Spirit-Wise Math: Two Examples from a Collection of Mathaphors

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The author is a retired Unitarian Universalist minister whose first career was teaching mathematics. Her dedicated website is www.PiZine.org.

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Spirit-Wise Math:  
Two Examples from a Collection of Mathaphors

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Synopsis

This article consists of two examples of loosely spiritual insights drawn from mathematics, both of which are from a work-in-progress – a collection of mathematical metaphors.

Introduction

Many years ago I left a career teaching mathematics (which ended when I was mathematics director at a small midwestern college) to begin a second career, one in ministry. I decided to take math with me into this new venture. As a result, I’ve spent much time and energy trying to understand the relationship between spirituality and mathematics. I wrote my doctoral thesis on this relationship, and it turned into a book, What Number Is God? Metaphors, Metaphysics, Metamathematics and the Nature of Things (SUNY Press, 1995). Eventually, I also developed/taught an award-winning course on “The Language of Mathematics”, learned how to give “math” sermons (you give them first, then tell what it was), and published extensively about the subject. I also coined the terms “matheology” and “mathaphor”, and I broadened the terminology “moral math”, which I first heard in personal correspondence with Sal Restivo, a founder of the modern sociology of mathematics. Now, as a retired Unitarian Universalist minister, I am sorting,

¹The author is a retired Unitarian Universalist minister whose first career was teaching mathematics. Her dedicated website is http://www.PiZine.org.
reviewing, and gathering together some of the best of the ideas I’ve worked with. What follows are samples from a collection of mathaphors, which are nothing more nor less than metaphors culled from the mathematics literature. As a self-defined mystic, I’ve tried to focus this collection on metaphors which inform positive social behavior (see, e.g. “Redemption”, *Still Points Arts Quarterly*, Fall 2016, pages 60-66) and on the spiritual journey of life (see, e.g., “The Miraculous in Number(s)”, *Parabola: The Search for Meaning*, Summer 2018, pages 78-83). What follows below are two examples of such mathaphors. In all of this work, I strive to use only user-friendly math so that “ordinary” folks might be included in my audience.

**Example 1**

**Real Number Line:** *There’s more irrationality than rationality.*

Hi, I’m Sarah, better known to the U.S. government as 270-53-8891. I live at 1991 N 93rd St. in zip area 68114. Contact me at 1-402-558-9311. I was born on 8-24-1945, the 3rd of 3 siblings. We lived on 140-acres on Rt. 307 in zip area 44010. I now have 3 children myself, ages 49, 45, and 40, plus 2 step-kids, 8 grands, and 5 great step-grands. I am 5’3, 135 lbs., and I am particularly fond of the number 1, from which you might infer that I like unity, oddity, and masculine energy (odd numbers are male; even numbers, female). You’d be right.

When we describe ourselves using as many numbers as possible, we create a picture considerably different from a description using mostly words. Some of these number-descriptions are remarkably informative. Sometimes, too informative. For example, in the self-introduction above, I occasionally changed some of the numbers from the “true” descriptors. The true numbers seemed too private and too risky to share.

While we tend to think of numbers as hard facts, most of us who are skilled at number manipulation (e.g., statistics) know how easy it is to mislead with numbers. Moreover, numbers are often emotion-laden in and of themselves. Ask yourself, are there some numbers you would rather not have associated with you? How does contemporary society respond to horoscopes, numerology, number divination, etc.? Are such things magic? Superstition? Evil? Helpful? Numbers are far more powerful than most people realize, and they are powerful in metaphorical ways most people overlook and/or underplay.
When he was in the sixth grade, I asked my grandson John (now in high school) if he knew what a number line was. He said yes, that he had studied it in the third or fourth grade, that he now knew about positive and negative integers, fractions, and other rational numbers, and that he currently was learning about irrationals. I was unsurprised by his response. He’d already taken some “advanced” classes in math, so I figured he would probably be conversant with this language of numbers.

John was a little younger than I was when I first encountered these ideas, but not much. I grew up, as did my age-mates, with this number-line image. What I did not realize until recently was that, although its first recorded use was by John Wallis in 1685,\(^2\) the number line wasn’t even a part of math education until about 1950. This discovery shocked me. Somehow, I had taken the existence of the number line, along with most of what I was taught as a child, as an incontrovertible fact that had been around more or less forever. If pressed, I’d acknowledge that the early civilizations probably had no idea what a number line entailed, but I thought it was an eternal truth, not something that was taught to children only after I was born. In addition to being shocking to me, this recent awareness was humbling, as such insights usually are. Time and again my assumptions and presumptions of “truth” have proven to be malleable.

