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A Holistic Mathematics Curriculum Revision: An Adelphi University Case Study[1]

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Synopsis

Join me as I take you on a journey with the faculty of the Department of Mathematics and Computer Science at Adelphi University during our two-year re-envisioning and implementation of our mathematics curriculum. From the beginning this involved a data-driven initiative that naturally led to the revisions. Here I describe in detail the process that our department followed. In closing I end with some recommendations for interesting research directions in the field of mathematics education.

1. Introduction and Background

1.1. Overview of the Paper

This paper serves as a case study for those interested in the experience of a department that initiated and completed a comprehensive restructuring of their mathematics program. The re-envisioning of the mathematics curriculum at Adelphi University (https://about.adelphi.edu/overview/quick-facts/fact-sheet/) was the culmination of a two-year department-wide movement to modernize all of our degree programs initiated by Dr. Christopher Storm and spearheaded by myself. (See [10] on the computer science curriculum revisions.) At the time, I felt that this restructuring was long overdue, but, as you will see, the timing could not have been better.

[1] I wish to acknowledge the other members of Adelphi University's Department of Mathematics and Computer Science who contributed to these curriculum revisions: Dr. William Quirin, Dr. Robert E. Bradley, Dr. Lee Stemkoski, Dr. Christopher Storm, Dr. Sarah Wright, and Dr. Branden Stone.

First, let's take a trip back to the 1960s to see how the mathematics programs were structured in my department.

1.2. History of the Mathematics Program at Adelphi University

Historically, the department had two undergraduate programs in mathematics: a B.A. in pure mathematics and a B.S. in applied mathematics (The B.S. in applied mathematics was similar in nature to the B.A.; however, it included a course in differential equations and two numerical analysis courses.) In the latter half of the 1960s, the B.S. in applied mathematics was eliminated and adopted the coursework from the B.A. in mathematics. The curriculum was separated into two tracks—non-education and education students. (See Table 1.)

Non-Education Students (40 Credits)	Education Students (31 Credits)
Analytic Geometry and Calculus I	Analytic Geometry and Calculus I
Analytic Geometry and Calculus II	Analytic Geometry and Calculus II
Calculus III	Calculus III
Calculus IV	Calculus IV
Modern Algebra I	Modern Algebra I
Modern Algebra II	Geometry I
Geometry I	Geometry II
Linear Algebra	Introduction to Probability Theory
4 Upper-division Mathematics Electives	1 Upper-division Mathematics Elective

Table 1: The Adelphi University Mathematics Major in 1969. Calculus IV is what would now be called a first course in real analysis.

The year 1971 saw the birth of computer science at Adelphi University. For about the next ten years, the faculty devoted their time to the development of computer sciences courses and the mathematics program remained unchanged. Here are some computer science courses that were developed during that time: Computer Programming, Applications of Computer Programming, Principles of Programming Languages, Combinatorial Computing, Compiler Construction, Discrete Structures, and Computers and Society. Additionally, along with the birth of computer sciences, the department developed, and began offering their first courses in statistics: Statistics for Biological, Management and Social Sciences, and Mathematical Statistics.

At the start of the 1982 academic year, the B.S. in Computer Science was formed at Adelphi University and all courses in computer science were given the designation: CSC. However, computer science and mathematics remained a joint department. In 1985, the mathematics program experienced its last major curriculum revision until 2014. (See Table 2.)

B.A./B.S. in Mathematics (45 Credits)
Analytic Geometry and Calculus I
Analytic Geometry and Calculus II
Calculus III
Introduction to Ordinary Differential Equations
Linear Algebra
Introduction to Computer Science I
Discrete Structures
Abstract Algebra
Geometry I
Introduction to Probability Theory
Analysis
Two Upper-division Mathematics Electives

Table 2: The Adelphi University Mathematics Major in 1985. Introduction to Computer Science I was a first course in computer programming.

From 1985 to the early-2000s many courses changed titles and designation numbers; however, the overall structure of the major remained pretty much unchanged.

In 2006, the department added a 3-credit Introduction to Proof and Abstract Reasoning course (MTH 301) to the major. The motivation for adding this course was the feeling from the faculty that our students needed preparation to enter a theoretical Real Analysis or Abstract Algebra course. The purpose of the course was:

> Learn to write and analyze formal mathematical proofs. Apply proof techniques including contrapositive, contradiction, cases, and induction. Justify proof techniques using truth tables. Express abstract ideas using constructs such as sets, Cartesian products, relations, functions, and equivalence relations. Prove theorems from number theory, geometry, real analysis, and abstract algebra.

