to mould the will insensibly, such as the fascination of noble music or the attraction of great literature" (*The Political Thought of Plato and Aristotle* (New York: Dover Publications, 1959), p.424).

[8] Mueller is relying here on passages from Aristophanes' *Wasps* and Plato's *Hippias Minor*, respectively.

[9] Mueller, *op. cit*., p.88. Thomas Heath thinks that arithmetic *was* included in children's education at an early stage. He says, "The main subjects [of elementary education] were letters (reading and writing followed by dictation and the study of literature), music and gymnastics; *but there is no reasonable doubt that practical arithmetic (in our sense), including weights and measures, was taught along with these subjects* [emphasis added]" (*A History of Greek Mathematics* (Oxford: The Clarendon Press, 1921, reprinted by Dover Publications, Inc., 1981), I, 18-19). However, the point is that, whether or not mathematics was included in the basic education of Athenian youth in fact, neither Protagoras nor Glaucon see it as an *obvious enough* component of elementary education to mention it in their descriptions; for them, it seems, "the three R's" of education were Reading, Rhythm, and wRestling!

[10] *Rep*.,326d.

[11] *Ibid*., 326eff.

[12] Plato, *Laws*, VII, 817e-818b. I mention, incidentally, that this Platonic dialogue, Plato's last dialogue and the only one in which Socrates neither appears nor is mentioned, also contains interesting observations about the use of games and play in the teaching of mathematics (VII, 819b-d).

[13] This is, naturally, an oversimplification to some extent. For one still considers leaders a *chosen* few. At the same time, we do see the reservoir from which we choose our leaders as much more extensive than one did in past times and we choose much later. Moreover, whereas Plato had in mind leaders in the narrow political sense, that is, leaders *as rulers*, we see leadership as a much more varied thing, and, accordingly, speak of not only political leaders, but also of intellectual leaders, spiritual leaders, moral leaders, and so on. One can see this wider view in Robert Ulich's definition of leadership in his essay, "Leadership and Education" (in *Vital Issues in American Education*, Alice and Lester D. Crow, eds. (New York: Bantam Books, 1963)): "We define a leader as a person capable of inspiring other people with a desire to follow his direction and example for the achievement of purposes considered desirable by those who represent the best of the conscience and consciousness of their community. Since a community is a part of humanity, and since humanity, despite all changes, has also an oral and written tradition, leaders do not merely come and then disappear from the eyes of mankind.. Rather they write the great documents, and are themselves the great monuments, in the evolution of human creativeness" (p.25).

[14] There is a third, more subtle, reason given in the passage from the *Laws* which I shall return to later.

[15] See, for example, *Rep*., 509a-511e. For a general discussion of this passage as well as Plato's other ideas about mathematics, see Robert S. Brumbaugh, *Plato's Mathematical Imagination: The Mathematical Passages in the Dialogues and Their Interpretation* (Bloomington: Indiana University Press).

[16] In other words, the use of mathematics is not so much in the definite knowledge of things it provides as it is in the way it trains one's thinking in general. This training the mind in some general way by all studies, as well as mathematics, of course, is really the theme of Bacon's essay "Of Studies." For Bacon, mathematics, in particular, makes one subtle and helps to make thought directed: "So if a man's wit be wandering, let him study the mathematics; for in demonstrations, if his wit be called away never so little, he must begin again." One finds this perspective also in Locke who writes in *Of the Conduct of the Understanding*: "I have before mentioned mathematics, wherein algebra gives new helps and views to the understanding. If I propose these, it is not, as I said, to make every man a thorough mathematician or a deep algebraist; but yet I think the study of them is of infinite use even to grown men" (quoted in Eva T. H. Brann, *Paradoxes of Education in a Republic* (Chicago: Chicago University Press, 1979), p.23). In more current material, one can see this view of mathematical thinking as something relevant to *other* kinds of thinking in, for example, the first three standards of the *NCTM Mathematics Standards* (1989): "Mathematics *as* problem solving," "Mathematics *as* communication," "Mathematics *as* reasoning [emphasis added]."

[17] See Gordon C. Lee, "Learning and Liberty: the Jeffersonian Tradition in Education," in *Crusade Against Ignorance: Thomas Jefferson on Education*, Gordon C. Lee, ed. (New York: Teachers College Press, 1967). In this connection Bernard Mayo, writes, "Prominent in this general reformation was [Jefferson's] plan for a broad system of public education, embracing free elementary schools, academies, and a university, with provision for the education of youths of genius at the public expense. This was a subject dear to Jefferson's heart. Throughout his life he was to 'preach a crusade against ignorance,' stressing the vital importance of general education. There could be no surer foundation for the preservation of the people's freedom and happiness. Only an educated people could understand their rights, maintain them, and provide for the successful functioning of a democratic republic, in which 'the influence over government must be shared by all the people'" (B. Mayo, *Jefferson Himself: The Personal Narrative of a Many-Sided American* (Charlottesville, Virginia: The University Press of Virginia, 1942), p.76).

[18] In Lee, *op.cit*., p.83.

[19] *Ibid*, pp.89-90.

[20] Contained in the "Report of the Commissioners Appointed to Fix the Site of the University of Virginia, &c." (in Lee, p.121).

[21] *Ibid*., p123.

[22] Robert O'Brien, *Machines* (New York: Time Incorporated, 1964), pp.167-168.

[23] *NCTM (1989) Standards for Grades K-4*, assumption 1.

