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Cover Page Footnote

Portions of this paper first appeared as "Mathematics Education and Early Abstract Art" (Stephen Luecking, Proceedings of Bridges 2013: Mathematics, Music, Art, Architecture, Culture).

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Visual Teaching of Geometry and the Origins of 20^{th} Century Abstract Art[1]

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Abstract

As a group, the artists educated near the turn of the 19^{th} and 20^{th} centuries possessed greater mathematical knowledge than expected of artists today, especially regarding constructive skills in Euclidean geometry. Educational theory of the time stressed such skills for students in general, who needed these to enter the workplace of the time. Mathematics teaching then stressed the use of manipulatives, i.e., visual and interactive aids thought to better fix the student's acquisition of mathematical skills. This visual training, especially in geometry, significantly affected the early development of abstraction in art. This paper presents examples of this visual mathematics education and samples its effects on the development of abstract art in the first decades of the 20^{th} century.

Keywords: *abstract art, visual manipulatives, algebraic models, geometric art, mathematics education, art education*

1. Introduction

Today's art student can train with nary a nod to mathematics. Although art education remains affected by the innovations of mathematically, especially geometrically adept artists from nearly 100 years ago, its application of mathematical elements requires no substantive experience with

[1]Portions of this article first appeared as "Mathematics Education and Early Abstract Art" in the *Proceedings of Bridges 2013: Mathematics, Music, Art, Architecture, Culture* edited by George Hart and Reza Sarhangi (Tessellations Publishing, Phoenix, Arizona, 2013); that article is available at http://archive.bridgesmathart.org/2013/bridges2013-35.pdf, last accessed on July 10, 2019.

mathematics. Instead the fledging artists handle these elements procedurally. Perspective, for example, was part of the genesis of a sophisticated new geometry of projection during the Renaissance, but students today learn to apply it by a set of procedures involving the determination of a horizon line and vanishing points and the construction of converging lines. Executing this set of procedures needs no knowledge of the underlying how and why of 3D objects mapped onto a 2D surface.

This was not always the case, as practical geometry, that is, geometry learned through constructive drawing, was once a more important technical skill than it is today and consequently received greater stress in education. In the development of abstract art geometric objects and patterning offered a ready-made and familiar category of abstract objects to which artists could refer. It helped, too, that the mathematical instruction of the pioneering abstract artist accentuated visual comprehension of principles.

2. Models of Abstraction

When the Russian sculptor Naum Gabo arrived in Munich in 1911 to study engineering, Germany was the ideal place to see physical models of algebraic surfaces, as they would appear when graphed into 3D coordinates. These were the products of model-making firms in Munich that marketed to universities worldwide. In the latter 19$^{\text{th}}$ century the use of visual learning tools dominated education there, where the philosophy of *anschaulich* held sway. *Anschaulich* can variously be described as "accessible to insight" or "imaginable", but the term has no direct English translation [15]. It carries the connotation of thinking by developing mental pictures of abstract relationships and then making these visible to the mind's eye.

One famous outcome of this educational philosophy was Albert Einstein, who learned under this system and credited his discovery of relativity to such mental visualizing. Equally famous was the influence of Froebel blocks (Figure 1), designed in the 1830s by German educator and founder of kindergarten, Friedrich Froebel, on the architect Frank Lloyd Wright (Figure 2). Late in life Wright wrote of that influence in his autobiography:

> That early kindergarten experiences with the straight line; the flat plane; the square; the triangle; the circle!
>
> . . . the square became the cube, the triangle the tetrahedron, the circle the sphere.

> These primary forms and figures were the secret of all effects ... which were ever got into the architecture of the world. [8]

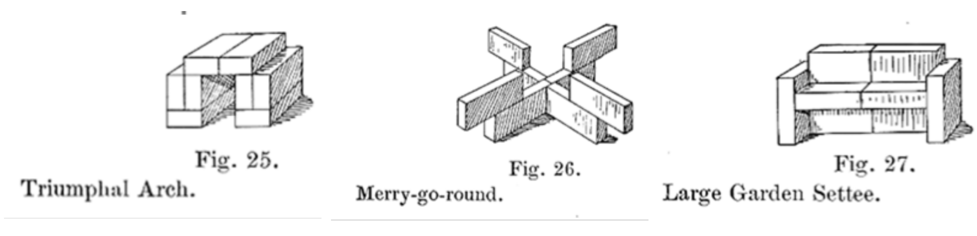

Figure 1: Friedrich Froebel, *Gift #4 Forms of Life*, included in Edward Wiebe's *Paradise of Childhood*, a how-to book from 1869 on the use of Froebel's blocks still in print today [22].

Froebel placed primacy on the child constructing a conceptual and visual architectonics of space. Wright's quote is an echo of Froebel who wrote:

> The importance of the vertical, the horizontal, and the rectangular is the first experience, which the child gathers from building; then follow equilibrium and symmetry. Thus the child ascends from the construction of the simplest wall with or without cement to the more complex and even to the invention of every architectural structure ... [7]

Figure 2: Frank Lloyd Wright, *Unity Temple*, 1904, Oak Park, lL. Library of Congress, Historic American Buildings Survey Philip Turner, Photographer June 1967.

Anna Wright purchased the blocks after seeing them at the Centennial Exposition of 1876 in Philadelphia [20]. At that time in the United States German educational tools were in demand and marketed here. The fame of these tools had spread worldwide. A show of surface models collected by Felix Klein for instruction at the University of Göttingen, for example, crossed the Atlantic in 1893 to be featured at the World Columbia Exposition in Chicago [16]. Klein and fellow mathematician Alexander Brill had earlier founded the best known of the model publishing firms in Munich, where they produced plaster, string and cardboard models for shipment throughout the Western world.