The typical outline of the course was as follows.

Course Outline for Introduction to Proof and Abstract Reasoning.

- Review of Sets and Logic. Analysis of Analysis of Logical Arguments.
- Proof Techniques—Direct Proof, Biconditional Proof, Proof by Contrapositive, Proof by Contradiction, Proof by Cases, and the Principle of Mathematical Induction (PMI).
- Relations and Equivalence Relations.
- Functions.
- Binary Operations and Abstract Algebra—Definition of a Binary Operation and Examples, Definition of a Group and Examples, Proofs in Group Theory, Definition of Ring and Examples, and Proofs in Ring Theory.
- Introduction to Real Analysis—Fields, Ordered Fields, Sequences, Functional Limits, and Continuity.

Additionally, the department introduced a year-long integrated calculus sequence for students that did not pass our calculus placement exam. Prior to this, we followed the model that students were either placed into Pre-Calculus (if they did not pass a placement exam) or Calculus I (if they passed the placement exam). This model switched to Calculus I (if they passed the placement exam) or Calculus 1A (MTH 130) and Calculus 1B (MTH 131) (if they failed the placement exam). MTH 130 and MTH 131 were intended to:

(130) Review the properties of polynomials and rational functions. Master the techniques of differential calculus as applied to these functions: limits, continuity, the derivative, and the antiderivative. Explore connections between the derivative and real world problems involving dynamics, the economy, material optimization, and related rates.

(131) Review the properties of trigonometric and exponential functions and their inverses. Master the techniques of differential calculus as applied to these functions: limits, continuity, the derivative, and the antiderivative. Explore connections between the derivative and real world problems involving dynamics, material optimization, and related rates.

Outline of Calculus 1A (MTH 130) (3 Credits).

1. Review and Linear Equations
2. Functions: Definitions, Transformations, Operations, and Inverses
3. Polynomial and Rational Functions
4. Limits
5. Differentiation: Definition, Tangent Lines, Derivative Rules, Implicit Differentiation
6. Applications of Differentiation (Optimization) and Curve Sketching

Outline of Calculus 1B (MTH 131) (4 Credits).

1. Review of Differentiation and Optimization
2. Antiderivatives, The Fundamental Theorem of Calculus, and Integration by Substitution
3. Exponential and Logarithmic Functions
4. The Calculus of Exponential and Logarithmic Functions
5. Trigonometric Functions
6. The Calculus of Trigonometric Functions
7. L'Hôpital's Rule

Finally, in 2006, the department needed to designate a "Capstone Experience Course," which was intended for university-wide general education assessment purposes. The department felt that none of the courses in the major was fit to be a capstone course because they did not produce an artifact for assessment. Therefore, the department replaced Differential Equations with Advanced Mathematical Modeling. (See Table 3.)

In 2009, the B.A. in Mathematics went into a dormant state from a lack of enrollment, which was due to a four-semester language requirement being added by the College of Arts and Sciences to all B.A. degrees. Thus, since 1985, the department had a one-size-fits-all major in mathematics.

B.A./B.S. in Mathematics (47 Credits)
Calculus I or Calculus 1A and Calculus 1B
Calculus II
Calculus III
Linear Algebra
Introduction to Computer Programming
Discrete Structures
Introduction to Proof and Abstract Reasoning
Abstract Algebra
Geometry I
Introduction to Probability Theory
Analysis
Advanced Mathematical Modeling
Two Upper-division Mathematics Courses

Table 3: The Adelphi University Mathematics Major in 2006. Following New York State Teacher Certification regulations, students wanting teacher certification for teaching mathematics grades 6-12 must take Mathematical Statistics and Geometry I as their two electives.

The majority of students that came to Adelphi and declared a mathematics major were planning for careers in mathematics education so it was not really an issue.[2]

However, near the late 2000s, that trend shifted dramatically. We experienced an increase in students who did not want to pursue education, but rather wanted to attend graduate school in mathematics, obtain a career in actuarial science, or statistics. Thus the department needed to seriously consider its major, because the one-size-fits-all structure could not provide both populations of students with the skill sets they needed for employment or graduate studies. Change was on the horizon and my background in mathematics education was going to be put to the test!

