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Book Review: What is a Mathematical Concept? edited by Elizabeth de Freitas, Nathalie Sinclair, and Alf Coles

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Synopsis

This is a review of *What is a Mathematical Concept?* edited by Elizabeth de Freitas, Nathalie Sinclair, and Alf Coles (Cambridge University Press, 2017). In this collection of sixteen chapters, philosophers, educationalists, historians of mathematics, a cognitive scientist, and a mathematician consider, problematise, historicise, contextualise, and destabilise the terms 'mathematical' and 'concept'. The contributors come from many disciplines, but the editors are all in mathematics education, which gives the whole volume a disciplinary centre of gravity. The editors set out to explore and reclaim the canonical question 'what is a mathematical concept?' from the philosophy of mathematics. This review comments on each paper in the collection.

What is a Mathematical Concept? Edited by Elizabeth de Freitas, Manchester Metropolitan University, Nathalie Sinclair, Simon Fraser University, British Columbia, Alf Coles, University of Bristol, Cambridge University Press, 2017, 288 pp., \$71.50 (hardcover) \$66.49 (kindle)

In this collection of sixteen chapters, philosophers, educationalists, historians of mathematics, a cognitive scientist, and a mathematician consider, problematise, historicise, contextualise, and destabilise the two longest words in the title question. It won't be a spoiler to reveal that none of them offers a crisp answer. This is hardly a surprise because, as Nietzsche observed, nothing that has a history can be crisply defined, and the words 'mathematical' and 'concept' have rich, interconnected, and unfinished histories [8]. The contributors come from many disciplines, but the editors are all in mathematics education, which gives the whole volume a disciplinary centre of gravity. There is also an agenda, which is set out with commendable frankness in the editorial introduction, "We conceived this book as a collection of essays exploring and in some sense reclaiming a canonical question—what is a mathematical concept?—from the philosophy of mathematics." (page 2). There then follows a thumbnail sketch of the history of the philosophy of mathematics that results in a three-way categorisation of philosophers of mathematics:

- *cognitivists* who think that mathematical concepts "exist in the mind and are created by the mind" (with Descartes, Locke and Kant offered as examples, with some reservations)
- *realists* who think that "mathematical concepts exist outside the mind and are independent of all human thought" (Plato, Frege and Gödel offered as examples, without qualification), and
- *nominalists* who think that mathematical concepts are merely symbols or fictions (no examples offered, curiously)

The volume editors add some qualifications to this categorisation and note that there is overlap and influence between members of the three groups. This then is the philosophy of mathematics from which the title question is to be reclaimed: a discipline that carves out its dialectical space through a succession of absolute oppositions: mental/non-mental, corporeal/non-corporeal, real/fictional.

In the next paragraph of the introduction (page 3), the focus of fire changes. A bridging paragraph on the allegedly ubiquitous influence of Kant on philosophers of mathematics prepares for a shift to a new opponent, perhaps the real opponent for these editors. Kant's view that mathematics is in some sense all in the mind is rediscovered in theories of learning (including, especially, constructivism) and cognitive psychology. In this Kant-derived outlook, concepts, whether they are acquired or constructed, are cognitive items. Therefore, "concepts are treated as abstractions that ultimately transcend the messy world of hands, eyes, matter and others" (page 3).

In contrast, the editors recommend learning theories that pay attention to "how concepts are formed in the activity itself rather than in the rational cognitive act of synthesizing them", and address, "the role of the body in coming to know mathematics" (page 5).

Turning to the contributions, the first is an oddly old-fashioned paper from the young historian of mathematics Michael Barany. He sets out his version of the 'strong program in the sociology of knowledge' (SSK) using a folksy metaphor from Barry Barnes's work of 1983 and the main mathematical case study from Lakatos's *Proofs and Refutations* [6] of 1976. In the references for this paper, there is nothing from the present century except one of his own papers and Ian Hacking's *Why is there philosophy of mathematics at all*? [5]. The references do include David Bloor's (1978) 'Polyhedra and the abominations of Leviticus' [1], in which Bloor uses Lakatos's dialogue about polyhedra as an example of the sort of thing SSK talks about. It's true that Bloor did it differently (by drawing on Mary Douglas's work on hygiene taboos) but nevertheless it is disappointing that Barany did not work up a new example. The suspicion has always clung to Lakatos's philosophy of mathematics that the Euler formula about polyhedra is a singular case that suits Lakatos's purposes precisely because it is untypical of most mathematics (see for instance Chapter 7 (pages 151–174) of [2]). Insofar as SSK accounts of mathematics rely solely on Lakatos's case study, the same suspicion clings to them.