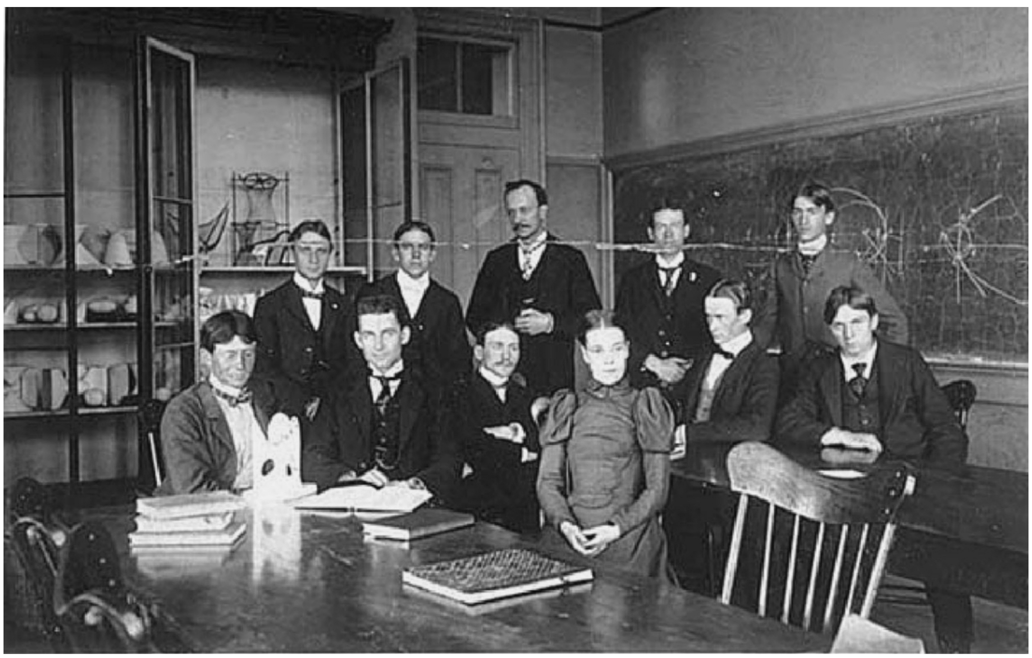

Figure 3: E. J. Townsend and Students at the University of Illinois, about 1900, courtesy of the University of Illinois Archives.

Nowhere in the U.S. were the educational innovations of Germany more sought after than in the mathematics departments of a then burgeoning university system (Figure 3). By 1893 Klein was no longer in the model publishing business, but had gained a reputation as the world's foremost mathematics educator. His travels in the U.S. on the occasion of the Chicago exhibition brought him to New York where he met with professors of mathematics from throughout the country. From this meeting emerged the American Mathematical Society.

3. Practical Geometry

The mathematics education of the early 20th century innovators of abstraction in art varied in degree, but not in the overall emphasis on practical geometry and on visual instruction. Since the early Renaissance practical geometry, in the form of compass and straightedge constructions of Euclidean geometry, was considered a necessary component of the fine artisan's training.

Albrecht Dürer addresses this in the introduction to his 1526 geometry text "Unterweisung der Messung mit dem Zirkel und Richtscheit":

> It is this skill, which is the foundation of all painting. For this reason, I have decided to provide to all those who are eager to become artists a starting point and a source for learning about measurement with rulers and compass. From this they will recognize truth as it meets their eyes, not only in the realm of art but also in their proper and general understanding ... [5]

A set of geometric drawing tools was part of the stock in trade of the established artist into the 19th century. Gilbert Stuart, internationally acclaimed as a portraitist and best known for his paintings of George Washington, acquired a fine set of tools after making his name during study in London (Figure 4).

Figure 4: Gilbert Stuart's magazine drawing kit, George Adams, London, ca. 1780. Collection of the Smithsonian Institute. https://ids.si.edu/ids/deliveryService?id=https://npg.si.edu/media/96D0010A_1.jpg (accessed 12/25/2018).

Stuart's set was known as a magazine and was crafted by the firm of George Adams, who were then scientific instrument makers to the court of King George III. A magazine was the most extensive kit offered by the firm and its use required a correspondingly extensive knowledge of geometry.

By the late 19th century an industrializing society reinforced the teaching of practical geometry in public schools (Figure 5), as the need promulgated for machinists, engineers, and other industrial craftsmen. Those inclined toward visual careers often began their higher education in technical and design institutes before switching to art. This was the case for a number of pioneering artists in the early decades of the 20th century. In such schools students learned the geometry of curves with structural applications, such as the catenary and parabola, and curves with mechanical applications, such as cycloids and pursuit curves [6]. Students learned to construct these using compass and straight edge (also see Figures 10 and 11).

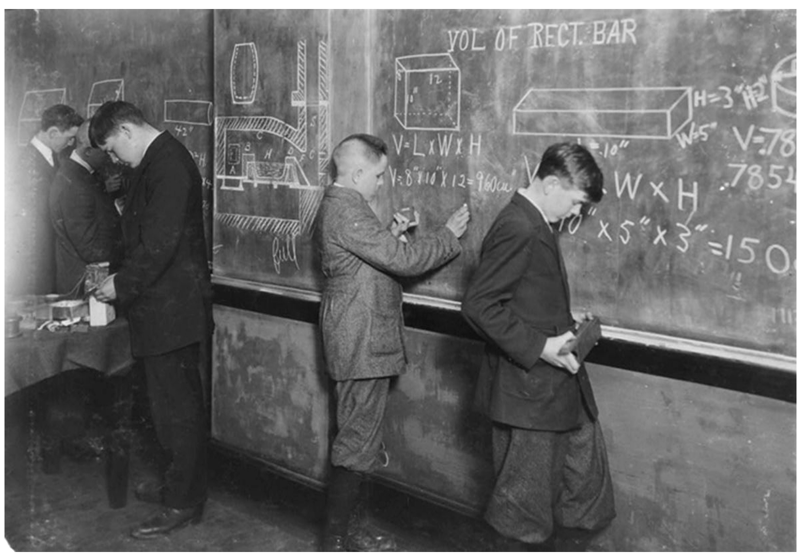

Figure 5: Lewis Wickes Hines, Vocational Printing Mathematics Class. Fall River, MA, photographic print, Library of Congress, 1916. Students are using manipulatives as part of their learning.

In England of the late Victorian a push toward phasing out the teaching of constructive methods in favor of teaching logical proofs was thwarted by an impassioned argument that its elimination would damage the moral character of British schoolchildren. The culture at large perceived constructive geometry as a character-building tool. The English language reflects this with more than its share of metaphors for expressing moral behavior in terms of geometry.

Trustworthy people are “straight” talkers and “upright” citizens who are “on the level”, while those who talk in “circles” lead us to wonder what their “angle” is. To be prepared is to be “squared” away.

Moreover, prevailing philosophy asserted that Euclidean geometry was not just epistemologically valid, but certain. Its truth went deeper than just common sense. Kant’s notion of a priori structures required that our minds be hard-wired for Euclidean geometry and that this geometry corresponds with the essential nature of space. At the physical scale at which life is lived the certainty of Euclid’s constructs had proven itself over and over again. At the scales — astronomical and atomic — toward which physics was trending, they did not work.