[2] Adelphi University also has what is called the Scholars in Teacher Education Program (STEP), which is a 4+1 program. Students gain a B.A./B.S. in their content area with a minor in education and gain a M.S. in one year.

2. The Revisions

2.1. The Longitudinal Study

In 2013, I conducted a statistical analysis using data from Fall 2000 to Fall 2011. The sample consisted of $n = 332$ mathematics majors. The analysis did not include transfer students. (Transfer students would bring nuisance variables that could not be controlled for, such as habits picked up from their prior institution and personal experiences at their prior institution.) We were looking solely at the influence that the university had on freshmen that started at Adelphi University. The purpose of this analysis was to examine enrollment trends, the influence of MTH 130/131 on the upper level calculus courses, and the influence of MTH 301 on the upper division mathematics courses. The report included quantitative and qualitative results.

2.2. Enrollments

The enrollment analysis involved the $n = 274$ Adelphi mathematics majors who entered Adelphi before Fall 2009 (thus omitting $n = 58$ students).

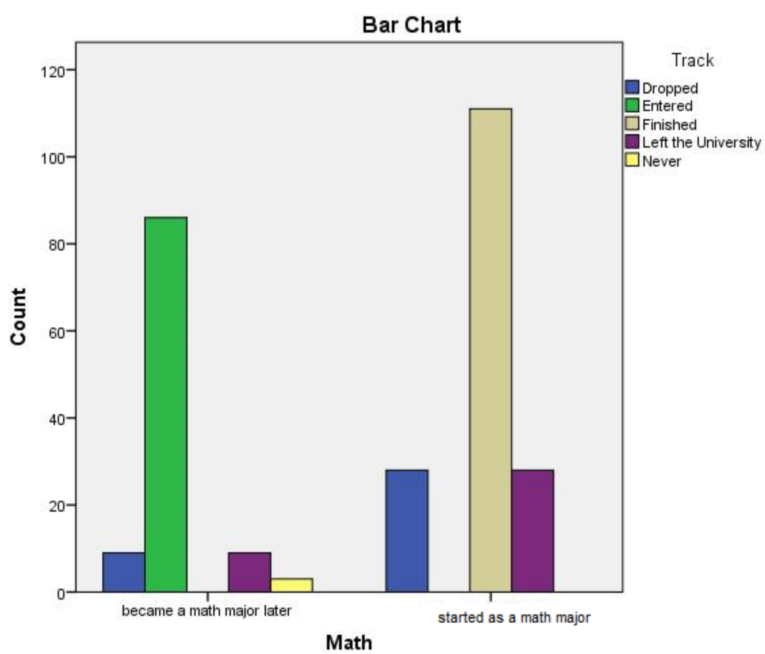

Figure 1: Enrollment trends from Fall 2000 to Fall 2011.

Of the 167 students who entered Adelphi as a mathematics major, 66% of them completed the major and 17% dropped the major. The majority of students who dropped the major did so after completing Discrete Structures, Calculus I, Calculus II, Calculus III and/or Linear Algebra. However, most of the students who dropped the major did not do so because of poor grades. Interestingly, 17% of these students not only dropped the major but also withdrew from the university. (See Figure 1.)

Of the 107 students who eventually declared mathematics as a major, 80% completed the major and 8% dropped the major. It appears that the majority of students who entered the major did so after completing Calculus II. Also, of the students that would eventually drop the mathematics major, most of them did so after Calculus II. Again, those that dropped the major also left the university. (See Figure 1.)

Finally, 3% of these students eventually declared the mathematics major; however, they never took a mathematics course and never completed the major. It should be noted that this 3% did not withdraw from the university. (See Figure 1.)

2.3. Consulting the CUPM

In 2015, the Committee on Undergraduate Programs in Mathematics (CUPM) published its *Curriculum Guide to Majors in the Mathematical Sciences* [5], which is published by the Mathematics Association of America (MAA). This guide provides recommendations for curricula, sample syllabi, assessment recommendations, uses of technology in the classroom, and methods for promoting undergraduate research in mathematics.

I kept the following vital recommendation from [5] in mind throughout the entire revision process:

> Major programs in the mathematical sciences should present the beauty, fun, and power of mathematics. They should be designed so that all students come to see mathematics as an engaging field, rich in beauty, with powerful applications to other subjects and contemporary open questions. Each department should create and maintain a community that welcomes and supports all students, including those from groups that have been traditionally underrepresented in mathematics [page 9].