Children today learn about the number line pretty much the way I did. Draw a straight line, pick an arbitrary starting point and an arbitrary unit and mark off the integers (1,2,3,4...) as on a ruler; add 0; add negative numbers and mark out some fractions and, eventually, convert them all to decimal form so you can find places for irrational numbers (such as pi, the square root of 2, or the cube root of 3). This process constructs a model for all the real numbers because there is a unique place for each and every real number on this line, which, of course, is isomorphic to any other scaled number line. The line is continuous, but can always be magnified to squeeze in any other real number desired. It’s an excellent model for the real numbers, especially for children, because it is intuitive and simple, and it provides a way of introducing arithmetic and ordering into the set of all decimal expansions.\(^3\)

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\(^3\) The real number line can also be understood as the field of real numbers, i.e., a set with arithmetic and ordering that satisfies a rather long list of real number axioms.
Intuitive as it is, the real number line developed slowly over a great deal of time and there are now several other models in use to explain the same things. In particular there are the Weierstrass-Stolz, the Dedikind, and the Meray-Conner models, all three of which depend on sophisticated mathematics usually introduced in advanced courses on analysis. It’s also worth mentioning that in the last few centuries mathematicians developed numbers which are not “real”, such as the complex numbers (which include both a real and an imaginary component) and transfinite numbers (which describe the relative size of infinite sets). What is important for this essay, however, is that the real numbers can be divided into two kinds of numbers —those which can be put in the form of a fraction and those which cannot be put in that form. The first are called rational numbers and the latter are known as irrationals, and there are many, many more of the latter than of the former.

As a mystic, I’ve come to appreciate this simple lesson —that there’s more irrationality than rationality — which I learned from my early training in mathematics. Granted, I am talking about naming more than about concept here; it may well be that the choice of the terms “rational” and “irrational” for certain numbers is purely happenstance. It may be that it is not happenstance, either. When I look back at my own spiritual journey into mysticism, I recognize that as a child I was sensitive, open, imaginative, and

This interpretation has the effect of divorcing the real number line from its geometric interpretations just as the non-Euclidean geometries did to the original Euclidian concept of space. See footnote 4 for further reading.

4 For an excellent description of these models and how they relate to the real number line, see T.W. Gamelin, “What Really Are Real Numbers?”, a handout for a UCLA course on the Teaching of Mathematics, web posting, November, 2006 http://www.math.ucla.edu/~twg, last accessed on January 29, 2019.

5 An excellent demonstration of the proof that there are more irrational than rational numbers can be found in a UMKC youtube: https://www.youtube.com/watch?v=mEEM_dLWY0g, last accessed on January 29, 2019.

6 Andrew May observes in an essay on “Mathematics and Mysticism” a fundamental dualism between the terms “rational” (which comes from a Latin root meaning to reason or calculate) and its opposite, “irrational” (which means “ineffable” or “beyond mortal ken” and is often used as a pejorative). He notes this same dualism in philosophy between the material and spiritual worlds, between the physical and the metaphysical, between science and religion, and between logic and intuition. In mathematics, he says, rational and irrational “have a different but curiously parallel meaning” [emphasis added]. Essay first published in British Mensa’s Aquarian newsletter, February 2006. Available at: http://www.andrew-may.com/mm.htm, last accessed on January 29, 2019.
highly creative, but that at least three of these four qualities were considered second-rate to rational, sensible, practical, and intelligent. And irrational was basically a total cultural shutdown. Kids went to school to refine their analytical skills, not to cultivate irrational thinking. The latter was a sure way to the nut house. The social message? Irrational: no, no, goodness, no.

In spite of this message, I grew up to be a sensitive, open, imaginative, and highly creative individual, what one of the staff in the pastoral department of the hospital where I currently serve as a contract chaplain calls “a gentle soul”. She’s right, too. Well, maybe my husband would not agree with her, especially not when I am angry about something he did or did not do! Basically, however, she hit the proverbial mystic right on the head. In my work as a chaplain, being a “gentle soul” has often proven helpful. Usually this comes about from some sort of “irrational” super-sensitivity which makes me unusually receptive to the individuals I am working with. I often intuit something they aren’t verbally saying, and sometimes my mirroring this back to them can have productive consequences. I don’t know precisely where this ability comes from, but I’m sure it is not the result of a careful rational calculation. I remember once, long ago when I was into such things, I consulted an astrologist, who established the various alignments of the stars at my birth and concluded that both my parents were “psychic” and so I must really be psychic, and (in response to my own question to her), she herself was certainly not psychic in the least: her skill at discernment was entirely a matter of proper (i.e., analytic) calculation of the stars. This entire scenario would have been impossible for me to claim, or even repeat, in my childhood because it was, of course, irrational and therefore bad.

The irrational has long been cast as undesirable. History traces this discrimination back to the ancient Greeks and a mathematical discovery which upset the mystical beliefs of the Pythagorean brotherhood that strongly influenced society at the time. The story is that one of the honored brotherhood discovered that the numerical length of the hypotenuse of a right triangle whose other two sides each measured one unit could not be expressed as the ratio of two whole numbers⁷ (See Figure 1). We now describe this situation by

⁷ Proofs of the irrationality of the square root of two can be found online. One site contains 28 proofs: A. Bogomolny, “Square root of 2 is irrational” from Interactive Mathematics Miscellany and Puzzles, http://www.cuttheknot.org/proofs/sq_root.shtml, last
saying that the length of that particular hypotenuse is the square root of 2, an “irrational” number. The unfortunate fellow who discovered this irrationality was, according to some accounts, tossed overboard and drowned for his insight.