Always attentive to educational innovations in Europe the American system of teaching geometry had also undergone challenges similar to those in England. The American response was to initiate a federal study into the rationale for teaching drawing in education. The timing of the study was important. Rebuilding after the civil war stimulated interest among American for their new and developing culture. Like Anna Wright the larger populace became attuned to world cultures due to the 1876 Exposition in Philadelphia.

In February of 1880 the US Senate passed a resolution tasking the Department of the Interior:

> ... to furnish ... a statement containing all of the information possessed by this Department relative to the development of instruction in drawing as applied to the industrial or fine arts... in the public schools and other institutions of the country with special reference to the utility of such instruction in promoting the arts and industries of the people ... [3]

Drawing here denoted constructive geometry and drafting as well as linear perspective fine art drawing. Congress wanted these investigated because it wanted an educated citizenry trained in technical thinking and possessing a keener regard for taste. They considered the drawing skills essential to training quality industrial craftsmen. They similarly saw a populace of consumers with sharpened tastes as creating demand for quality. The overall goal was to achieve cultural parity with Europe and to enhance the desirability of American industrial exports.

The "statement" demanded five years of effort by the Committee for Education (at the time education was under the purview of the Interior). The document presented to the Senate ran to over 1100 pages. It supported not only teaching this broad spectrum of drawing in the public school, but proposed that drawing be required and allotted instructional time comparable with reading and writing:

> . . . universal teaching in all public schools of the elements of "industrial drawing" — meaning by that, an orderly progressive course of drawing based on geometry — is an essential part of any general system of the public education of a people . . . [3]

4. Visual Manipulatives

The field of geometry had in the meantime advanced well past its Euclidean origin and mathematics educators began to lobby for a more analytic approach to geometry that emphasized algebraic over visual study. They sought to provide a base for later study in higher geometry. As seen above resistance was strong and visual manipulatives remained the standard in education. Joshua Holbrook introduced one popular set of visual models used in the United States for elementary education in 1833 (Figure 6). By 1870 its use was mandated by law in over 2000 schools [13].

Figure 6: Holbrook School Apparatus Manufacturing Company, Holbrook's Geometrical Forms and Arithmetical Solids, 1859. Accessed on 12/28/2018 at http://americanhistory.si.edu/collections/search/object/nmah_690329

This date, 1870, was also the date when public schools in Massachusetts and New York began requiring geometric drawing. The success of these programs for industry was likely the primary stimulus for the above Senate report [3].

One set of teaching models that particularly stirred critics as being too objective and distracting from formulation were those made by W. W. Ross, superintendent of public schools in Fremont, Ohio from 1864 until his death in 1906. Ross's models addressed geometry in higher grades and used dissection to demonstrate the origin of curves and surfaces normally studied analytically. Ross' set was extensive, comprising 18 plane figures and 23 solids, half of which were dissected. In the introduction of his manual, "Mensuration Taught Objectively, with Lessons on Form," he avers:

> ... every ordinary operation in the mensuration of surfaces and solids ... can be taught objectively and illustratively so that the pupils shall perceive the reasons of the steps from the first, and the operations themselves shall become the permanent property of the reason rather than the uncertain possession of the memory. [18]

Arguably, the first mathematical objects to appear in 20$^{\text{th}}$ century abstraction were the geometric solids represented in sets like those of Holbrook and Ross. Giorgio DeChirico, for example, populated many of his paintings with manikins, whose body parts referenced these models. De Chirico described his imagery as metaphysical, an effect augmented by his allusions to mathematics. DeChrico's geometric shapes even bore inscribed lines reminiscent of those appearing on the instructional models used in primary and secondary schools; an example of DeChrico's work is displayed on the next page as Figure 7. Shapes like cones and spheres often featured curves to define important sections, such as engravings of circles, ellipses, parabolas and hyperbolas on a cone. The Mr. Woodman series of photographs by Man Ray (one is displayed on the next page as Figure 8) included the actual manipulatives.

Strong visuals as mnemonic devices were not just the purview of sculpted models, but appeared in texts as well. Most noted in this regard was Oliver Byrne, whose 1847 adaptation of the first six books of Euclid minimized text and labels in favor of brightly colored visuals (see Figure 9 in the following pages). Covering Euclid's exposition of plane geometry and proportion, Byrne restricted his colors to the three artistic primaries, red, yellow and blue, and black and white [2]. Byrne's diagrams looked much like the paintings of Constructivist and De Stijl art of the coming century.

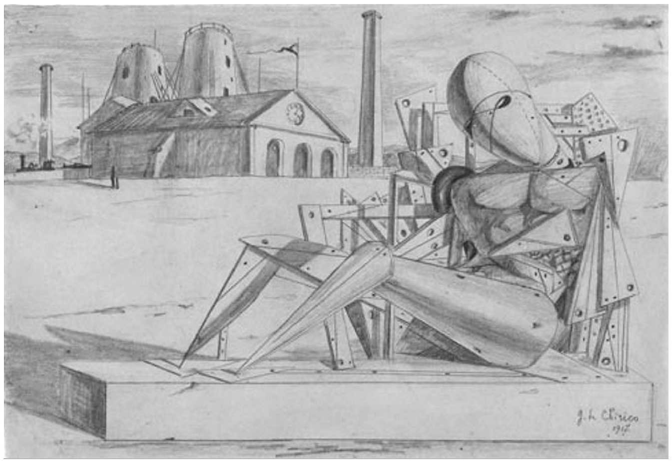

Figure 7: Giorgio de Chirico, *Solitude*, 1917, Pencil and wash on paper, 8 1/4 x 12 5/8" Gift of Abby Aldrich Rockefeller (by exchange) and Purchase. © 2008 Artists Rights Society (ARS), New York / SIAE, Rome.

Though Gabo could view numbers of algebraic models during his studies in Munich (where he also met De Chirico) actual production of models had almost ground to a halt at the time of his residency there. By the 1930s such models had fallen out of favor, rarely used for instruction and even more rarely crafted by geometers.

Figure 8: Man Ray, *Mr. Woodman*, photograph, ca. 1925. Artists' manikins are commonly used to determine posing and proportioning in studies for paintings. Both De Chirico and Man Ray used these as surreal geometric representations of the human figure.