Barany's paper permits an observation about the field in general. At the back of the SSK is a reading of the later Wittgenstein. SSK has this in common with a lot of sociology of knowledge and educational theory. Whatever the merits of these approaches derived from Wittgenstein, they do not (mostly) fulfill the promises of the volume editors to get into the messy world of hands, eyes, matter, others and the body. While Wittgenstein tied meaning ultimately to our animal nature ("if a lion could speak" etc. [7, page 223]), he did not give much sense of the lived-in body in his writing. For example, he imagined a community that communicates in commands for slabs and beams [8, §2], but we never gain from him a sense of the heft of a slab or the texture of its surface. Presumably, habitual slab-deliverers would feel an anticipation in their muscles and an expectation in their fingers that, for them, might be part of the meaning of the command 'slab!'.

Wittgenstein's descriptions of language-games and forms of life may be embodied, but they are not bodily. For that, one has to look elsewhere, to other, less fastidious phenomenologists.

The second paper, by Reviel Netz, takes as its case study the Archimedes palimpsest. Before he gets to work on the mathematics, Netz argues that the term 'concept' is "an artefact of twentieth-century philosophy" (page 36), arising from Brentano's observation that human thinking is always about something and Frege's distinction between sense and reference. It's not obvious how this claim about the history of the word 'concept' would mesh with the history of cognitivism sketched in the editorial introduction. In any case, the thrust of Netz's argument is that Brentano's *Beziehung* and Frege's *Sinn* are not "historical agents" (page 48), because "Science is what people do together, not what they each have privately, in their heads. Indeed, the main line of historical change involves, quite simply, the different ways in which things are done together—the transitions of scientific practice, which have nothing to do directly with concepts" (page 49). In short, his answer, as a historian, to the title question is: an irrelevance. He illustrates this by showing that ancient Greek mathematicians may not have had the concept of set, but they had practices that involved actual infinities. So it is not true to say that they could not have understood the actual infinite character of the natural number sequence.

The third paper, by Juliette Kennedy, is the first one to engage with the promise to explore mathematics as something done with the body and with material artefacts. She considers the role of drawings and diagrams in mathematics, both the diagrams that play essential roles in proofs and also the incidental drawings that mathematicians create on the fly to help their thinking. She argues that in order to understand the role of either kind of picture, it is necessary to reconsider the distinction between syntax and semantics. Kennedy's paper is also a rare moment in this collection where the opposition gets a fair shake. She goes to some trouble to explain how and why the distinction between syntax and semantics arose, and why Pasch and Bolzano tried to push spatial intuition out of mathematics. The editorial introduction speaks dismissively of "all-too-easy dualisms" and "conventional approaches" (page 6). Kennedy shows that the distinctions between syntax and semantics, and between mathematical reasoning as Bolzano understood it and spatial intuition, are not easy conventions. She shows that they were hard-won cognitive achievements, even as she argues against them.

She draws on Ken Manders' work on Euclid, and on studies of architectural drawing, because, "Architectural practice has not otherized the body in its critical discourse, as the epistemology of mathematics has done" (page 69). She is alive to the "problem of developing contentual inference" (page 72), more specifically of working up a notion of inference that involves "manual/practical agency" (page 61). She sees clearly that it is all very well to situate the body in mathematics and mathematics in the world, but this entails a requirement to show that some of these bodily doings can constitute mathematical proof, properly so called.