![Figure 1: The Pythagorean triangle described.](image)

True or false, the story itself shows how upsetting the idea of irrational numbers was to the Pythagorean society. The reason it was so upsetting is that the Pythagorean brotherhood (which was highly secretive) had built up an entire theory of the universe based on the lovely harmony found in all numbers and all things, and really all things essentially were numbers (“All is number.”) and wasn’t it just splendid how you could chart the relationship between the earth and the moon and the stars and always, always, you found this incredible harmony? Just as numbers determined the relationship between the beautiful chords of music, so, too, did numbers determine the harmony of the skies. Until, of course, someone realized that some numbers didn’t work that way, and that meant that everything the society taught was basically invalid.\(^8\) No wonder irrational got bad press!

The history of mathematical knowledge is rife with such occurrences: think of the discovery of non-Euclidean geometries, for instance,\(^9\) or the idea that some things can not be proven true or false in math.\(^10\) What often happens in these and similar instances is that there’s a great deal of initial uproar during which the discoverer of “heretical” insight(s) often suffers undue con-

\(^8\) For more about the discovery of irrationals, see “The Unspeakable Tragedy” from *String, Straightedge, and Shadow*, by Julia E. Diggins, Viking Press, New York, 1965.

\(^9\) Gamelin (ibid.,“What Really Are Real Numbers?”) notes that “We may compare the divorce of the construction of the real numbers from geometry to the divorce of the foundations of geometry from its origins in the Euclidean geometry of space” (page 11).

\(^10\) Cf., Gödel’s incompleteness theorems.
sequences. This period is then slowly followed by a general acceptance of the notion, the result of which is new insight into the way the world works, at least mathematically.

Today, mysticism seems to have two primary meanings, one related to religious thought, where it is based on non-rational communion or unity with the divine (now sometimes referred to as resolution of “the subject-object dichotomy”), and one related to secular thought, where it refers to obscure or irrational cognition. My life-span and culture fall into an era which, overall, has favored clear and analytical cognition over anything too emotional, sentimental, occult, supernatural, or otherwise non-cognitive. Like the Pythagoreans, I learned to champion the rational over the irrational, but, unlike these early Greeks, I have come to value both kinds of entities as part of the same continuum of real existence. Getting to this perspective has not always been a simple procedure.

For instance, when I was a young mother, my son Willi, then five or six, was looking forward to a family trip where he was going to fly in a small airplane with his father, a new pilot, to visit his aunt. He was incredibly excited. One day, shortly before the trip, we were talking on the phone with this aunt, who informed us that she had recently received a phone call from Willi. She relayed detailed parts of their conversation, including a discussion about the upcoming flight with his father. When, later, we asked Willi about the call, he denied making it (this was before cell phones, of course) and he continued to deny it until, after several additional calls to his aunt to determine if she might have been mistaken, I pressured our beloved child into owning up to making the call. “If you don’t tell us the truth, Willi, then you won’t be able to go.”

Almost immediately, I felt badly about this coercion. I felt even worse when our phone bill later showed not a hint of any such call being made from our home. I was caught between wanting to believe my son (whom I suspected wasn’t actually old enough to know how to dial a long-distance call) and my sister-in-law (who would never lie about such a thing). Ultimately, I mentally filed the whole inexplicable episode under “irrational events”.

There have been other inexplicable events in my life. After the unfair pressure I exerted on my son to “fess up” (and he did!), I grew reluctant to pass judgment on what might seem to be an irrational experience.
Overall, this reticence to discount the irrational has served me well in the ensuing years. Once, for instance, in the early years of my ministry I visited a parishioner who was a self-acclaimed atheist. She told me that not long after her husband had died years before, he had shown up by her bedside one night. They had conversed. A dream? Not according to her. Because of my experience with Willi, I found I had no difficulty accepting the truth of her situation. What still bothered me, however, was how she could be so positive about the event and still remain ardent in her atheism.

Now, years later, I think that perhaps if she had been steeped in the curiosities of mathematics she, too, would have come to appreciate the occurrence of the analytic or rational event as the rarity in life rather than the preferred. Think of rational events as the occasional diamonds on an endless necklace, each surrounded by countless irrational pearls. The set as a whole is a precious craft of art. It bears a seeming magic about it in that whenever you grab a portion of it to examine more closely, you discover you’re holding just as many previously unseen jewels as you saw from a distance. The necklace defies analysis, yet endlessly expands its worth. It stretches, like elastic, yet ever retains its jeweled pattern — diamonds linked by pearls, rationality merged in irrationality — and both create more beauty together than either could bring forth alone.

While the mystical realm as I understand it manifests in both rational and irrational ways of thinking, we tend in ordinary life to focus more on the rational than the irrational. What would happen if we changed that tendency? When mathematicians did this, whole new mathematical fields and technologies arose. Today, our able young people are systematically introduced to these mathematical visions and possibilities at ever younger ages. Imagine a world where we were similarly encouraged to develop our subtle energies right along with our rational intellect.