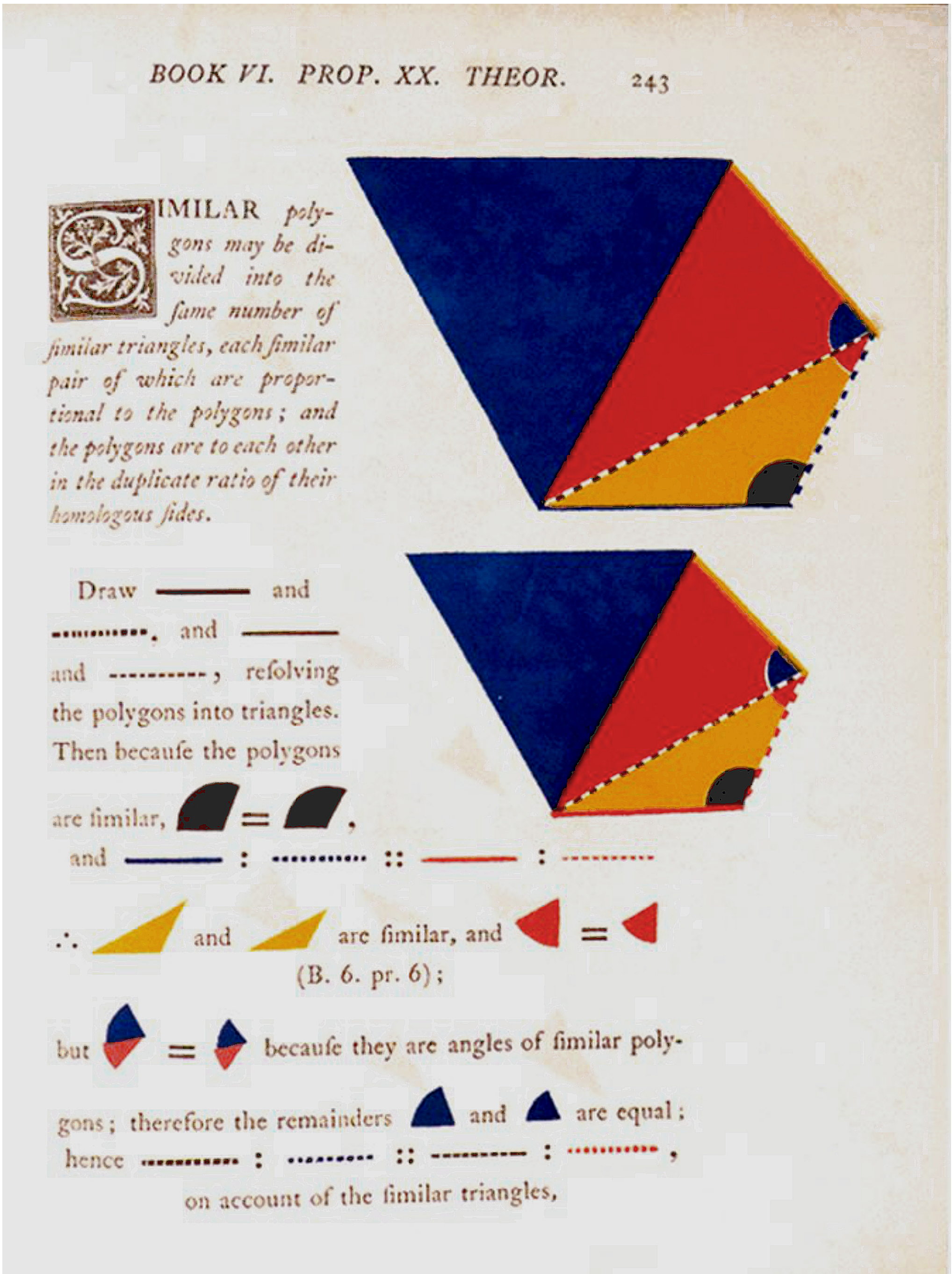

Figure 9: Oliver Byrne, page from *The First Six Books of the Elements of Euclid*, William Pickering, London, 1847 [2].

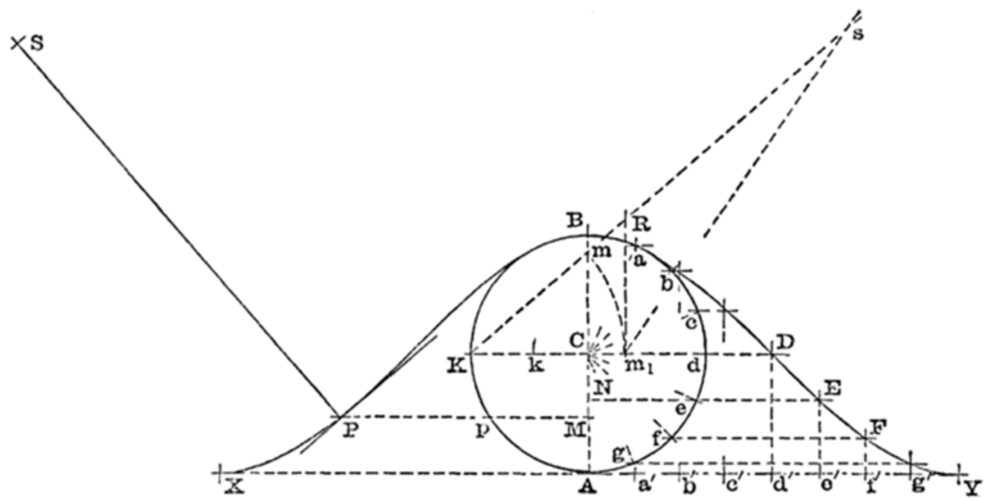

Figure 10: T. H. Eagles, *Cycloid construction*, from *Constructive Geometry of Plane Curves* [6], a text used at the Royal Indian Engineering College. This example demonstrates the point-wise construction of a cycloid. More impressively, it provides constructions that, for any point P on the cycloid, gives the tangent, normal, and radius of curvature at that point.

Ironically the 1930s were the beginning of the surface models' greatest impact on art. Sculptors typically responded to these models not as mathematics, but as the reifying of an order embedded in nature. As such, artists saw in these models parallels to the other natural objects sharing shelf space in museums.

Even those artists, who are categorized as among the most subjective and alogical of modern sculptors, procured inspiration from these models. In commenting on the collection of models that Max Ernst suggested he view at the Institut Henri Poincaré in Paris, the Surrealist Man Ray (see Figure 11 on the next page for one of his paintings) succinctly stated the prevailing attitudes of artists toward the models:

> The formulas accompanying them meant nothing to me, but the forms themselves were as varied and authentic as any in nature. [9]

The Institut's collection inspired Ray to produce numerous photographs (an example is given in Figure 12) and a subsequent series of 20 paintings. While these works accurately delineate the models they bear no specific mathematical meaning. They do, however, attest to the aesthetic power of mathematical form and to the intuition that these forms underlie the beauty of nature.

