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Gunhan Caglayan
New Jersey City University

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Recommended Citation

Gunhan Caglayan, "Pattern Blocks Art," *Journal of Humanistic Mathematics*, Volume 10 Issue 2 (July 2020), pages 511-526. DOI: 10.5642/jhummath.202002.23. Available at:
<https://scholarship.claremont.edu/jhm/vol10/iss2/23>

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Pattern Blocks Art

Günhan Caglayan

Mathematics Department, New Jersey City University, USA
gcaglayan@njcu.edu

Synopsis

Pattern blocks are versatile manipulatives facilitating connections that can be made among various strands of mathematics such as number sense, algebra, geometry and measurement, spatial reasoning, probability and trigonometry. This note focuses on an artistic interpretation of the pattern blocks with primary focus on convex polygons made with pattern blocks, and describes five mathematically rich activities using them.

Pattern blocks are versatile manipulatives facilitating connections that can be made among various strands of mathematics such as number sense, algebra, geometry and measurement, spatial reasoning, probability and trigonometry. The six pattern blocks are:

- an equilateral triangle (green),
- a blue rhombus,
- an isosceles trapezoid (red),
- a regular hexagon (yellow),
- a square (orange),

and

- a white rhombus.

All sides of all pattern blocks are congruent, which are considered to be of 1 unit length in this exposition (Figure 1).

Figure 1: Pattern blocks.

Region relationships between the yellow hexagons, red trapezoids, blue rhombuses, and green triangles are observed as follows.

1. One yellow hexagon is equal *in area* to two red trapezoids (or equivalently, three blue rhombuses; or six green triangles; or one trapezoid–rhombus–triangle combination).
2. One red trapezoid is equal in area to three green triangles (or equivalently, one rhombus–triangle combination).
3. One blue rhombus is equal in area to two green triangles (Figure 2).

The orange square and the white rhombus are not expressible as an integer multiple (in area) of a green triangle, although the area of one orange square equals twice the area of one white rhombus, which can be demonstrated merely via area conservation principle (compare the pair at the bottom right of Figure 2).

This note focuses on an artistic interpretation of the pattern blocks with primary focus on important notions related to convex polygons made with pattern blocks such as interior angle sum formula, equiangular vs. equilateral polygons, and congruent polygons. Further connections to ways of generating equilateral convex polygons of unit length are also illustrated.

Figure 2: Region relationships between pattern blocks.

In the rest of this article we describe five mathematically rich activities that can be incorporated into mathematics classes for a wide range of courses, including but not limited to enrichment courses for elementary school students as well as college-level courses for pre-service teachers.

Activity I: Discovering the Interior Angle Sum Formula in an Inductive Approach

Using the fact that the orange square has four right angles and the green equilateral triangle has three 60 degree angles, it is possible to determine the interior angles of the remaining pattern blocks via different techniques (Figure 3).

Figure 3: Determining the interior angles of pattern blocks.

In an inductive reasoning approach, it is possible to determine the interior angle sum for various convex polygons (each made with pattern blocks) with number of sides $n = 3, 4, 5, 6, 7, 8, 9, 10$ (see Figures 4 and 5 on the next page). Every time a polygon is made, these values could be recorded to obtain Table 1.

Number of Sides	Interior Angles	Interior Angle Sum
3	$60^\circ, 60^\circ, 60^\circ$	180°
4	$90^\circ, 90^\circ, 90^\circ, 90^\circ$	360°
5	$60^\circ, 150^\circ, 90^\circ, 90^\circ, 150^\circ$	540°
6	$90^\circ, 150^\circ, 90^\circ, 150^\circ, 90^\circ, 150^\circ$	720°
7	$90^\circ, 150^\circ, 150^\circ, 90^\circ, 150^\circ, 120^\circ, 150^\circ$	900°
8	$150^\circ, 120^\circ, 150^\circ, 120^\circ, 150^\circ, 120^\circ, 150^\circ, 120^\circ$	1080°
9	$150^\circ, 120^\circ, 150^\circ, 150^\circ, 120^\circ, 150^\circ, 150^\circ, 120^\circ, 150^\circ$	1260°
10	$120^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 120^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ$	1440°
11	$150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 120^\circ$	1620°
12	$150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ, 150^\circ$	1800°

Table 1: Interior angle sum, inductive approach.

Patterns on the last column of the table emerge: (1) As the number of sides increase by 1, the increase in the interior angle sum is 180° (2) All numbers in the last column seem to be a multiple of 180° .

Figure 4: Interior angle sums of convex polygons ($n = 3, 4, 5, 6, 7, 8, 9, 10$).

Using these observations, the interior angle sum formula as $I = (n - 2) \times 180^\circ$ is established for $3 \leq n \leq 12$ sided convex polygons.

Figure 5: Interior angle sums of convex polygons ($n = 11, 12$).

Activity II: Equiangular vs. Equilateral Polygons

Convex polygon designs made with pattern blocks could also help understand and distinguish between the adjectives equilateral (all sides congruent) and equiangular (all interior angles congruent); and that an equilateral polygon is not necessarily an equiangular one and vice versa. For instance, although all convex polygons illustrated in Figure 4 are equilateral, only the triangle and the square are equiangular; which makes them regular polygons (i.e., polygons that are both equilateral and equiangular). As another example, the dodecagon in Figure 5 is both equilateral and equiangular which makes it a regular dodecagon. Using pattern block combinations, it is also possible to build equiangular polygons that are not equilateral for $n = 4, 6, 12$ (Figure 6).