The fourth paper is jointly written by two of the editors, Elizabeth de Freitas and Nathalie Sinclair. They appeal to the work of Gilles Châtelet to argue that mathematical concepts are not abstracted from sense experience, are not fixed essences and are not historically inevitable. Mathematics is present in shared practices, it is generative and it might have been different from the way it is. The word they use they explain how mathematics is present in shared empirical reality without being merely empirical is 'virtual'. "We believe that mathematics cannot be divorced from reality and it is the virtual dimension of matter that animates the mathematical concept" (page 80). As well as Châtelet, they draw on Gilles Deleuze, thus, "A concept brings with it an entire 'plane of immanence' on which we operate, explore and create" (page 87). Writing as educationalists, they aim to "rescue mathematical concepts from the staid curricular lists" and "promote awareness of the vitality and indeterminism that lie at the source of curricular concepts." (page 87). One cannot help wondering whether Platonism is the principal culprit for the apparent staidness, fixity and moribund quality of the lists of concepts on mathematical curricula. Perhaps some blame must also lie with credentialism, managerialism, the cult of league tables, bureaucratic inertia, and the determination of education ministers to take their own formative experiences as perfect models for future generations.

This paper, like the editorial introduction, is marked by some partisan rhetoric. For example, "Miraculously, the material sensation becomes immaterial (a dualism of mind and body). This approach remains committed to an abstract/concrete binary that ultimately abandons mathematics to a realm of the inert and disembodied" (page 77). Abandonment is bad, but no-one is named and no sympathetic reconstruction of the motives for the condemned approach is undertaken. The reasons that drove Plato to think of mathematics as having one foot in the realm of unchanging, intelligible forms are not explored. Since his name appears here (page 78), it is worth recalling that Plato did not think that mathematics was the wholly intellectual study of something perfectly inert and disembodied, precisely because it starts from empirical experience and has recourse to sensory models of its objects (see *Republic* 510-511, 533 [9]). In the *Republic* and elsewhere, he worried as earnestly as anyone in this volume about the problem of getting from empirical experience to knowledge of the pure forms. In any case, the relation between body and mind is hardly an unsolved problem, so a philosopher who brackets it in order to say something specifically about mathematics is not necessarily trafficking in miracles. Later, we learn that, "Galileo courageously proposed an alternative to Aristotle's teleological explanation" (page 79). No account is given of why this was courageous on Galileo's part.

Deleuze appeared in the fourth paper, and he dominates the next two. In the fifth paper, Arkady Plotnitsky first sets up the definition of philosophy that Deleuze and Guattari offer in *What is Philosophy?* [3]. According to them, philosophy is the activity of inventing new concepts (with all the important words in that definition subject to explication and refinement—they don't just mean spinning new notions out of nowhere). The project of Plotnitsky's paper is to show that something like the same creative activity happens in mathematics. This goes beyond and in some way against what Deleuze and Guattari had in mind, because they were trying to identify what is special about philosophy. As he explains, "I argue that creative *exact* mathematical and scientific thought is defined by planes of immanence and invention of (exact) mathematical and scientific concepts, the architecture of which is analogous to that of philosophical concepts in Deleuze and Guattari's sense" (page 99). Plotnitsky evidently understands that in order to deploy Deleuzian notions such as the plane of immanence in the case of mathematics, one has to show that such terms properly apply to mathematics. He chooses Riemann as his mathematical case study, because Riemann emphasised conceptual understanding over axiomatics. Like de Freitas and Sinclair, he wants mathematics education to be a pedagogy of creativity. Perhaps there might be a way of saying this without demanding that readers get to grips with Deleuze and Guattari. There better had be, if this thought is to have any hope of catching on.

The sixth paper is also a piece of Deleuze scholarship. In principle, mathematicians can explore any mathematical structure that they like. In reality, they study structures that are interesting. 'Interesting', here, covers the whole history of how problems and questions give rise to ideas and eventually precipitate 'interesting' mathematical structures. Such histories are always richer, messier, and more dogged by contingency than one might think, looking at the tidy accumulations of mathematical theory that make up curricula. The problems, questions, and intuitions present in the history of a piece of mathematics may well continue to shape the field as mathematicians continue to work on it. Simon B. Duffy explains how Deleuze, drawing on Lautman and Cavaillès, embeds this fact in his larger philosophical programme. Whether this tells us anything new about mathematics is a question for Deleuze specialists.