Figure 11: Man Ray, *Return to Reason*, 1921, oil on board, 14-5/8 x 9-7/8 in. The Margaret G. Deal Fund in honor of Gertrude C. Deal, Harrison H. Deal © Man Ray Trust / Artists Rights Society (ARS), New York / ADAGP, Paris. Before turning to art, Man Ray studied architectural engineering. Some of his works incorporate curves, such as the conic sections, not typically used by less technically trained artists.

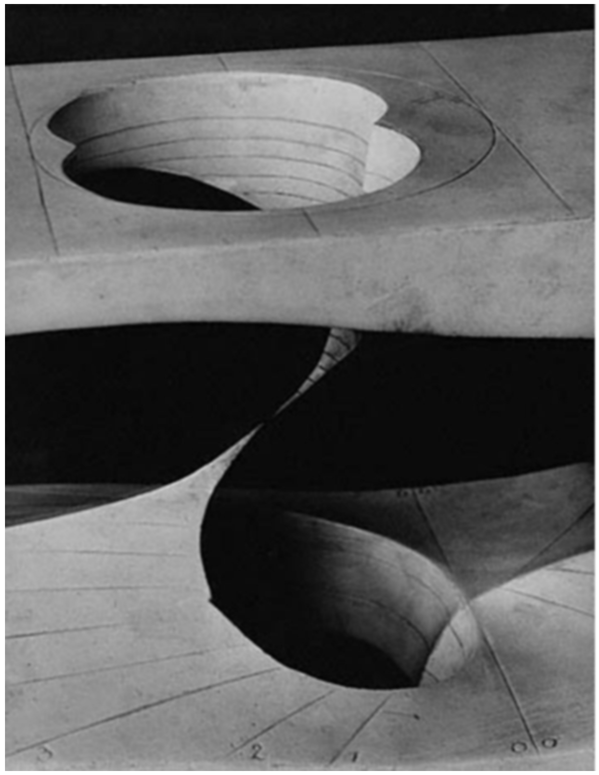

Figure 12: Man Ray, *Mathematical Object*, 1934, gelatin silver print, 11 13/16 x 9 3/16 in., J. Paul Getty Museum © Man Ray Trust ARS-ADAGP. The formula for this model's surface is $((4z^2 - r^2)(x^2 + y^2) - r^2z(2x + z))^3 - 27r^4 4z^2 y^2(x^2 + y^2 - z^2)2 = 0$.

5. Higher Dimensions

By the late 19$^{\text{th}}$ century advanced geometry had re-routed into higher dimensions. These were abstract spaces that beings stuck in mere three dimensions could only describe analytically, with numbers and equations. Nevertheless, the idea of a fourth dimension of space had gained popular appeal, to the point that *Scientific American* sponsored an open contest in 1909 seeking the clearest and simplest exposition of the fourth dimension. The winner, using the nom de plume of Tesseract, was the noted architect Claude Bragdon, who republished his prize essay in 1913 as the text *A Primer of Higher Spaces* [1].

Bragdon's explanation relied on a Euclidean approach to the fourth dimension using the technique of dimensional analogy. An example of this strategy was the unfolding analogy that took geometric forms from higher dimensions and unfolded them into the next lower dimension. This played out as follows:

a square unfolds into four line segments, a cube unfolds into six squares and a hyper-cube unfolds into eight cubes. The unfolded array of cubes can be evidenced in three-dimensions while the hyper-cube cannot. Popular lecturers on the fourth dimension used this same tact, as they could rightly assume that their audiences had sufficient visual knowledge of Euclidean geometry to understand the analogies.

The same year that Bragdon published his text saw the initial production of the futurist opera *Victory Over the Sun* in Saint Petersburg, Russia. The set design by Kasimir Malevich features arguably the first depictions of four-dimensional objects by an artist. The set features windows (see Figure 13 below) as they would appear in a four dimensional figure. If a window in three dimensions is a rectangular panel, then, by dimensional analogy, its panels form a cube in four dimensions. These windows offered a view of the future through the fourth dimension.

Figure 13: Kasimir Malevich, sketch for *Victory Over the Sun*, 1913, PD-Russia. Rather than the rectangular window of a three-dimensional room, Malevich's window is a cube with each plane of the cube opening into another sector of four-dimensional space.

There is considerable circumstantial evidence that Bragdon's text had reached Russia and could arguably have influenced Malevich's efforts. Most telling was the fact that Bragdon could read and write Russian and that as a noted theosophist [14] he was familiar with the famed Russian theosophist P. D. Ouspensky.

Theosophy, like other related spiritualist belief systems of the late 19$^{\text{th}}$ century, was not a religion but an esoteric "science". Such science believed spirits to be a physical phenomenon and could be studied as such. A common notion of these belief systems was the actual existence of a fourth spatial dimension, since a being in such a dimension could easily act like a spirit in three dimensions [1]. Artists who shared spiritualist ideology turned to geometric imagery, which they could harvest from centuries of hermetic symbols to depict metaphysical concepts. Wassily Kandinsky and Piet Mondrian were two pioneering abstractionist affected by this tradition.

Perhaps the best example was the Swedish artist Hilma af Klint, who was wholly given to the esoteric traditions, with membership in theosophy, anthroposophy and Rosacrucianism. Her series of 1300 paintings began in 1905, but were not exhibited until 1986. One of her works, *Altarpiece No. 2*, is displayed in Figure 14 on the next page.

6. Influence on Modern Art Education

6.1. The Bauhaus

Even with mathematical training all but dismissed in current art training, that training still bears the impact of the visual mathematical training of late 19$^{\text{th}}$ and early 20$^{\text{th}}$ century education. Many, if not most, first year programs at universities and art schools are largely modeled on the Vorkurs of the Bauhaus. Its founder and first director, the architect Walter Gropius, melded the Bauhaus from two prior schools: one of fine art, the Weimar Academy of Fine Arts, and one of craft, Grand-Ducal School of Arts and Crafts. By larding the Bauhaus faculty with notable avant-garde artist from Europe and Russia, Gropius succeeded in institutionalizing Constructivism, a notably geometric style of abstraction.

Johannes Itten (see Figure 15, displayed in the following pages, for an example of his work: *Tower of Fire*), was the most experienced teacher invited to the Bauhaus, and he designed the cornerstone first-year course of study [4].