Figure 6: Building equiangular polygons that are not equilateral.

Activity III: Congruent Polygons

Given a certain polygon made of pattern blocks, it is possible to generate congruent polygons using a different set of pattern block pieces. Figure 7 illustrates congruent polygon pairs with number of sides $n = 3, 4, 5, 6, 7, 8, 9$.

Figure 7: Congruent polygon pairs with number of sides $n = 3, 4, 5, 6, 7, 8, 9$.

It seems to be the case that as the number of sides increases, more possibilities become available for congruent polygons. Figure 8 on the next page depicts fifteen congruent dodecagons each made with differently arranged sets of pattern blocks.

Activity IV: Generating Noncongruent Equilateral Convex Polygons of Unit Length

Another interesting activity could be the generation of equilateral convex polygons of side length 1. In this activity, the length of a side of an orange square is assumed to be equal to 1. The guiding question could be something like: Where n denotes the number of sides, for a given n , how many distinct (noncongruent) equilateral convex polygons of side length 1 exist? Cases for $n = 3$ and $n = 4$ are obvious and the case $n = 5$ was already shown above (see the pair at the bottom right of Figure 2) so we go on with $n = 6$.

Case $n = 6$: The first column of Figure 9 depicts four distinct non-congruent equilateral convex hexagons of unit length. Each row demonstrates congruent hexagons for each equilateral hexagon.

Figure 8: Congruent dodecagons ($n = 12$).

Figure 9: Case $n = 6$.

Case $n = 7$: Although there are multiple ways of making it, there exists only one equilateral convex heptagon of unit length (Figure 10).

Figure 10: Case $n = 7$.

Case $n = 8$: This case seems to be a rich one as there are three distinct non-congruent equilateral convex octagons of unit length. As before, each row demonstrates ways of generating congruent octagons for each equilateral octagon (Figure 11).

Figure 11: Case $n = 8$.

Case $n = 9$: Even though there are multiple ways of building it, there seems to be only one unique equilateral convex nonagon of unit length (Figure 12).

Figure 12: Case $n = 9$.

Case $n = 10$: Attempts of generating distinct non-congruent equilateral convex decagons of unit length failed. However many ways I tried, I ended up with the same decagon (Figure 13).

(For students of mathematical proofs, there is an opportunity here: to see if they can prove the above uniqueness claims!)

Figure 13: Case $n = 10$.

Activity V: Evolutionary Patterns

The previous activity proved beneficial in understanding the evolutionary patterns associated with non-congruent equilateral convex polygons of unit side length built with pattern blocks. Figure 14 depicts a possible arrangement describing the evolutionary relationships among cases $n \in \{5, 6, 7, 8, 9\}$.

Figure 14: Evolutionary patterns.

Cases $n = 6, 7, 8, 9, 10$ are further analyzed below:

Case $n = 6$: Figures 15a and 15b depicts the parent figure and the resulting hexagon for each parent figure. For example, combining the pentagon with the green triangle – white rhombus combination in two different ways, we obtain two distinct equilateral hexagons (Figures 15a and 15b). As another example, combining the pentagon with a single green triangle, we obtain a different equilateral hexagon (Figure 15d).

Figure 15: Case $n = 6$.

Case $n = 7$: Figure 16 depicts three different parent figures and the resulting heptagons for each parent figure. For example, combining two distinct hexagon parent figures with the green triangle – white rhombus combination, we obtain the same equilateral heptagon (Figures 16a and 16b). Similarly, combining the pentagon parent figure with the blue rhombus – orange square combination, we obtain the same equilateral heptagon (Figure 16c).

Figure 16: Case $n = 7$.

Case $n = 8$: There are many ways to obtain the desired octagons, either by using a hexagon or a heptagon as parent figure (Figure 17). It is interesting to note that using the specific hexagon (white rhombus – square – blue rhombus combination) as the parent figure, it is possible to generate the three distinct non-congruent equilateral convex octagons of unit length (see Figures 17g, 17h, 17i).

Figure 17: Case $n = 8$.

Case $n = 9$: Proceeding in a similar manner as before, we obtain the unique nonagon either by using a specific hexagon (as in Figures 18a or 18b), heptagon (as in Figures 18c or 18d), or octagon (as in Figure 18e) as parent figure.

Case $n = 10$: Finally, Figure 19 depicts the evolutionary relationships between the equilateral convex decagon of unit side length and various shapes as parent configuration. In particular see Figures 19a, 19b and 19e for constructions with an octagonal parent, Figures 19d and 19f for constructions with a heptagonal parent, and Figure 19c for a configuration obtained from a nonagonal parent.

Figure 18: Case $n = 9$. Figure 19: Case $n = 10$.

Case $n = 11$: Of particular interest, we note in Figure 20 the impossibility of obtaining an equilateral convex hendecagon of unit length.

Figure 20: The impossible case $n = 11$.

Wrapping Up

There are many others in the world of mathematics who work (and play) with pattern blocks. Interested readers might find handouts and other related activities at [1, 4, 5, 6]. Researchers find working with pattern blocks a productive teaching activity (cf. [2, 3]).

In my own teaching, I have been using a variety of math manipulatives (including pattern blocks), mostly with preservice mathematics teachers. I have not yet had the chance to implement all of the activities in this note with my own students — most of them are new.

I am currently working on the $n=12$ (dodecagon) case which by itself is prone to many interesting results and observations. I do hope this short note will encourage others to start playing with pattern blocks and exploring mathematics in an artistic way.

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