Chapters seven and eight shift away from Deleuze into the worlds of type theory and arithmetic geometry. David Corfield begins his paper with a useful discussion of varieties of mathematical concept, "there does seem to be a difference between ... a core constituent of the visionary outlook of a large scale research programme, like that of Langlands, and ... some stably defined kind of entity, such as a group. There is also a difference between the 'concept of (a) group' and the 'concept of symmetry'" (page 128). Later (on page 137), he mentions Dudley Shapere's point that the persistence of a word (such as 'atom') does not guarantee the stability of a concept, nor does a change of vocabulary necessarily indicate a sharp break in the development of ideas. The bulk of his paper is devoted to showing that homotopy type theory can express what Corfield calls the vertical integration of mathematics. Children learn to add and multiply numbers at an early age, and a little later they learn about raising numbers to powers. Some of them will go on to learn that all sorts of objects can be added and multiplied. A few will learn to think of exponentiation in terms of mappings between sets. Even at the most abstract level of mathematics, mathematicians think about analogues to primary-school addition and multiplication. Homotopy type theory can express this thought in precise mathematical terms. As with arithmetic, so with geometry; there are circles all the way up (page 138). This vertical unity of concepts offers a response to a criticism one might make of the emphasis in other contributions to this volume on the bodily quality of mathematical practice. The examples offered in support of this claim are almost always drawn from very elementary mathematics, perhaps because the proponents are mostly in mathematics education and have school mathematics in mind. Sure, the objection runs, for children, arithmetic means manipulating bricks and geometry means drawing shapes, but in what sense is (for example) the theory of modular forms immanent in material doings and goings-on? Corfield's argument for the vertical unity of concepts suggests a direct way that the rudimentary mathematics of blocks and string connects to the most abstract mathematical notions, and not merely as intuitive or heuristic resonance, or as part of the problem-history.

Peter Scholze has just been awarded the Fields Medal for his development of the theory of perfectiod spaces. Chapter eight is a valiant attempt by Michael Harris to lay out the mathematical background to this development, to explain Scholze's achievement, and to defend his judgment that, "perfectoid geometry provides *the right* framework for thinking about a number of central questions in algebraic number theory, and secondarily, that the cohomology of perfectoid spaces has *the right* (p-adic integrality) properties for applications to such questions. . . . and . . . that number theorists had made *the right* decision to devote time to learning Scholze's new framework even before it had been shown to have important applications to their field" (page 156; emphases in original). This is in some tension with the problematising, anti-canonical spirit of the editors. Harris goes on to observe that "It is possible to talk sensibly about convergence without succumbing to the illusion of inevitability" (page 157). The state of the discipline in 2010, its open questions, its established methods, and overall aims jointly posed a question. According to a unanimous consensus of experts, perfectoid geometry is the answer, and the chief reason for that, according to Harris, is that it solves many problems in other fields that were not among the targets when it was developed. (With Lakatos in mind, we might call this 'quasi-empirical progress'.) Harris's paper is the peak of technical difficulty in this book, by some distance.

The chapters reviewed so far are grouped in the table of contents into pairs: history, the mathematical hand, Deleuze, high-level research mathematics (my labels). The next group is a trio of papers on school mathematics in culture and politics. Like others in this volume, Heather Mendick is frustrated by the oppressive dreariness of mathematics education (she writes in the UK context). Rather than turn to the abstractions of Deleuzian philosophy, she takes up the more bodily resources of queer theory. A queer approach refuses definitions of its own and seeks to unearth the means by which definitions and oppositions are established and discourses shaped. Having learned to do this in the case of some basic binary oppositions (male/female, heterosexual/homosexual), queer theorists now apply this approach to any other concepts, including mathematics. Mendick discusses the way in which the shaky distinction between conceptual understanding and rote-learning in mathematics education comes to be entrenched and then gendered, with boys associated with the former and girls with the latter—in spite of test results that show no such thing (page 167). Part of her complaint is about the bogus quality of mathematics as experienced by school students, especially in the form of word problems. She compares school mathematics to Christmas, quoting Eve Kosofsky Sedgwick, "The depressing thing about the Christmas season... is that it's the time when all the institutions are speaking with one voice" (page 167).

In the next paper, Richard Barwell and Yasmine Abtahi apply discursive psychology to explore the way in which Canadian mass media frame developments in school mathematics as a stand-off between (good old fashioned, effective, understandable, parent-friendly) procedures to be learned by rote and (new-fangled, confusing, parent-baffling) conceptual understanding to be learned by discovery. This polarisation was politically charged: preferring the good old ways was presented as the natural stance of cultural conservatives, while trusting children to discover mathematical concepts was obviously part of a constellation of liberal-left attitudes. Once the question of how to educate children in mathematics (which one might think would demand nuance) has been polarised and politicised, there is little hope of doing it better. This is not only a Canadian phenomenon. A mathematician once remarked to me that you can know a lot about a British person's politics if you know their views about the teaching of long division.