Subtle energies. When I was 40, I’d never heard the phrase. Thirty-some years later, I link it with myofascial, craniosacral, and other non-mainstream medical therapies, acupressure, acupuncture, Reiki, meditation, vital forces such as chi or qi, psychic insight, various understandings of consciousness, neurofeedback technologies, and new possibilities for individual and collective

\[11\] This is not intended to be exclusive. Just as there are other kinds of numbers besides real numbers, there are likely additional ways in which the mystical realm manifests.
wholeness. Subtle energies, in short, include many things often associated with mysticism, but which may be presented under the umbrella of scientific interest in all forms of healing energy.

When I first encountered the notion of subtle energies (under this coinage, anyway), it was through a traditionally trained allopathic physician who also utilized less conventional homeopathic treatments in his practice. This physician was open in his outlook. At one point he referred me to an intuitive (in this case a “phone” psychic) whom he knew to be helpful to people dealing with health problems and difficult life journeys. He also introduced me to a study group for subtle energies, an organization now nearly 30 years old whose goal is to serve as “an open forum for scientific and intuitive exploration of integrative healing, applied spirituality, and the subtle realms”.

In the 1980s and 1990s such efforts to merge the study of science and spirit blossomed. Helped by the development of the Internet, some of these efforts have continued to grow and integrate these two often-separated disciplines. I am indebted to my homeopathically-inclined doctor (now deceased), because he validated ideas I’d had a difficult time affirming. He legitimized for me what was often looked upon with deep skepticism and suspicion by most Western medical providers of the era, and by many other scientists and religious folks as well. Mathaphorically (i.e., having to do with metaphors drawn from mathematics), these things were the irrational numbers on the real number line. They augmented the outstanding logic of the pure rational numbers. Together with the force of beloved scientific tradition, the subtle energies created the holistic integration of the “real” number-line world.

Example 2

42: *The answer to everything is pretty simple.*

When Douglas Adams’ fictional hitchhiker (*Hitchhiker’s Guide to the Galaxy*) approached the great computer, Deep Thought, for an answer to the ultimate question of life, the universe, and everything, the computer replied that there was, indeed, an answer, but it would require some time to determine it.

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The hitchhiker asked how long Deep Thought would need. “Come back in 7.5 million years”, the computer replied. And so the hitchhiker’s expectant ancestors returned in 7.5 million years, only to have Deep Thought tell them they probably weren’t going to like the answer, which was “42”. “For a moment, nothing happened”, Adams wrote. “Then, after a second or so, nothing continued to happen.”

What happened in real life, however, was that Adams (1952–2001) turned a totally ordinary, apparently meaningless number into a now well-known symbol for the answer to the ultimate question of life, the universe, and everything else. It is true that people with no sense of humor usually don’t care much for this symbol, but, fortunately, humor often lives on after everything else passes away. And embedded within humor there often lies a significant kernel of truth. In this case, the truth is that the answer to the meaning of life, the universe, and everything else is really pretty simple. But given that simplicity, I assure you that most answers, or attempts at answers, tend to look anything except simple. Here are two illustrations, both relying heavily on a mathematics far more complex than “42”.

The first may be found in a 2014 book called Our Mathematical Universe, by MIT physics professor Max Tegmark. In four hundred pages of intriguing, inspirational, scientifically-sophisticated narrative, he essentially addresses the very question which Douglas Adams set forth in his sci-fi spoof. Tegmark’s answer, which is based on the most contemporary scientific truth about our cosmos, rests on the notion that “the ultimate nature of this [the universe’s] strange physical reality” is mathematics. At the end of each of his thirteen chapters, Tegmark sets forth, in what he calls “The Bottom Line”, a list of significant ideas he’s covered (see Figure 2, Excerpts from Tegmark’s “The Bottom Line”).

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14 Check out the University of California TV’s video production about Douglas Adams (https://www.youtube.com/watch?v=OHiJiLrDzYm0) and/or recent youtube variations (deviations) on this scene (e.g., https://www.youtube.com/watch?v=aboZctrHfK8), both last accessed on January 29, 2019.
**Figure 2**
Excerpts from Tegmark’s “The Bottom Line”