Having a doctorate in mathematics education, the following statement from [5] struck close to home:

> The adoption of the Common Core State Standards by many U.S. states, for instance, will significantly change the preparation of incoming college students and will necessitate changes in the preparation of pre-service teachers [page 2].

The CUPM [5] stated that students' interests in mathematics have become more diverse and a mathematics department must be able to adapt to these interests. This was my chance to implement a change in our program that I knew would benefit our students. It was decided that we needed two different majors in mathematics: a B.S. in Mathematics, which will be a "design your own major" and a B.A. in Mathematics, which will be intended for future educators.

For the revision of our B.A. in Mathematics, the department consulted resources from the National Council of Teachers of Mathematics (NCTM). We utilized these documents to ensure that our content aligned with the standards that are required for future teachers. Specifically, we examined the *Standards for Mathematics Teacher Preparation* from the Council for the Accreditation of Education Programs (CAEP) [6] and *The Common Core Mathematics Standards: Transforming Practice Through Team Leadership* by Hull, Miles, & Balka [8].

2.4. Overview and Process

The curriculum revision was a faculty-driven initiative, in which we followed a top-down approach. We looked at the major as a whole and slowly worked our way down to the course level. The work began in the summer of 2014 with the task of revising our mission as a mathematics program and what learning goals we wanted to fulfill for our students. The mission mtatement we came up with is presented in Section 2.5 and the program learning goals we settled on are discussed in Section 2.6.

After completion of the mission statement and program goals, in September 2014, we continued with a detailed review of the courses that we were offering at the time. The review focused on a few key aspects: learning objectives per course, relevance of the learning objectives in the context of the Mission, sequencing of courses, overlap with other courses, and aligning with the CUPM guidelines.

We finished the revised majors in April 2015. The proposals were submitted for review by the academic governance processes in May. The internal governance process was completed in March 2016. New York State approval for the launch of the revised program was received in August 2016, and we started advising students into the new program as of the start of the 2016-2017 academic year.

2.5. Our New Mission Statement

The hallmark of mathematics is its devotion to abstraction, generalization, and rigor. The mission of the department is to develop our students' abilities to use critical thinking, quantitative reasoning, and problem solving skills to solve mathematical problems and to bring a critical mindset to other disciplines. Students' abilities to express mathematical ideas (both orally and written) is embedded throughout the major. These skills and mathematical content are brought together through a final project housed in the department's capstone course.

Upon graduation the undergraduate mathematics majors will have the knowledge and skills for gainful employment in industry, or for graduate study in mathematics or closely related fields such as: sciences and engineering, economics or finance, or for a career as a mathematics educator. The degree is designed to provide each student with a foundation in several branches of mathematics (including calculus and analysis, algebra, and logic). In addition, the department also provides the opportunity to choose additional branches of mathematics appropriate to a students' interests and career goals, such as actuarial mathematics, and probability and statistics.

2.6. Program Learning Goals

By the start of the 2014 academic year, we had our program goals established. When the CUPM guidelines were published in early 2015, we updated the language of each of the goals to meet the cognitive and content goals set out by the CUPM. Here are our program goals:

Mathematics Goal 1. Mathematics majors will develop the computational skills necessary for more advanced work in mathematics.

Mathematics Goal 2. Mathematics majors will be conversant with the results and theorems from certain key branches of mathematics

Mathematics Goal 3. Mathematics majors will be proficient in reading, constructing, and critiquing mathematical proofs.

Mathematics Goal 4. Mathematics majors will be able to communicate mathematical ideas in oral and written forms.

Mathematics Goal 5. Mathematics majors will gain employment in their respective area of mathematics or mathematics education, or pursue graduate studies in mathematics or a related area.

Each of these goals were refined further into operational objectives, which, in turn, informed the critical review of each course, and to the decision to redesign existing courses, eliminate them, or introduce new material.

To assess how well we achieve the goals that we created for the programs we created a curriculum map that would detail where assessment would take place. Specifically, we determine if a core course introduced the goal, reinforced the goal, or if students were expected to obtain mastery of the goal. We created assessment rubrics which were then applied to a sample of artifacts from these courses, and the outcomes form the basis of our assessment. Assessment is repeated annually and will lead to curriculum reviews every four years.