Figure 14: Hilma af Klindt, *Altarpiece No. 2*, Group X, 238 x 179 cm, oil and metal leaf on canvas, 1915. (photo copyright Jerry Hardman-Jones) A dedicated theosophist af Klint, more than any other artist, sought to portray her beliefs in esoteric science.

Despite his distinctly expressionist values, Itten premised the Vorkurs on using geometry to seek out and learn visual relationships. Prior to becoming an artist, Itten had trained as a teacher, especially in the use of Froebel blocks.

At the Ecole des Beaux-Arts in Geneva, Itten worked under the influence of Eugene Gilliard. Gilliard's teaching methods fit especially well with Itten's experience with Froebel blocks. Gilliard's technique had students build the painted image by first laying down an armature of basic geometric shapes and then elaborating these into representational forms.

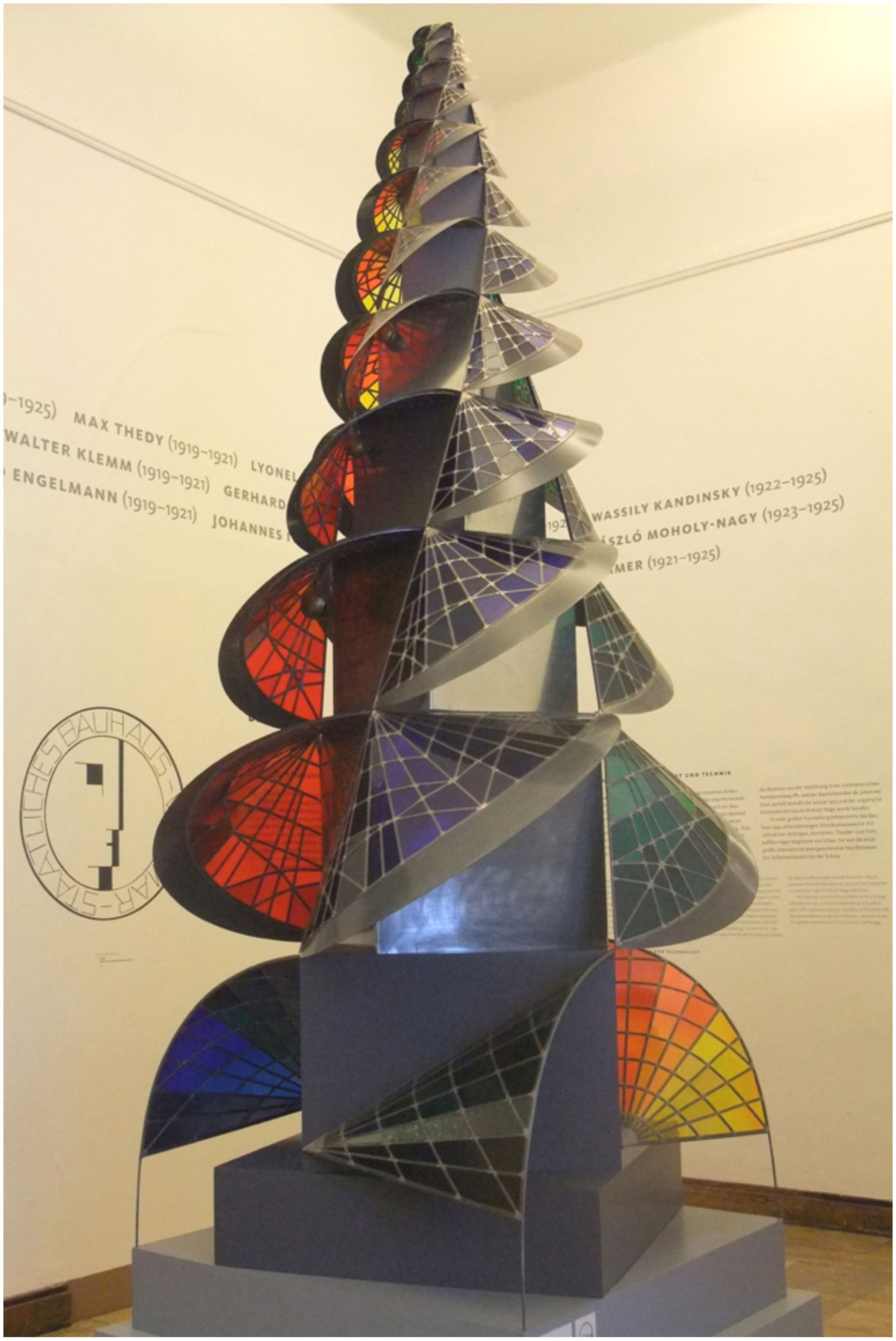

Figure 15: Johannes Itten, *Tower of Fire*, 1920, copied in 1995/1996 by Michael Siebenbrodt (© VG Bild-Kunst, Bonn 2010).

The geometry of the Vorkurs was, like the procedural instruction of perspective, not taught mathematically, but as a set of visual tools for propagating aesthetic research. This geometry resembled mathematics to the degree that it was formalized rather systematically (Figure 16), but this formalization was predicated on organizing perceptual principles, sometimes referred to as visual logic or as a visual grammar. Later advances in perceptual psychology supported such analytical approaches to art. Gestalt psychology, especially, demonstrated a predilection for the mind to organize visual data into geometric configurations.