In the third paper of this group, Tony Brown appeals to Althusser, Žižek and Badiou to argue that the production of mathematical concepts is a kind of commodification. He is on his strongest ground in his discussion of school mathematics, because it is clear that whatever the forces are that shape the school curriculum, they are not solely the kind of internal mathematical reasons that Harris describes in articulating the *rightness* of perfectoid spaces. Nor, evidently, is the curriculum driven principally by the educational needs of children. That said, one might wonder whether we need Žižek to tell us that in the secondary schools of a country that has a national curriculum and an aggressive regime of school inspection and ranking, it's the government that decides what is mathematics.

His wider argument for the relative arbitrariness of mathematical concepts is less successful:

> Geometrical constructions ... are routinely built into our physical landscape such that we do not notice them any more. We become accustomed to moving around such landscapes and those ways of moving become part of who we are. For instance, ... we use *circle* as a concept in building our world, and as a result circles become materialised or absorbed in the very fabric of our physical and conceptual world. Stellated octahedrons, in contrast, have been denied that level of intimacy and familiarity with humans. Geometrically speaking, there is no reason as to why one might be privileged over the other. Circles have been reified not because of any essential difference between them and say, stellated octahedrons, but because of merely historical and political reasons. (page 193)

If you throw a stone into a still pond, it creates circular ripples. Whirl a slingshot or a bull-roarer round your head, and it describes a circular path. The sun and the full moon are, to the human eye, circular disks. Circles really are fundamental in nature in a way that stellated octahedrons are not. Moreover, circles turn up in mathematics unbidden in ways that stellated octahedrons do not. What are the roots of unity? Points on a circle. There are reasons—historically and politically situated but ultimately mathematical reasons—why Corfield can show that type theory has circles all the way up. The circle is one of *the right* canonical shapes.

In the twelfth chapter, the third editor, Alf Coles, argues that "there is considerable pedagogical advantage in viewing *every* mathematical concept as a relation" (page 205; emphasis in original). He begins with basic arithmetic, drawing on the approaches of Caleb Gattegno and Vasily Davydov. Rather than starting children in arithmetic by counting, these curricula start with comparisons of lengths. By playing with coloured rods of various lengths, for example, children can see that two of these end-to-end are as long as one of those. The numbers 2 and 1/2 thus appear, simultaneously, as relations between rods. This, it is argued, is less mysterious than requiring children to abstract two-ness from a pair of blocks and then, having achieved this feat, learn a whole new way of thinking about numbers when they meet fractions.

Coles goes on to give similar re-framings of area, algebra and complex numbers as relations. He makes a plausible argument that this relational view helps to resolve the headache of how embodied creatures can know abstract mathematics. Relations are non-material, yet are unproblematically real. Small children know about relations like 'is a parent of' and 'is a sister of'. They are interested in who is older and who is taller. As he puts it, "The relation 'double' does not exist *in* either the smaller or the larger rods, but arises through a human comparison between them. In this sense, relations and differences are *always already* abstracted from the objects that give rise to them" (page 215; emphases in original. This is one of two occurrences of 'always already' in this volume—the other is on page 199). Moreover, making a comparison is an action, so the relational view that Coles advocates dovetails with the enactivist philosophy of mind that is a theme of this book. Interestingly, a view very like this has been argued from different sources already by Robert Thomas, who claims that mathematics is, "*the science of detachable relational insights*" [10].

In the next chapter, Wolff-Michael Roth treats concepts as classifications—in terms of philosophical logic—as sortals. Thus, he excludes Coles's relational view of mathematics. He argues that we learn sortal concepts by using them rather than by grasping their essences, and describes a class designed on enactivist principles in which children are to learn the meaning of 'cube'. He argues, drawing on Vygotsky, that there is nothing more to a concept than its extension. One wonders what an enactivist account of learning the meaning of 'perfectoid space' would be like.