2.7. The Core Courses

The core courses in the B.A. and B.S. include a Mathematics Orientation Seminar (see [12] for information on the Mathematics Orientation Seminar), Calculus I, Calculus II, Bridge to Higher Mathematics, Analysis, Abstract Algebra, and Senior Seminar I and II. (See Table 4.)

B.A./B.S. Core Courses (22 Credits)
Mathematics Orientation Seminar
Calculus I
Calculus II
Bridge to Higher Mathematics
Analysis
Abstract Algebra
Senior Seminar I
Senior Seminar II

Table 4: The Core of the B.A./B.S.

We made no changes to our Calculus I & II, Analysis, and Abstract Algebra courses because they all aligned with the CUPM guidelines.

2.7.1. The End of the Integrated Calculus Sequence

The first major revision was the end of the integrated calculus sequence. Let's examine why it was removed.

Bressoud, Johnston, Murphy, Rhea, Williams, and Zorn [2] stated that this integrated calculus model was first started at Moravian College and also added that:

> This is a successful alternative to inserting a precalculus class because students are constantly encountering new more challenging material while getting the support they need.

However, the longitudinal study revealed a different story for the students at Adelphi University.

The following regression models were created:

- Model 1: Predicting Calculus II final score using MTH 130 and MTH 131 as predictor variables.
- Model 2: Predicting Calculus III final score using MTH 130, MTH 131, and Calculus II as predictor variables.

Neither model was statistically significant, indicating that MTH 130 and MTH 131 were not significant contributors to Calculus II and Calculus III final grades.

A multiple linear regression model was calculated predicting Calculus II final scores from Pre-Calculus and Calculus I final scores. A significant regression equation was found ($F(2, 66) = 37.292, p < .001$), with R^2 of 0.531. The fitted model was (Calculus II final score) $= -0.787 + 0.637$(Pre-Calculus) $+$ 0.495(Calculus I).

A second multiple linear regression model was calculated predicting Calculus III final scores from Pre-Calculus, Calculus I, and Calculus II final scores. A significant regression equation was found ($F(3, 60) = 7.552, p < .001$), with R^2 of 0.274. However, Calculus II final scores showed to be a true predictor above and beyond the contributions of Pre-Calculus and Calculus I. Finally, a simple linear regression model was calculated predicting Calculus I final score from Pre-Calculus final score. A significant regression equation was found ($F(1, 81) = 33.698, p < .001$) with an R^2 of 0.294. (Calculus I final score) $= 0.382 + 0.772$(Pre-Calculus).

Overall our integrated calculus course appeared not to offer positive outcomes for our students proceeding forward in the major. There is no evidence that the course content or instructor of the course were a contributing factor to this; however, it does appear that student attitudes and perceptions were the key to this sequence not being a success.

The majority of the students who enrolled in Calculus 1A and Calculus 1B were first year students at Adelphi University. The Research & Planning Group for California Community Colleges [16] states that student attitudes are correlated to their first year GPA. Nist and Holschuh [11] are much more specific when they state that high school students are prepared to form an attitude that college courses move at a much faster pace, i.e., semester courses instead of year-long courses. It is quite possible that the integrated calculus sequence sent contradicting messages to students.

From the qualitative results of the longitudinal study, stretching Calculus I over a full-year had negative impacts on our students. The fall offering of Calculus II primarily had students from the integrated calculus sequence. Both students and instructors experienced frustration with this course. Students developed a feeling that when the material got challenging, the instructor would slow down the pace of the course, and this feeling remained through the entire mathematics major.

For the most part, the integrated calculus sequence created two different populations of students:

1. Those that either did the Pre-Calculus and Calculus I model and were prepared for the semester-length rigor,
2. Those that went through the integrated calculus sequence and were not prepared for the semester-length rigor and had a rougher transition into the major.

Therefore, we decided to revert back to the Pre-Calculus and Calculus I model, which we had prior to 2006. We also decided that we should add advanced topics to Pre-Calculus, such as parametric equations and conic sections.

2.7.2. From Introduction to Proofs to Bridge to Higher Mathematics

The qualitative results seemed to imply that the purpose of MTH 301, our Introduction to Proofs course, was not clearly perceived. A majority of students viewed MTH 301 as a course that was meant to be an introduction to abstract algebra and real analysis. Additionally the quantitative results indicated that MTH 301 had no significant influence on final grades in abstract algebra or analysis. Student performance in our geometry course did improve due to the influence of MTH 301; even though there were no geometry proofs in MTH 301, the emphasis on proof-based work probably helped students with them in our geometry course.