[24] Harold and Loretta Taylor, *George Pólya: Master of Discovery*, (Palo Alto, California: Dale Seymour Publications, 1993), pp.83-84. Pólya himself says in the preface to his book *How to Solve It* (Princeton: Princeton University Press, 1945) that "Although the present book pays special attention to the requirements of students and teachers of mathematics, it should interest anybody concerned with the ways and means of invention and discovery. Such interest may be more widespread than one would assume without reflection" (p.vi).

[25] I have in mind, first of all, Leibniz who dreamed of a language that would turn all inquiries in mathematical problems that could be solved by calculation: "...if we could find characters or signs appropriate for expressing all our thoughts as definitely and as exactly as arithmetic expresses numbers or geometric analysis expresses lines, we could in all subjects *in so far as they are amenable to reasoning* accomplish what is done in Arithmetic and Geometry.

"For all inquires which depend on reasoning would be performed by the transposition of characters and by a kind of calculus, which would immediately facilitate the discovery of beautiful results. For we should not have to break our heads as much as is necessary today, and yet we should be sure of accomplishing everything the given facts allow.

"Moreover, we should be able to convince the world what we should have found or concluded, since it would be easy to verify the calculation either by doing it over or by trying tests similar to that of casting out nines in arithmetic. And if someone would doubt my results, I should say to him: 'Let us calculate, Sir,' and thus by taking to pen and ink, we should soon settle the question" ("Preface to the General Science" [1677] in *Leibniz: Selections*, Philip P. Wiener, ed. (New York: Charles Scribner's Sons, 1951), p.15).

It is worth pointing out that Leibniz was clearly an inspiration to Pólya. Indeed, Leibniz is one of the four mathematicians to whom Pólya dedicates a separate entry in his "Short Dictionary of Heuristic," in *How to Solve It*.

[26] Eva T. H. Brann, *Paradoxes of Education in a Republic*, p.30.

[27] See Lee, *Crusade Against Ignorance*, p.22. One should also, at this point, return to Bacon whom we quoted above, for in the passage in "Of Studies," where Bacon mentions mathematics, mathematics is listed among other studies no less important: "Histories make men wise; poets witty; the mathematics subtile; natural philosophy deep; moral grave; logic and rhetoric able to contend [note: "logic" is *not* related to mathematics...]." Mathematics has no monopoly on thought and ways of thinking.

[28] I am, to a certain extent, paraphrasing Jacob Klein here. Klein begins his book (one of the most brilliant books on the history of mathematics, in my opinion) concerning the conceptual foundations of symbolic mathematical thought, *Greek Mathematical Thought and the Origin of Algebra* (originally published as "Die griechische Logistik und die Entstehung der Algebra " in Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik, Abteilung B: Studien, Vol. 3, fasc.I (Berlin 1934); fasc.2 (1936)). English translation by Eva T. H. Brann, Cambridge, Massachusetts: The M.I.T. Press, 1968), by observing the inability of separating the form and content of mathematical physics. Thus, Klein writes, "After three centuries of intensive development, it has finally become impossible to separate the content of mathematical physics from its form. The fact that elementary presentations of physical science which are to a certain degree nonmathematical and appear quite free of presuppositions in their derivations of fundamental concepts (having recourse, throughout, to immediate 'intuition') are still in vogue should not deceive us about the fact that it is impossible, and has always been impossible, to grasp the meaning of what we nowadays call physics independently of its mathematical form. Thence arise the insurmountable difficulties in which discussions of modern physical theories become entangled as soon as physicist or nonphysicists attempt to disregard the mathematical apparatus and to present the results of scientific research in popular form. The intimate connection of the formal mathematical language with the content of mathematical physics stems from the special kind of conceptualization which is a concomitant of modern science and which was of fundamental importance in its formation" (pp.3-4).

[29] Theon of Smyrna quoted by Heath in *A History of Greek Mathematics*, I, 246.

[30]The word in the passage translated as 'craftsman' is, in the Greek, *architecton*, but it might as well be the synonym, *technites*, that is, a *technologist*.

[31] Brann has this to say about texts and textbooks: "Textbooks, then, are opposed to works that are original in both senses of the term, in being the discoveries or reflection of the writer himself, and in taking a study to its intellectual origins, using the original language of discovery. In sum, textbooks follow primarily a scheme of presentation; texts convey the order of inquiry" (*Paradoxes of Education in a Republic*, p.100).

[32] In "Uses of Great Men."

[33] Naturally, I realize that this requires some technical background, just as reading poetry does. But as one learns versification in order better to appreciate verse, one should study technical things in mathematics in order better to appreciate the thought in mathematical works. In other words, technical background should always be just that, *background*.

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[34] I do not belittle this recognition; most people see imagination as the property of the arts and something removed from mathematics and science. In this connection, I fully agree with Jacob Bronowski when he says, “We do great harm to children in their education when we accustom them to separate reason from imagination, simply for the convenience of the school timetable” (“The Imaginative Mind in Science,” in *The Visionary Eye: Essays in the Arts, Literature, and Science* (Cambridge, Mass.: The M.I.T. Press, 1978), p.21). One way to ameliorate this situation is to return mathematical work to the work of real human beings.

[35] Thomas Tymoczko, “Humanistic and Utilitarian Aspects of Mathematics” in *Essays in Humanistic Mathematics*, Alvin M. White, ed. (Mathematics Association of America, 1993), p.13.

[36]Robert Maynard Hutchins, *The Great Conversation*, (Chicago: The Encyclopaedia Britannica, Inc., 1952).