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Mathematics Versus Statistics

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Synopsis

Mathematics and statistics are both important and useful subjects, but the former has maintained prominence in the American education system. On the other hand, statistics is more prevalent in daily life and is an increasingly marketable subject to know. This article gives a personal history of one mathematician's bumpy road to learning and teaching statistics. Additionally, arguments for how and why to include statistics in the K-12 and college curricula are provided.

Mathematics = {science of numbers: operations, interrelations, combinations, generalizations, and abstractions}

Statistics = {branch of mathematics: collection, analysis, interpretation, and presentation of masses of numerical data.}

These definitions, based off of those given by Merriam-Webster, indicate a clear relationship between statistics and mathematics: $\textsc{Stat} \subset \textsc{Math}$. However, many statisticians would disagree with that conclusion. Cobb and Moore state, "Statisticians are convinced that statistics, while a mathematical science, is not a subfield of mathematics" [2, page 814]. Instead, statistics is viewed like other science discipline such as physics or chemistry, which make use of the tools and language of mathematics but are their own fields of study. So, who is right? Is statistics a branch of mathematics? Is statistics its own discipline? Does it matter?

I argue that it does matter. People, perhaps without realizing it, are bombarded with statistical information and conclusions on a daily basis. Whether one is looking at election predictions, a medical study, or a sports bracket, statistics is the subject providing those results. The general population would benefit from a basic understanding of statistics, but many people never take a statistics class. On the other hand, everyone learns some mathematics and some science. Statistics should be taken, distinct from mathematics classes, at the K-12 and collegiate levels.

This paper is part reflective and part argumentative. As a trained theoretical mathematician, I tell the story of how I transitioned from teaching abstract mathematics to also teaching introductory statistics. I follow up with arguments of why and how statistics should be included in secondary and post-secondary curricula.

A Mathematician's Path to Statistics

When it comes to mathematics people usually want to know "What is this useful for?". I never really cared about the answer to that question, though. In high school, college, and beyond, I enjoyed mathematics for its own sake. Solve for x? Sure, that sounds fun. Prove a theorem? Yes, please. Mathematics was a fun puzzle, and I was ready to put the pieces together.

Before the fall semester of my first year in college, I took a summer course in introductory statistics. I would say more about that experience, but honestly I do not remember what I learned. Fast forward through four years of undergraduate mathematics courses, with an emphasis on pure mathematics. It did not occur to me to take additional statistics courses.

In graduate school, I continued to study theoretical mathematics, although the program requirements forced me to take a few applied mathematics courses after the first year. However, I was not sure that a background in pure mathematics alone would make me an attractive job candidate in academia. Coming from a smaller high school and college, I wanted to teach at a small or mid-sized university. Faculty at such schools often have to teach a variety of classes outside of their specialty, perhaps including statistics. So, I found myself in a statistics classroom again.

These two graduate-level statistics courses had fill-in-the-blank notes and primarily multiple-choice exams. I came out of them knowing a good deal more statistics than I did going in, but not necessarily understanding the mathematics behind the formulas or all of the applications of the methods and tests. Very few examples in those courses used current events, elections, sports, etc. As I delved into my final year of study, focusing on my dissertation in algebraic topology, I failed once again to use and retain the statistical knowledge I had gained.

Luckily, I did get a permanent position at a mid-sized Midwestern university. The university is small enough that I teach a range of classes, from Calculus and Finite Mathematics to Abstract Algebra and Topology. However, my department is large enough that I do not need to teach the applied upper-level mathematics courses. When my first sabbatical approached, I considered what I needed most for professional development and research. What I came up with was ... more statistics.

After graduate school I changed research areas from topology to mathematics education and the Scholarship of Teaching and Learning (SoTL). Education research draws heavily on statistics. Unfortunately I had forgotten much of my graduate-level statistics. I had managed to get by through collaborating with others, and by using the basics I remembered about t-tests. This deficiency was hampering my research, though. I decided to make my sabbatical project re-learning, and learning more, statistics. I sat in on one of the department's 300-level STAT courses, turned in homework, and enjoyed being a student again.

Through this experience I finally appreciated how foundational statistics is for citizens who want to engage in news and society. One of the aspects of this latest course that made it great was the instructor's use of real data. Whether the data was from a class experiment or an outside data set, it was more interesting to analyze. We also had to write our conclusions and analyses in the context of the problem, forcing us to make connections to real-world applications. While taking the class, I was also finishing a research project. It was a wonderful experience to have a research question, realize that a statistical method/test from the course fit the question, and apply that knowledge successfully. Sometimes we would learn the right method in the same week I needed to use it. In my earlier statistics courses, I had missed making these connections to examples or problems that I found interesting.

Since my sabbatical plan was to learn more statistics, I was recruited to teach our introductory statistics course in the following semester. I was excited to share my newfound passion for statistics. My path to statistics was certainly non-linear. One could say it was not even functional.

However, my winding road brought me to the realization that while I think mathematics is beautiful and fundamental, I also think that statistics is essential.