Therefore, I decided that the department should not attempt to revise MTH 301. Instead we needed to focus our attention on creating a new transition course that concentrated on building skills needed to read, write, and truly understand proofs (see [13] for the details on our transition course journey.) Bridge to Higher Mathematics was our newly designed transition course. Here is how we described the purpose of this new course:

> Students are introduced to the writing of mathematics proofs, where an emphasis is placed on precise thinking and the written and oral presentation of mathematical results. Students will express abstract ideas using constructs such as logic, set theory, relations, functions, axiom systems, number theory, probability, combinatorics, and graph theory.

The course content includes:

Course Outline for Bridge to Higher Mathematics.

- Set Theory
- Mathematical Logic and Quantifiers
- Logical Arguments
- Combinatorics
- Basic Probability Theory
- Direct Proof, Contradiction, Cases, Induction
- Formal Axiom Systems
- Equivalence Relations and Functions
- Cardinality
- Graph Theory (or other advanced topics, which is not Abstract Algebra or Analysis)

2.8. The Capstone Course

Finally, Advanced Mathematical Modeling was removed as the capstone because it was strictly producing applied mathematics artifacts. We needed something more flexible. In the end, our capstone experience courses were inspired by the research of Johnston, Webster, and Wilson [9] and Buck, Grabner, and Roberts [4].

Senior Seminar I stands as an introduction to our capstone course Senior Seminar II. This course aims to introduce students to research in pure mathematics, applied mathematics, and statistics. Students select a topic of interest that they will do formal research on during Senior Seminar II.

Senior Seminar II serves as the capstone experience for the major. Students experience a genuine research experience in mathematics, where they see first-hand the beauty of the subject. Through this, students solidify the mathematical knowledge they have gained during their academic career. In particular, students learn how mathematics evolves as a discipline, how to read and find relevant research papers, identify problems, do actual research, learn to write up mathematical results, and improve their LaTeX skills.

Students learn the skills mentioned above by developing a written thesis. This thesis consists of either original work in a field of mathematics in which the student is interested, or a vertical development of a topic, which was inspired by the work of Sally and Sally [14].

Students learn the breadth of the mathematical field through attending at least two instructor-approved seminar talks from outside speakers. The Department of Mathematics and Computer Science currently has two series of talks that bring in outside speakers: the American Mathematics Society Student Chapter Seminar Series and the Pohle Colloquium Series on the History of Mathematics. Through this opportunity, students gain an exposure to the types of mathematics outside what is currently offered at Adelphi University.

Students learn to present mathematics via poster and/or oral presentations. Along with the thesis, the students are required to create a poster detailing the problems and results of the students' explorations. These posters are presented at the end of the semester and at Adelphi University's Research Day. See `https://aurc.adelphi.edu/2019-conference-details/` for more details on Research Day.

Our department owes thanks to Dr. Branden Stone for developing this year-long capstone experience for our students. The content of this section of the paper is fully credited to Dr. Stone.

2.9. The B.S. in Mathematics

The revised B.S. in Mathematics became a "design your own major." (See Table 5.) The B.S. in Mathematics includes a standard Calculus III course and Linear Algebra course, neither of which required revisions based on the CUPM guidelines [5].

B.S. in Mathematics (49 Credits)
Calculus III
Linear Algebra
Introduction to Computer Programming
Complete either Analysis II, Abstract Algebra II, or Mathematical Statistics
4 Additional Upper-division Courses

Table 5: Revised B.S. in Mathematics.

What makes this truly a "design your own major" is the five open electives. One of the electives is set: students must select one course to complete a year-long sequence in either algebra, analysis, or probability and statistics. This year-long sequence gives students the opportunity to specialize in some area of mathematics, which was strongly recommended by the CUPM [5]. With this specialization students can then choose courses to fit their career plans. For example, a student who wishes to enter a career in statistics would complete the year-long sequence in probability and statistics, and take electives in the field of statistics, such as data visualization or regression analysis.

2.10. B.A. in Mathematics

The revised B.A. in Mathematics became a "breadth major" in mathematics, which was designed for students who wish to enter the field of mathematics education. (See Table 6.) This program aligns with the requirements for New York State Certification and the recommendations provided by the NCTM.