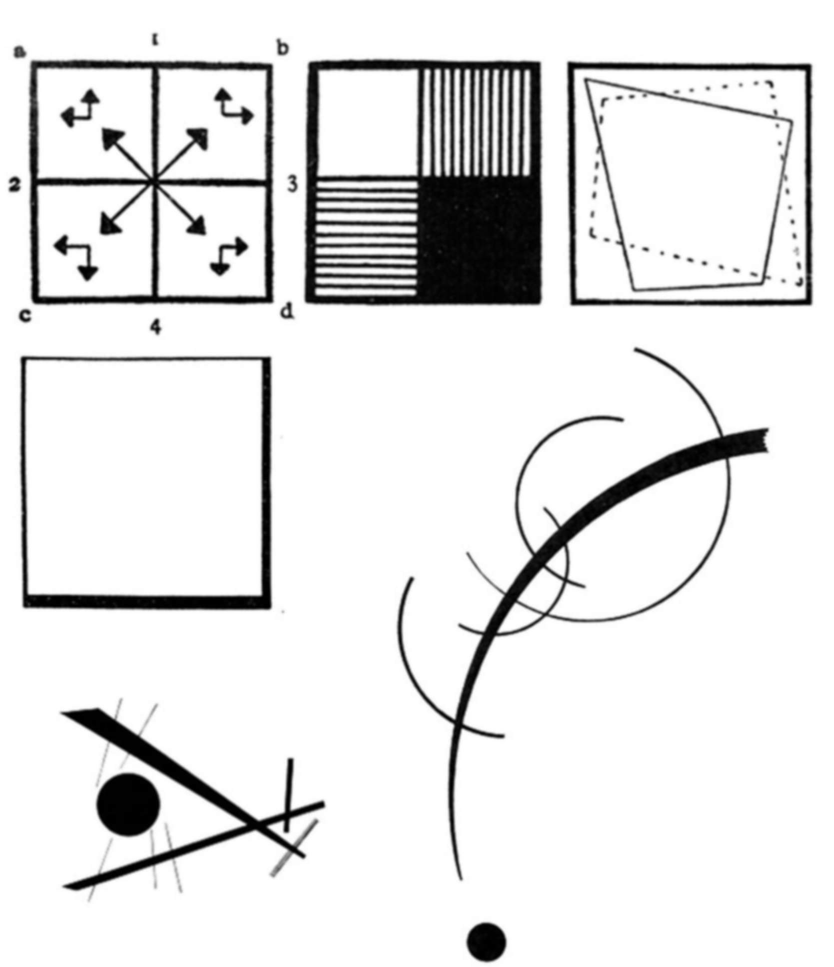

Figure 16: Wassily Kandinsky, illustrations from *Point and Line to Plane*, 1926 [11]. The first four diagrams above analyze the perceptual forces between the picture's surface and its edges. The other two focus on line relations. Kandinsky wrote this text for the Vorkurs of the Bauhaus.

6.2. The Inkhuk

The state art schools of Russia, the VKhUTEMAS, were founded contemporaneously with the Bauhaus and shared similar principles. As Commissar of Education for the fledgling Russian republic, the painter Wassily Kandinsky kept lines of communication open with the Bauhaus [13]. Within the VKhUTEMAS system, Moscow's Inkhuk (Institut Khudozhestvennoy Kultury or Institute of Artistic Culture) expounded an even more formalist and analytic approach than did the Bauhaus. Kandinsky initiated the Inkhuk in May 1920 as a school of theory to focus on formal analysis, but it had by December of that same year taken an even more analytic bent. Inkhuk's director Alexei Babichev, a mathematician turned sculptor, introduced a reorganization of the school to that end. In one position paper Babichev declared:

> ... the form of the work and its elements are the material for analysis, and not the psychology of creation ... [19]

Another sculptor Alexandr Rodchenko was more direct in a similar position paper when he wrote:

> Art is a branch of mathematics, like all sciences. [17]

More than any other instructors at the Inkhuk, sculptor Karel Ioganson (Figures 17 and 18) and Rodchenko (Figure 19) practiced these ideas in their work. The emphasis on the artist as a design scientist and researcher led to the label Laboratory Constructivism, and these sculptors were up to the task. Both worked in a style dubbed Linearism by Rodchenko, which employed only compass, straight edge, and colored pencil to created diagrammatic images. Furthermore both created sculptures whose only referents were their own geometric structure. In doing so, Ioganson exhibited the first known example of a tensegrity structure, almost 40 years before its re-introduction by sculptor Kenneth Snelson [10].

There was a key social dimension to the Inkhuk's conflation of art and mathematics. It was believed that this would yield an art of a universal logic, like that of mathematical formalism. In part due to decree by Lenin, this was to be an art of the collective, impersonal and objective, and not of individual expression; it was to be an art created in the factory and not at the easel. The abstract artists' purpose was that of pure theory, validated by its eventual application to product and graphic design. Regarded, then, as more research than art, this movement came to be labeled Laboratory Constructivism.

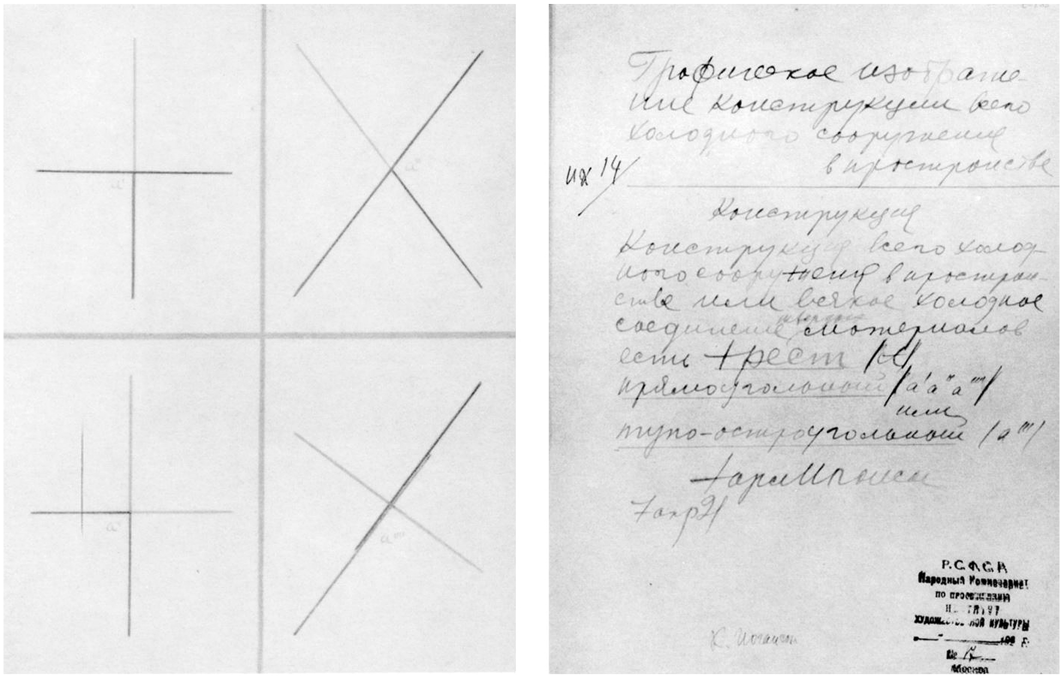

Figure 17: Karel Ioganson, recto and verso of a drawing from the portfolio of the Inkhuk collected by Babichev, ca 1921. From the catalog of the George Costakis Collection, Solomon R. Guggenheim Foundation. The notes on the back of this drawing reveal something of the desire to equate visual and mathematical formalism: "... any cold combination of hard materials [...] is a *Cross* (A): *right-angled* (a', a", a"") or *acute- and obtuse-angled* (a"')." See [10] for this translation and more on Ioganson.