<table>
<thead>
<tr>
<th>Excerpt</th>
<th>Page/Chapter</th>
</tr>
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<tbody>
<tr>
<td>*We’ll... examine the ultimate nature of this strange physical reality, investigating the possibility that it's ultimately purely mathematical, specifically a mathematical structure that's part of a fourth and ultimate level of parallel universes. (p14, Chapter 1, “What is Reality?”)</td>
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<tr>
<td>*The very fabric of our physical world, space itself, could be a purely mathematical object in the sense that its only intrinsic properties are mathematical properties – numbers such as dimensionality, curvature and topology. (p33, Chapter 2: “Our Place in Space”)</td>
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<td>*This entire history of our Universe is accurately described by simple physical laws that let us predict the future from the past, and the past from the future. These physical laws that govern the history of our Universe are all cast in terms of mathematical equations, so our most accurate description of our cosmic history is a mathematical description. (p67, Chapter 3: “Our Place in Time”)</td>
<td></td>
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<tr>
<td>*Precision cosmology has revealed that simple mathematical laws govern our Universe all the way back to its fiery origins. (p94, Chapter 4: “Our Universe by the Numbers”)</td>
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<td>*Inflation [the leading theory for our cosmic origins] generically predicts that our space isn’t just huge, but infinite, filled with infinite galaxies, stars and planets, with initial conditions generated randomly by quantum fluctuations. (p118, Chapter 5: “Our Cosmic Origins”)</td>
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<tr>
<td>*Eternal inflation predicts that our Universe... is just one of infinitely many universes in a Level I multiverse where everything that can happen does happen somewhere. Inflation converts potentiality into reality; if the mathematical equations governing uniform space have multiple solutions, then eternal inflation will create infinite regions of space instantiating each of those solutions – this is the Level II multiverse. (p153, Chapter 6: “Welcome to the Multiverse”)</td>
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<tr>
<td>*Everything, even light and people, seems to be made of particles. These particles are purely mathematical objects in the sense that their only intrinsic properties are mathematical properties – numbers with names like charge, spin, and lepton number. (p183, Chapter 7: “Cosmic Legos”)</td>
<td></td>
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<tr>
<td>*The wave function and Hilbert space, which constitute arguably the most fundamental physical reality, are purely mathematical objects. (p230, Chapter 8: “The Level III Multiverse”)</td>
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<tr>
<td>*The mathematical description of the external reality that theoretical physics has uncovered appears very different from the way we perceive this reality. (p242, Chapter 9: “Internal Reality, External Reality, and Consensus Reality.”)</td>
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<td>*With a sufficiently broad definition of mathematics, the [External Reality Hypothesis] ERH implies the Mathematical Universe Hypothesis (MUH) that our physical world is a mathematical structure. This means that our physical world not only is described by mathematics, but that it is mathematical (a mathematical structure), making us self-aware parts of a giant mathematical object. (p271, Chapter 10: “Physical Reality and Mathematical Reality”)</td>
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<tr>
<td>*The MUH implies that it’s not only spacetime that is a mathematical structure, but also all the stuff therein, including the particles that we’re made of. Mathematically, this stuff seems to correspond to “fields”: numbers at each point in spacetime that encode what’s there. The MUH implies that you’re a self-aware substructure that is part of a mathematical structure. (p318, Chapter 11: “Is Time an Illusion?”)</td>
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</tr>
<tr>
<td>*[A]ll structures that exist mathematically exist physically as well, forming the Level IV multiverse... The MUH implies that most of the complexity we observe is an illusion, existing only in the eye of the beholder, being merely information about our address in the universe. (p357, Chapter12: “Testing the Level IV Multiverse”)</td>
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<tr>
<td>*On the largest and smallest scales, the mathematical fabric of reality becomes evident, while it remains easy to miss on the intermediate scales that we humans are usually aware of... If the ultimate fabric of reality really is mathematical, then everything is in principle understandable to us, and we’ll be limited only by our own imagination. (p398, Chapter 13: “Life, Our Universe, and Everything”)</td>
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</tbody>
</table>

Mixed in with scientific explanations for everything from how we measure the age of the universe to what we know about cosmological inflation, we find again and again this simple idea that mathematical existence equals physical existence.
To the casual reader, Tegmark’s Mathematical Universe Hypothesis (MUH) can seem to be as clear as Moisturized Uniplanar Dirt (MUD). Four hundred pages of explanation don’t necessarily change this perception. Still, as astrophysicist Mario Livio (author of *Is God a Mathematician?*) put it in the “Praise” for *Our Mathematical Universe*, “Max Tegmark says that the universe is mathematics. You don’t have to necessarily agree to enjoy this fascinating journey into the nature of reality.”

My point, of course, is that our quest for understanding about the meaning and nature of our existence can be greatly gifted by mathematics. Tegmark offers a thought-provoking, detailed illustration of this idea, which is Platonic in perspective. To him, reality is a four-level nested hierarchy of increasing diversity such that everything, in theory, is some form of *mathematical structure* (emphasis added), by which Tegmark means a *set of abstract elements with relations between them* (again, the emphasis is added). The mathematics exists “out there” whether or not we recognize it, which is why it is a Platonic view. The postulated equivalence between physical and mathematical existence means that “if a mathematical structure contains a self-aware substructure [such as ourselves], it will perceive itself as existing in a physically real universe”. In other words, we are really self-aware mathematics — we just perceive ourselves as something else.

My second illustration has a compatible hypothesis, but it is interpreted through non-numerical symbols which look like two truncated edges of a two-dimensional box, (□), rather than the usual symbols of today’s mathematical language. In the 1960s the British mathematician G. Spencer-Brown (1923-2016) produced a small book called *Laws of Form* which he described as “a text book of mathematics, not of logic or philosophy, although both logic

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16 Cover material, *Our Mathematical Universe*.