Geometry I did not require any revisions based on the CUPM guidelines. There were two revisions made to the Probability Theory and Mathematical Statistics courses; however, the content of the courses matched the CUPM guidelines. First, there were requirements of adding labs/projects into both courses that had the students start working with real data. Also, Mathematical Statistics was moved to a computer lab, where SPSS became a vital technology piece added to the course. (Technically, the statistical software used is at the discretion of the instructor. However, in practice all instructors utilize either the Statistical Package for the Social Sciences (SPSS) or R.)

B.A. in Mathematics (41 Credits)
Multivariable Mathematics
Geometry I
Introduction to Probability Theory
Mathematical Statistics
Computer Applications in Mathematics
Mathematics for Elementary School Teachers or Mathematics for Secondary School Teachers

Table 6: Revised B.A. in Mathematics.

There is a hybrid Calculus III and Linear Algebra course called Multivariable Mathematics. According to Tentomas [15], the geometry content in elementary and secondary schools is becoming more sophisticated, such as introducing spatial and projective geometry skills. Browning, Edson, Kimani, and Aslan-Tutak [3] state:

> studies suggest that pre-service teachers enter their mathematics content preparation programs with limited geometry and measurement experiences, experiences chiefly focused on manipulation of formulas. Work using a van Hiele model for geometric learning indicated PTs were at different levels for different concepts, they tended to be at lower levels of geometric understanding, and they were not ready for a formal deductive geometry course (Mayberry, 1983) [page 345].

Our Multivariable Mathematics course allows students interested in secondary education to study more deeply the ideas covered in a high-school mathematics curriculum from Calculus III and Linear Algebra, while more applications to the sciences and engineering remain the focus of the already existing required courses for the B.S.. This course has also been very beneficial for our Computer Science majors pursuing Game Programming.

There is also a requirement for students to explore elementary or secondary mathematical content from an advanced standpoint: Mathematics for Elementary School Teachers or Mathematics for Secondary School Teachers. The goal of both of these courses is to teach the theory behind the mathematics taught at the elementary and secondary levels. Students who have an in-depth understanding behind mathematics they are going to teach will teach it better, which is supported by authors such as Ball, Hill, and Bass [1]. The content is taught with an emphasis on how it relates to both other topics in education curricula and within the framework of advanced mathematics. As a consequence, students who learn this context will be more effective classroom instructors. For example, in Mathematics for Elementary School Teachers, students will gain a deeper understanding of the Pythagorean Theorem using Euclidean Constructions. In Mathematics for Secondary School Teachers, for example, students will gain a deeper understanding of the geometry that they will teach by exploring topics in topology.

Finally, there is a Computer Applications in Mathematics course. This course is designed to teach students about computer programming and the programming of computer algebra systems. Additionally, other recent technologies utilized in the field of mathematics/mathematics education are explored, such as graphing calculators, GeoGebra, and Geometers Sketchpad.

3. Conclusions and Recommendations for Future Research

The sun set on the old mathematics major at the end of Spring 2020. Due to the difficulty brought on by the COVID-19 pandemic, there was no complete review of the new major. Some initial results of the first two years were explained in [12]. One aspect that was inspired by a colleague of mine, Dr. Kees Leune, was how the distributions have changed in students declaring double majors and minor before and after the curriculum revisions. Studies, such as the one done by the Educational Advisory Board (EAB) [7], state that students that select double majors (minors) with interests in their future career goals are more likely to complete their undergraduate degree in four years. As indicated in Table 7, students with double majors in Math and CS has increased and total students with non-math (or CS) minors has increased. The greater increased flexibility has given our students the motivation to pursue other interests.

I end here with some questions that might lead to other case studies or research opportunities. In either case, I hope that this journey through our curriculum revision process has been insightful and inspiring.

1. Have institutions that have adopted the integrated calculus model done quantitative studies to measure the effectiveness of this model? If so, what did they find? If not, would you consider conducting such a study?

	Spring 2016	Spring 2018
Mathematics Enrollment	103	81
Dual Major (Math and CS) Enrollment	8 (7.8%)	17 (21%)
Total with CS Minor	2 (1.9%)	3 (3.7%)
Total with Non-Math Minor	11 (10.7%)	22 (27%)

Table 7: Minors and Dual Majors.

2. Have institutions that have adopted this model done a qualitative analysis of students and faculty to see what the perceptions are with respect to the big picture of mathematics? If so, what did they find? If not, would you consider conducting such a study?

3. Do you have an interesting curriculum revision story to share?

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