Some of the artists who believed in personal expression and mystical content, Kandinsky and Marc Chagall among them, eventually emigrated from Russia. (Kandinsky would replace Itten at the Bauhaus.) Ten years later Lenin banned abstract art of any sort, as did Hitler with his disbanding of the Bauhaus. The resulting diaspora of avant-garde artists was to spread the new ideas in art and art education to North America.

7. Conclusion

Like other educated people of their time, artists benefiting from late 19$^{\text{th}}$ century mathematics education possessed an appreciation for geometry that considerably exceeded that expected today. Consequently geometry provided source materials for the development of modern abstraction.

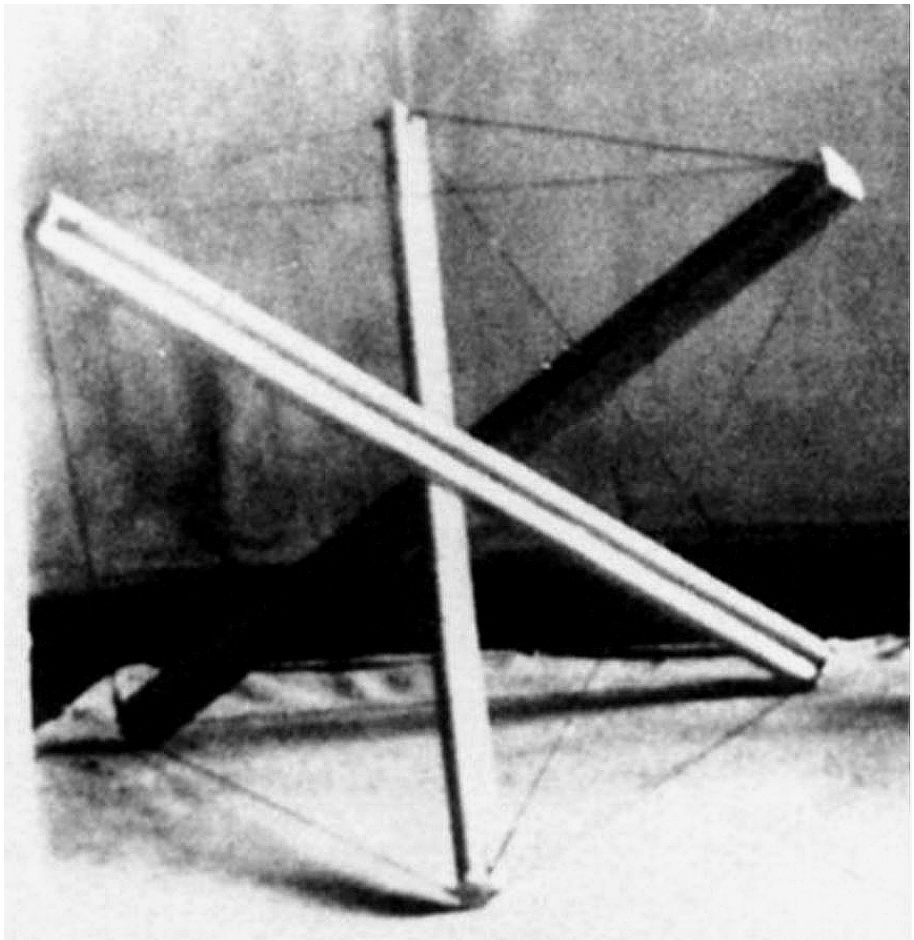

Figure 18: Karel Ioganson, *Linear Construction*, 1921, wood and wire cable, PD-Russia. Ioganson created the drawing in Figure 17 while researching structure for his sculptures [10].

Geometry books by noted mathematicians David Hilbert, Henri Poincaré, and H.P. Manning became best sellers, as did more occult books by P. D. Ouspensky, in which the latter argued for a very real fourth dimension from whence occult phenomena emanated. Popular lecturers on topics of higher space geometry could and did expect a working knowledge of geometry from their audiences.

One outcome was the relatively quick popular reception of Einstein's ideas about time and space as functions of one another. Another was the quick adoption of alternative geometries by avant-garde artists.

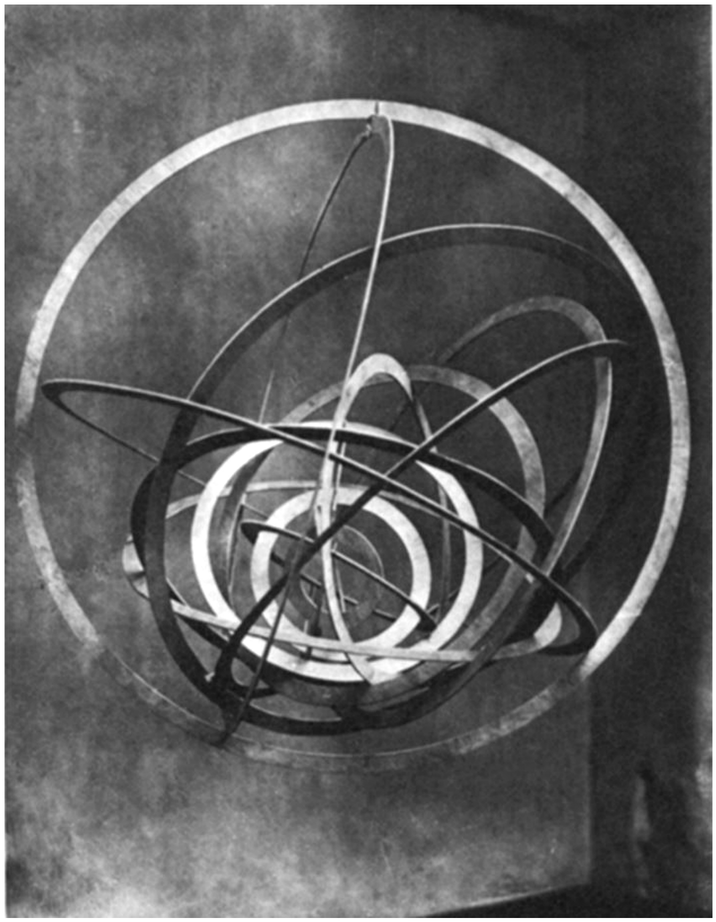

Figure 19: Alexandr Rodchenko, *Hanging Construction*, 1921, plywood, PD-Russia. Photo by the sculptor.

The new ideas about Non-Euclidean geometries and the geometry of time found receptive eyes and ears in the more theoretical of these artists. In return geometries new and old gifted them with vistas into spaces previously unimagined.

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