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Teaching History of Mathematics: A Dialogue

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Teaching History of Mathematics: A Dialogue

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Synopsis

Many colleges and universities offer a course in the history of mathematics. While the potential benefits for students taking such a course might be apparent, it is often less clear how teaching a history of mathematics course can be a transformational experience for faculty. We present a dialogue between the authors regarding their experiences teaching history of mathematics courses, including their motivation for doing so, the impact these experiences have had on their classroom practices and assessment methods, and the opportunities history of mathematics courses offer for incorporating social justice, equity, and inclusion into the study of mathematics. Our goal is to shine additional light on unexpected ways that teaching the history of mathematics can transform faculty perspectives and practices.

Ben

I have always loved history and politics—I applied to college with a plan to double-major in history and political science. I ended up graduating with a double-major in English composition and mathematics, but my strong interest in history remained. As an undergraduate, I wanted to take Truman State University's history of math sequence but did not have time in my schedule. When I started my position at the University of Kentucky, I was eager to teach a history of mathematics course as I felt I had missed this

part of my education. In addition, teaching history of math would provide me with an opportunity to live a double-life as a secret English composition professor, incorporating writing assignments in a math course where they were a natural fit.

Eric

As a faculty member at Bloomsburg University teaching a broad array of mathematics classes and interacting with a student population composed of diverse backgrounds and interests, I continually confronted the realization that although I could answer many of my students' "what", "how", and "why" questions concerning content, I had difficulty responding to "who", "where", and "when" content questions. Further, I had minimal ability to answer any of these six types of questions concerning the developmental process of the subject. In short, I was capable in only the three "what", "how", and "why" questions (admittedly, the most common ones for mathematics) but none of the six when put into historical context.

Reflecting on my own mathematical development, I realized this gap stemmed at least partially from my approach to my mathematical experience. The core outcome of my undergraduate studies at Kenyon College, rather than developing a broad foundation of theoretical mathematics as I thought at the time, was a deep appreciation of and love for the language, methods, and breadth of mathematics. My graduate student experiences went deeper, building a robust foundation in mathematics before specializing in a research area. At the end of the process, I felt I had accomplished the task of preparing myself to be an effective faculty member at an institution of higher education; I believed being granted the terminal degree in my field justified this opinion. I now realize that this belief was partially correct but underdeveloped; I had developed a solid base of mathematical knowledge and a deep understanding of my particular research problems, but my understanding lacked any motivating thesis for how mathematics fit into the contexts of societal and intellectual development. My choice to create and ultimately teach a history of mathematics course was in large part a response to these realizations.

Ben

My experience as an undergraduate English composition major was a critical influence on my understanding of the broader social and intellectual contexts for mathematics, and it also had a strong influence on my view of how stu-

dents develop their mathematical knowledge and practices. One of the core lessons I learned in my English composition studies is that written communication is inseparable from the act of reading. The process of revising one's own understanding is facilitated through the acts of reading, writing, revising, re-reading, re-writing, revising, etc. Viewing creative and intellectual processes in mathematics through this lens had been valuable to me as a graduate student, and continued to be valuable to me as a professor. I wanted to teach my students how to effectively communicate and understand mathematics, to write and speak about mathematics, to continually revise and deepen their own knowledge. In order to do this, I had to first help them learn how to effectively read. For my history of math courses, I aspired to assign readings to the students that were meaningful, selecting texts that were written with the purpose of being read by students, and I wanted to hold students accountable for engaging with their texts.

I have learned three major lessons from my experiments using reading assignments in my history of mathematics courses. First, it is pointless to expect that every student will complete a thorough reading before class. The goal is to get a critical mass of students to complete the reading, and then to let that critical mass drive class discussion and help engage the others. Some students are simply not interested in learning the course material; there are many (legitimate!) reasons why someone will take a course with the goal of passing, despite my desire for everyone to love the content. Second, it isn't enough to have students read; they must be required to act upon that reading. Assigning critical reviews of text passages as homework, focusing classroom activity on small group discussions with designed prompts that force students to engage with their reading, and designing homework problems that require engagement with course texts are all important ingredients for meaningful experiences with reading. Third, students (even successful ones) typically have had little to no guidance regarding how to effectively read mathematics. It is important to spend a lot of time in class querying students regarding how they read, what order they read passages in, what parts they skipped, where they got stuck, where they gave up, and it is important to make these discussions public in order to highlight the ordinary nature of the challenges of reading.