The Place of Statistics in Education

K-12 Education

As a mathematics enthusiast, I do not need to be convinced that algebra and calculus are important subjects. However, in looking at employment trends we see a growing demand for statisticians.[1] Additionally, there is a need for well-informed citizens. Rumsey [6] describes statistical citizenship as "the ability to function as an educated person in today's age of information". I believe that the average high school student would be better served by learning statistics as opposed to more commonly taken mathematical subjects. As Arthur Benjamin says in a popular TED talk, "The mathematics curriculum is ... building up towards one subject, and at the top of that pyramid is calculus ... That is the wrong summit of the pyramid. The correct summit, that every high school graduate should know, should be statistics" [1]. For those that do not continue to college and/or a STEM field, knowing how to calculate the rate of change of a function is not essential. But to be able to understand the data, graphs, and analysis that are reported in the news, media, weather, sports, or business is an indispensable skill.

Historically, statistics has not been emphasized at the K-12 level. In the United States, statistics emerged as a distinct discipline in the 1940s and 1950s following expanded uses of the subject during the war years [7]. Personal computers further advanced the field, but it is still less prominent than other mathematical subjects such as geometry and calculus. In 2009, 11% of high school students in the US had taken a Statistics/Probability course, compared to 76% percent for Algebra II and 35% for pre-Calculus [9].

Statistics is a growing field in primary education, in part due to many states' adoption of the Common Core State Standards (CCSS). These standards promote statistics and probability at the middle and high school levels [3].

[1] `http://www.worldofstatistics.org/employment-outlook/`, last accessed on July 12, 2019.

CCSS treat statistics as a topic within the mathematics curriculum. Usiskin and Hall in [10] discuss the advantages of mixing statistical ideas, such as linear regression, into mathematics courses. Since mathematics is going to be taught and tested, it makes sense to incorporate statistics when possible. On the other hand, these authors also recognize that statistics "has its own body of knowledge separate from mathematics," including sample selection, advanced data analysis, confidence levels/intervals, and so on [10, page 10].

Instead of being primarily imbedded into mathematics, statistics can be taught as a separate course, or within science and humanities courses. Ideally, both would happen. For example, in a History course studying the American Revolution, students could be presented with graphs contrasting the number/percentage of American lives lost in combat then (8000 soldiers) compared to other wars.[2] It is a surprisingly low number, but the restriction of "combat" leaves out 17,000 other soldier deaths; there is also the context of total U.S. population size to consider. In this case, students could see that "facts" are not always what they seem. The Guidelines for Assessment and Instruction in Statistics Education (GAISE) recommendations for K-12 cover a variety of interesting examples, from obesity in America to growing radishes [4]. This sort of analysis would break down the silo of statistics, which I experienced as a student, and showcase the interdisciplinary nature of the subject.

Statistics in Higher Education

The current status of statistics in higher education is conflicted. From 2005 to 2010, the number of students who took an introductory statistics course in post-secondary education increased by 34.7% [5]. Statistics-heavy majors, such as Data Science, have also emerged. On the other hand, despite 30 years of recommendations from the Mathematical Association of America (MAA), most undergraduate programs in mathematics do not require (and often do not offer) an applied statistics course. The MAA's most recent Curriculum Guide (2015) claims that "such a course would serve the vast majority of our students far better than one additional theoretical math elective" [8, page 36]. This statement, akin to my belief that most high school graduates

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[2] `https://en.wikipedia.org/wiki/United_States_military_casualties_of_war`, , last accessed on July 12, 2019.

need a statistics course over additional mathematics courses, underscores a distinction between mathematics and statistics and emphasizes the merit of exposure to the latter.

Although statistical theory has a mathematical foundation, there are different questions and details to consider. For example, the context of a problem influences the choice of data collection, measurement methods, and analysis. A statistics student must consider bias and extraneous variables. Answers are not always clear, as it can be a judgment call when looking at factors like strength of results or importance of outliers. Mathematics depends on deductive reasoning while statistical reasoning is more inductive. A college student taking calculus does not experience the same reasoning exercises since most problems in those courses are formulaic with specific solutions. But many situations in research and everyday life do not have clear-cut answers. People would profit from learning to deal with uncertainty, randomness, and data.

An introductory or applied statistics course is useful to students, but only if taught effectively. When real data is employed, students see benefits through: reading with understanding, recognizing patterns, identifying essential features, and applying appropriate methodologies [8]. Instructors, to make statistical education more meaningful, should "refrain from teaching our students merely how to follow recipes and should instead teach them how to really cook" [5, page 13].

As a student I experienced cut and dried statistics courses as well as investigative statistics with real data and I saw the advantages of the latter. There are many published resources available related to teaching statistics, but as a singular source the overarching ideas and suggestions are summarized in the GAISE (2016) report [5].

Conclusion

The title of this article, "Mathematics Versus Statistics", is perhaps a bit misleading. These subjects are not battling with each other. In fact, they complement each other. However, in the hierarchy of education, mathematics is currently the obvious winner. Every student learns mathematics. Many students learn a lot of mathematics. As the world becomes more data-driven and digital, we need to get to a point where every student also learns some statistics, if not a lot of the subject.

Statistics is gaining prominence in high schools through the CCSS and in colleges through the growing number of majors requiring it. As I found as a student, though, simply taking a statistics course may not be enough. There are many resources and suggestions available to statistics educators, from data sets to specific exercises. Incorporating real data and interesting examples, whether into a separate statistics course or within the context of another subject, is a great way to expose students to the critical thinking skills required of statistical analysis. They can then use the skills in other courses, research, or simply everyday life.

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