17 On page 323, Tegmark presents a diagram (which resembles a flow chart) of the relationships between most of the mathematical structures mathematicians are familiar with: real numbers, complex numbers, vector spaces, topological spaces, etc. This is just a small sample, he suggests, of a “full family tree”.


and philosophy can of course benefit from its application”. In this work, the author — known as a polymath for his skill not only as a mathematician, but also as a psychotherapist, engineer, inventor, and poet (under the name James Keys) — produced a 76-page treatise which elucidates the primary, non-numerical arithmetic of Boolean algebra, a system of logic developed by George Boole (1815-64) and now used extensively in theoretical computer science. This basic section of the book begins by defining the idea of “distinction” along with the two axioms from which the laws of form are then developed. The result, a “calculus of indications” with just two initial (or “primitive”) equations, ultimately leads to a wide variety of sophisticated mathematical and non-mathematical ideas.

In an even shorter (29 pages) set of “Notes”, Spencer-Brown describes the first 76 pages of his “non-numerical arithmetic” in ordinary, if somewhat enigmatic, words; he intends these “Notes” to serve to some extent as a personal guide to the mathematical text, which, like all such texts, is “not an end in itself, but a key to a world beyond the compass of ordinary description”, an initial exploration of which “is usually undertaken in the company of an

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21 According to Spencer-Brown, distinction “is perfect continence. That is to say, a distinction is drawn by arranging a boundary with separate sides so that a point on one side cannot reach the other side without crossing a boundary. For example, in a plane space a circle draws a distinction. Once a distinction is drawn, the spaces, states, or contents on each sides of the boundary, being distinct, can be indicated. There can be no distinction without motive, and there can be no motive unless contents are seen to differ in value. If a content is of value, a name can be taken to indicate this value. Thus the calling of the name can be identified with the value of the content” (from Laws of Form “Chapter 1: The Form”, page 1). The two axioms are the law of calling (The value of a call made again is the value of the call, page 1) and the law of crossing (The value of a crossing made again is not the value of the crossing, page 2).

22 These are equations of 1) “number” or, alternatively, the form of “condensation” and 2) “order” or the form of “cancellation”). Axiom 1 is frequently written \( \overrightarrow{\overrightarrow{\cdot}} = \overrightarrow{\cdot} \) and Axiom 2 appears as \( \overrightarrow{\overrightarrow{\cdot}} = \cdot \). I find it helpful to think of the symbolization \( \overrightarrow{\cdot} \) as a shorthand notation for a box \( \Box \), which is a distinction drawn in a plane similar to that of a circle. Thus, for example, if you cross the boundary from the space outside one of the boxes into the box \( \Box \), then cross back out of it to the plane and then cross again into another identical box \( \Box \), the result is as though you had just crossed once into the original box. \( \Box \Box = \Box \).
I am taken with this little volume by G. Spencer-Brown, not only for its subject matter (which is at once both utterly simple and awesomely complex), but also for the subtle undercurrent of spiritual intrigue which pervades his writing, particularly in his descriptive “Notes”. (See Figure 3: Selected Quotes from *Laws of Form*). For me, he calls forth a kind of scientific mysticism which invites the reader to apply the laws of form to our own existence.

In his introductory “Note on the Mathematical Approach”, for instance, he begins by saying that the “theme of this book is that a universe comes into being when a space is severed or taken apart. The skin of a living organism cuts off an outside from an inside. So does the circumference of a circle in a plane. By tracing the way we represent such a severance, we can begin to reconstruct [...] the basic forms underlying linguistic, mathematical, physical, and biological science, and can begin to see how the familiar laws of our own experience follow inexorably from the original act of severance.”

In a longer excerpt from the final section of his “Notes”, he adds:

> Let us then consider, for a moment, the world as described by the physicist. It consists of a number of particles which, if shot through their own space, appear as waves and are thus [...] of the same laminated structure as pearls or onions, and other wave forms called electromagnetic which it is convenient, by Occam’s razor, to consider as travelling through space with a standard velocity. All these appear bound by certain natural laws which indicate the form of their relationship.

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23 G. Spencer-Brown, *Laws of Form*, from “A Note on the Mathematical Approach”, page xxix. Interestingly, the author notes elsewhere that the primary form of mathematical communication is largely injunction rather than description (see pages 77-81.)

24 While taking quotes out of context and rearranging them carries with it the inevitable possibility of making inferences the author would not embrace as his own, I have tried to be attentive to the sense (at least as I have understood it) of what Spencer-Brown is writing. In particular, I lean for permission on his own directive: “What the mathematician aims to do is to give a complete picture, the order of what he presents being essential, the order in which he presents it being to some degree arbitrary. The reader may quite legitimately change the arbitrary order as he pleases.” page 79.