Eric

Interacting with students through reading assignments has reaffirmed my

opinion that learning is an active rather than passive endeavor. Students who engage the readings by reviewing passages that are of interest or difficulty multiple times, and by looking up and reading material on related topics, tend to communicate their ideas in a more polished manner. The experience of actively reading better prepares them to discuss the mathematical content and has a clear impact on their conveyance skills. Although likely present in my math content classes focused on direct connections to and applications in the physical sciences, in my history of math class the impact of active reading on learning is prevalent and easily recognized.

As a result, one of my goals in developing assignments for my history of mathematics course was to emphasize similar engagement with writing. I wanted to create periodic assignments that required initial engagement with the material, a form of reflection and critique, and a secondary engagement with both the material and with the writing process. As a result, my weekly writing assignments typically ask students to investigate mathematicians who developed aspects of, or political events that affected, topics from the reading. As students use sundry resources leading each to find slightly different material, the topics of the papers are varied in addition to containing differing opinions and themes. I often have the students do peer reviews during class which are turned in alongside the papers. I provide written feedback on both the paper and the reviews and return everything to the author of the paper so she or he can see the difference in opinions and how I responded to the student responses. On occasion, peer reviews occur outside of class; in these cases I ask reviewers to take a classmate's paper home with them and to write comments and a review as a homework assignment due the following class period, to be returned to the original author of the paper for consideration. I have found this process helps reviewers offer more meaningful critiques but the tradeoff is that a single assignment can stretch beyond one week of class time.

Ben

Most of my ideas for using writing assignments were (again) strongly influenced by my experience in English composition courses. A book by Donald Murray titled *The Process of Revision* was particularly influential, and I wrote about this in an article published in PRIMUS [3]. After repeated experiments using writing in math courses, I became more aware of the large body of literature on student learning outcomes and psychological aspects

of learning, both social and cognitive, and the role writing plays in these. Overall, the most substantive influences on my design and use of writing assignments have come from the humanities and social sciences.

I have also used these resources directly in my history of math courses. I have experimented with assigning chapters from James Loewen's book *Lies My Teacher Told Me: Everything Your American History Textbook Got Wrong* to explicitly introduce ideas from history and sociology. I regularly include substantial readings from *The Crest of the Peacock: Non-European Roots of Mathematics* by George G. Joseph, a history of math book that directly confronts Eurocentric perspectives. Using this text creates a natural context in which to discuss ideas from the social sciences in regards to the culture and development of mathematics. It also provides a counterweight to the 'mythologies' of mathematics frequently presented in history of math books.

Along these lines, one of my favorite aspects of the history of math course is how many connections there are to other disciplines and to current events. I am a current events junkie, and whenever I am teaching history of math and see an interesting article in the news about the Middle East, North Africa, India, China, or other areas and countries connected to the course, I will start class by showing the article and mentioning the geographic/political/historical connection to the development of mathematics. I know that for at least some of my students, this has a big impact; students have often mentioned that they have never considered how their mathematics courses might actually be connected to the world at large.

Eric

I too have found current events enter my classroom—regular mathematics-related occurrences like Leap Days and exceptional ones such as the detection of gravitational waves have spawned everything from short conversations to full day lectures. Such inclusions into the classroom help break down the (unfortunately) common way my students have mostly experienced mathematics as a top down subject: anything they do has been previously completed by a professional and sent to them through a professor who already knows the answer. This interpretation contributes to a development of boredom when students are studying mathematics as opposed to English or history where it is expected of them to create an original analysis of the material and often relate it to other events.

At least part of the emphasis embedded in the study of these other disciplines is on creation. My hope is that students leave my course with a sense of how the development of mathematics has similarities to the development of other academic disciplines in that mathematics requires creativity. Mathematics is not developed in a linear manner by professionals; periods of progress and stagnation are affected by outside influences of support and suppression; ideas are forgotten and then redeveloped (or sometimes possibly not); good ends are proven by false means to be properly justified only years later. These are all natural occurrences in mathematics. And perhaps equally important, non-mathematicians have contributed as much as, if not more than, professional mathematicians to the discipline throughout history.