The final pair of papers take up metaphors of evolution and growth. Like the other educationalists in this volume, their authors are looking for theories and metaphors to disrupt and resist the stultifying dreariness of school mathematics curricula and required teaching methods. Brent Davis offers a mixture of Richard Dawkins's meme theory of culture, complexity science, and embodied cognition. He uses these materials to make a useful point against Lakoff and Nuñez's account of mathematical cognition, "complexity thinking cautions against conflating the original parts with the emergent possibility" (page 243). Mathematical thinking may start with material metaphors, but that does not entail that it must be material metaphors all the way up (and yes, the use of 'up' in this sentence is an example of the sort of thing Lakoff and Nuñez talk about). There is an interesting moment in this paper where Davis announces that, "in the humanities, a century of psychoanalytic, phenomenological, and structuralist thought has redefined *idea* as a contingent, situated, and volatile form that exists in the material world of action and agency" (page 238). This may be true of most of the humanities, but the mainstream tradition in English-speaking philosophy still stems from the Fregean arguments that Netz briefly mentions in his contribution (see especially [4]).

Similarly, Ricardo Nemirovsky reaches for the metaphor of crystalline growth, because this is marked by contingency and relational properties—how a crystal grows at a point depends on what is going on around it, chemically and physically. Crystal growth depends on impurities like a pearl needs a speck of grit. Nemirovsky uses this image to argue that Cantor's *Grundlagen* of 1883, in which he introduced transfinite ordinal numbers, is better understood in these terms. "I propose that the *Grundlagen* was a singularity for the growth and dissolution of the number concept. The more general point is that Cantor authored a singularity, not a concept" (page 262).

The final contribution is a collection of miscellaneous reflections on the themes and contributions of the volume by David Pimm, chiefly attending to linguistic clues that suggest semantic layers and movements. For an example of the style, he notes three occurrences of the -oid suffix in this book: cuboid, groupoid, and perfectoid, and plays with the etymological root, the Greek *eidos* (page 278). In one sly observation (page 279), Pimm notes that, "A goodly number of the mathematicians mentioned [in the book] are nineteenth-century German-speakers, while many of the philosophers drawn on are twentieth-century French-speakers. Bolzano, Cantor, Dedekind, Frege (?), Riemann, ..., Althusser, Châtelet, Deleuze, Derrida, Foucault, Lacan, ... Why not the other way round?"

There is one other issue I want to raise. It's notable that the contributors from mathematics education often mention the views of great dead philosophers without giving any references. This happens a lot in the editorial introduction. At the other end of the volume, Pimm mentions, twice, "Kant's striking assertion that 'Mathematics is pure poetry"' (page 274). I should have liked to read that in context, but Pimm gives no clue. Occasionally, a great dead philosopher is given a reference but in a perfunctory fashion. For example, another of the authors wrote, on page 223:

> In the classical, constructivist approach, empirical concepts are derived from experience by means of abstraction (Kant, 1956).

I'm not sure that Kant said exactly that. My own reading of Kant makes me want to check. Which page of Kant's extensive writing did our author have in mind? Turning to the list of references, I read:

> Kant, I. (1956). Werke [works]. Wiesbaden, Germany: Insel.

What is going on here? I suspect that Kant has been tagged with a view that did not even exist in his day, just as Plato is tagged with 'Platonism' and Adam Smith is tagged with the ideological programme of the Adam Smith Institute. But I can't check, because classical constructivism is described in a single phrase (which incidentally makes for a very easy victory), and the reference to Kant's works is wholly unhelpful. Perhaps this does not matter for the purposes of research into mathematics education, because the gesture in the direction of canonical philosophers is simply for orientation and throat-clearing. On the other hand, when a project is set up in opposition to a long-established orthodoxy, the orthodoxy had better be precisely described and located in specific texts. Otherwise, there will be no meeting of contending perspectives. Undergraduates in the humanities are required to give references, with page numbers, to show that their scholarly opponents are not made of straw.

Overall, this is a rich and interesting book written by people who love mathematics and wish it were better appreciated for its vitality and its value as a human activity, both in philosophy and in school classrooms. Many—perhaps all—of the contributors have taught mathematics at some time and through the book, a teacherly delight in sharing knowledge alternates with frustration that the teaching of mathematics could be better, more fun, and perhaps less implicated in the systems and discourses that categorise children crudely and reductively.

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