25 Spencer-Brown, page xxix.
Selected Quotes from Laws of Form, Arranged to Demonstrate a Continuity of Thought

1. The discipline of mathematics is seen to be a way, powerful in comparison with others, of revealing our internal knowledge of the structure of the world, and only by the way associated with our common ability to reason and compute. xxii

2. [The primary form of mathematical communication is not description, but injunction. In this respect it is comparable with practical art forms like cookery, in which the taste of a cake, although literally indescribable, can be conveyed to a reader in the form of a set of injunctions called a recipe. Music is a similar art form, the composer does not even attempt to describe the set of sounds he has in mind, much less the feelings occasioned through them, but writes down a set of commands which, if they are obeyed by the reader, can result in a reproduction, to the reader, of the composer's original experience. 77

3. A recognizable aspect of the advancement of mathematics consists in the advancement of consciousness of what we are doing, whereby the covert becomes overt. Mathematics is in this sense psychedelic. 85

4. [We have a direct awareness of mathematical form as an archetypal structure. xxiv

5. Although all forms, and thus all universes, are possible, and any particular form is mutable, it becomes evident that the laws relating such forms are the same in any universe. xxix

6. It is only by fixing the use of [the constellar principles by which we navigate our journeys out from and in to the form] that we manage to maintain a universe in any form at all, and our understanding of such a universe comes not from discovering its present appearance, but in remembering what we originally did to bring it about. 104

7. Understanding has to do with the fact that whatever [sic] is said or done can always be said or done a different way, and yet all the ways remain the same. 96

8. There is a tendency, especially today, to regard existence as the source of reality, and thus as a central concept. But as soon as it is formally examined, ... existence is seen to be highly peripheral and, as such, especially corrupt (in the formal sense) and vulnerable. 101

9. [What is commonly now regarded as real consists, in its very presence, merely of tokens or expressions. And since tokens or expressions are considered to be of some (other) substratum, so the universe itself, as we know it, may be considered to be an expression of a reality other than itself. 104

10. An observer, since he distinguishes the space he occupies, is also a mark. 76

11. Any evenly subverted equation of the second degree... is thus informed in the sense of having its own form within it, and at the same time informed in the sense of remembering what has happened to it in the past. 100

12. It seems hard to find an acceptable answer to the question of how or why the world conceives a desire, and discovers an ability, to see itself, and appears to suffer the process. That it does so is sometimes called the original mystery. Perhaps, in view of the form in which we presently take ourselves to exist, the mystery arises from our insistence on framing a question where there is, in reality, nothing to question. 105

13. We cannot fully understand the beginning of anything until we see the end. 79

Figure 3: Selected Quotes from Laws of Form.

Now the physicist himself [sic], who describes all this, is, in his own account, himself constructed of it. He is, in short, made of a conglomeration of the very particulars he describes, no more, no less, bound together by and obeying such general laws as he
himself has managed to find and to record.

Thus we cannot escape the fact that the world we know is constructed in order (and thus in such a way as to be able) to see itself.

This is indeed amazing. [. . .]

But in order to do so [i.e., to see itself], evidently it [the world we know] must first cut itself up into at least one state which sees, and at least one other state which is seen. In this severed and mutilated condition, whatever it sees is only partially itself. [. . .] In this sense, in respect of its own information, the universe must expand to escape the telescopes through which we, who are it, are trying to capture it, which is us. The snake eats itself, the dog chases its tail.

Thus the world, whenever it appears as a physical universe, must always seem to us, its representatives, to be playing a kind of hide-and-seek with itself. What is revealed will be concealed, but what is concealed will again be revealed. And since we ourselves represent it, this occultation will be apparent in our life in general, and in our mathematics in particular.\(^\text{26}\)

Taken together, these two authors (Tegmark and Spencer-Brown) point to a world formed of recursive, self-repeating, self-reproducing units which are to some extent self-aware and which are based-on and/or equivalent-to various mathematical structures. These math structures reveal themselves through tokens or marks or symbols. And sometimes through mathaphors.

Have you ever noticed how the sky is full of moods? Sometimes it is bright and cheery, with gates of joyous light inviting you to enter soft spaces. Other times it is dark and ominous, warning you to stay away. I’ve seen all sorts of creatures and things communing in the sky, too, plus the promise of the rainbow and the rage of thunder and wind. I’ve observed these things, and felt their emotions, yet all the while I’ve known that if I stick my fingers in the sky, I will touch nothing tangible, though I saw these things clearly with my own eyes. But though the sky is illusive in its scenery, it is perfectly

\(^{26}\) Spencer-Brown, pages 105-106.
able to communicate its emotions as clearly as the musician who follows the marks on a music script recreates specific sounds, or as a cook who obeys the injunctions of a written recipe produces particular tastes, or as a mathematician who follows the commands of a particular set of symbols experiences a bit of wonder and wisdom which something else has previously indicated.

And we? We are like the sky, full of moods and inclinations. We are nothing more, nor less, than constant in our form and ability to commune one with another, and with the universe itself, and with everything alive and otherwise. In this sense, we are perfectly simple.

Some years ago, I composed a letter in response to a message on a list-serve I accessed through my computer. A list-serve was a precursor of blogs and tweets and Facebook and Skype and texting, none of which, including the list-serve, was around when I was born. So much new, in such a short time-space. At the time I reproduce it here, the letter is still accessible on the Web, but that will not last, because, well, because everything changes. Yet, throughout the change, an identity of sorts always remains the same. That is the mystery in the mystic.