Ben

I have found that many of my students are surprised even by the contributions of professional mathematicians, as strange as that is to say. For example, students know the names of Newton and Leibniz, but they have no idea what they really did. Students are always amazed to hear that the people who made profound contributions to mathematics struggled with understanding and developing their own theories and solutions, at times making mistakes. I try to emphasize this idea of 'productive struggle' as a central aspect of doing mathematics in general.

Many of my students are equally fascinated to hear about people whose stories are not told as part of our mythology of mathematics. For example, I like to point out that Stewart's calculus book does not include any women in the index (except for Agnesi, but that is indexing only the curve and not the person). However, many contributions to calculus and calculus-based physics were made by women such as Maria Agnesi, Laura Bassi, Mary Sommerville, and Emilie du Chatelet. In Reuben Hersh and Vera John-Steiner's book *Loving and Hating Mathematics: Challenging the Myths of Mathematical Life*, there are lovely and thought-provoking passages about mathematicians whose stories are not told in our "standard mythologies". In many of my recent courses, I have included the Lathisms [5] and Mathematically Gifted and Black [7] projects to provide context for discussions about diversity, inclusion, and equity.

Eric

I have found some success in using the writing process to introduce many

of these issues into my history of mathematics classes. To emphasize the process of how to review and revise an argument we looked at drafts of the Declaration of Independence [9] and Franklin Delano Roosevelt's "Day That Will Live in Infamy" [1], along with other major documents and speeches. Students were amazed at two aspects of these documents: the extent to which the authors and editors, men and women considered of such historical significance, thought revision necessary—deletions, additions, and rewordings utilized through the writing process; and the impact revision had on the direct and implied meanings of these works. In addition to demonstrating expectations for significant revision of their own writing, this activity reinforced to my class the need to read math texts critically, ask questions, seek clarification in additional sources, consider what is left unsaid, and remain aware of prevalent accepted social norms and biases of the author's time.

Connecting my course to the specific issues of social justice and diversity through content in an organic manner has been challenging. However, I do believe this goal is attainable; one potential approach is through the peace and women's suffrage movements of England during World War I. Bertrand Russell's work as a peace advocate and the societal pressures dampening these movements highlighted in Adam Hochschild's *To End All Wars* resonated with me as an ideal connection. An assignment focused on reading and analyzing just the relevant chapters in this text might be beneficial. Unfortunately, I have not reached this period in history with enough time in the semester to give it a proper development during class.

Ben

Viewing our discussion from a larger perspective, I think one of the things that history of mathematics courses provide is the opportunity to experiment with strongly student-centered teaching methods. In every math course there is a tension for the instructor between their fidelity to the mathematics being learned and to the students who are learning. For me, I view my primary responsibility as being with the students. As Federico Ardila has said [2], "mathematics needs users, fans, and ambassadors". When we view our courses as primarily focused on mathematics, we run the risk of mistreating our students; as a result, both our students and mathematics are worse off. When we view our courses as being primarily focused on our students, and view mathematics as the setting for our common discourse, curiosity, and passion, then the potential for transformative experiences with mathematics

exists. My history of mathematics courses are the context in which I can most comfortably experiment with new ideas and approaches to cultivating authentic mathematical and personal experiences for all of my students.

Eric

Although rarely noted for his mathematical contributions, for example his work on magic squares [8], I believe the following quote mistakenly attributed to Ben Franklin encompasses much of my philosophy on education: 'Tell me and I will forget, teach me and I may remember, involve me and I will learn.' The benefits of active learning experiences in the mathematics classroom have been emphasized by the Mathematical Association of America (MAA) since the 1980's; a brief summary of their support can be found on MAA past-president David Bressoud's *Launchings* blog [4]. History of mathematics courses are a natural place to emphasize this engagement with the subject, especially when considering the MAA 2015 CUPM curriculum guide goes so far as to list as their first cognitive goal for majors in the mathematical sciences:

> **Cognitive Recommendation 1:** Students should develop effective thinking and communication skills [6].

My history of mathematics courses require students to engage with mathematics in a deep and meaningful way that for many will be novel. I hope that after this experience, seeing and communicating mathematics within the world at large and viewing the mathematical community from a global and diverse perspective will be more natural for my students.

Remark

For readers interested in learning more about the variety of history of math courses that have been developed, we recommend the Course Report on History of Mathematics developed for the 2015 MAA CUPM Curriculum Guide:

https://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/cupm/2015-cupm-curriculum-guide .

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