Here is the letter:

Subject: Re: Meta 056: The Loom of God: Gödel’s Proof

To Billy Grassie, Clifford Pickover, and the MetaList in general:

I’ve always been a sucker for flattery, so when Clifford Pickover wrote (Meta 056) that [I] had raised some excellent points in [my earlier post (Meta 045)] about the relationship between mathematics and religion, I admit to being hooked. Just which points did you like best, Clifford?

As for "I wonder what she thinks this relationship might be fifty years from now," I can only reiterate that the metaphorical connection between the two subjects has been around for a long, long time (see What Number Is God?) and it isn’t likely to disappear soon. On the other hand, sometimes I have trouble predicting

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27 See https://www.mail-archive.com/matematika@warnet.unpar.ac.id/msg00044.html, last accessed on January 29, 2019.
what’s going to happen five minutes from now, let alone fifty years, so I’ll not stick my prophetic neck out too far, thank you. However, when it comes to Dr. Pickover’s invitation to comment on Gödel’s mathematical proof of God, I’ll venture a bias. Personally, I find revelation more persuasive than logic. Although logic may in fact lead to revelation, it hasn’t ever happened that way for me. I prefer poetry, as in Symbols:

We are the words She writes
by joining cells one to another
as we set letters side by side, form shapes
that stand for meanings rarely understood.

Like marks that decorate
the sheets of dictionaries,
we hold no weight, bear no substance,
live lives as simple symbols
strung together into lines —
ever-changing colloquialisms
reflecting patterns
we call definitions, and yet

sometimes we rearrange ourselves
in ways that please Her eye:
sentences in books that charm,
turn abstracts into loved designs
soon viewed as wondrous tales.

Michael Guillen (Five Equations that Changed the World, page 2) has indicated that “In the language of mathematics, equations are like poetry”. So, perhaps I shouldn’t make such a distinction between Gödel’s “poetry” and mine. It’s all symbols, anyway.

Of course, both poetry and logic probably most often follow rather than precede revelation, sort of the way a lot of modern scientific exploration is funded only after the desired outcome is fairly well-established. Some of the newer conjectures regarding quantum mathematics and consciousness, such as that
consciousness appears in a kind of back-action from some future event to some past event (see http://listserv.arizona.edu/lsv/www/quantum-mind.html), may actually establish credibility for a whole new temporal relationship between revelation, logic and poetry. That may take time, though — say fifty years or so — to fully unfold.

In Douglas Adam’s terms, perhaps that should be 7.5 million years.

Overview

Below is a more comprehensive list of loosely spiritual insights which I have drawn from mathematics. You might recognize the two examples above as #6 and #21 below. Number 1 and #2 are the basis for the two published articles I mentioned in the Introduction. The rest are in various stages of completion.

I envision this collection as a gathering together of mathematical “snow globes”, each of which tells a spirit-wise math story. When I was a child I was fascinated with snow globes and the exotic stories I saw in each one of them. Today, as a “fifth seasoner”, I am equally entranced with these math-aphorical stories and am excited to collect and explore each of them with ordinary words. Please feel free to contact me if you have thoughts you’d like to share about this project.

1. \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \) (Snell’s law)
   When light travels from one medium to another, it generally bends or refracts.

2. \( e^{\pi i} + 1 = 0 \) (Euler’s formula)
   Everything important is connected.

3. **Definite integral of calculus**
   The one is equivalent to the many. Yes, that’s paradox. Yes, that describes relationship. Yes, that’s how God is.

4. \( 1 = .9999999 \cdots \) (repeating continuously)
   Things that appear to be very different may really be the same.

5. **Gödel’s incompleteness theorem**
   You can have consistency or completeness, but not necessarily both at the same time.
6. **Real number line**  
There’s more irrationality than rationality.

7. **Complex numbers**  
The real always has zero imaginary component. The complex is more fun.

8. **Transfinite numbers**  
The Infinite is strangely ordered.

9. **Bucky balls**  
Resilience depends upon structure depends upon relationship.

10. **Chaos theory**  
Chaos and order is a chicken and egg dilemma, but chaos deserves fresh PR.

11. **Reducing fractions**  
(as in $6/8 = 6/8 = \varepsilon/\varepsilon$)  
Humor makes everything easier.

12. **Network theory**  
To make a difference, target the hubs.

13. **Thirteen**  
is just another number; nonetheless I’m definitely not going to stop here.

14. **Statistics**  
You can lie effectively, or you can tell the truth. You may know which you are doing.

15. **Inequalities**  
Some things are greater than other things.

16. **Entanglement**  

17. **Mathematicians**  
There’s a little mathematician in each of us, and in other “lesser” creatures, too. God is a mathematician. God is love, and other stuff, too. But God is definitely a mathematician.

18. **N dimensions, hyperspace**  
Abbot had it right: a flatlander can’t fully comprehend sphere-land. But math, a “mystical” language, helps us grasp dimensions of existence that otherwise we’d mostly deny.
19. **Beginning Algebra**  
Algebra is an under-recognized source for comprehending the Golden Rule. Do unto one side as you do unto the other.

20. **Set of all sets**  
A “religion of all religions” is just as flawed and just as true as any other religion, but it is structurally different and refreshingly interesting.

21. “**42**”  
The answer to everything is pretty simple.