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Beyond the Classroom: Mathematics in Service

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Synopsis

Mathematical expertise demands effective thinking and learning methods, and these techniques transfer well to other domains. In this article, I discuss how my own training as a pure mathematician influenced my performance in three disparate domains: electrical engineering, art appreciation, and learning Italian. In electrical engineering, the focus is on how mathematical reasoning and thinking processes impact knowledge acquisition and problem solving. Appreciating and analyzing art raises the question, “How do we know for certain?” Acquiring fluency in another language is akin to gaining mathematics proficiency, and here, I explore the human side of persistence. The article combines narrative, reflection, analysis, and teaching ideas to suggest how, when teaching our subject, the mathematics community might pass on our core strength: our thinking and learning methodology.

1. Introduction

“My mathematics background is saving me.”

I told myself this every time I submitted homework for a free online Digital Signal Processing course. Although the two leading professors steeped the course in mathematics, the course targeted electrical engineers. For ten weeks, I scoured the web to learn assumed, basic engineering concepts; upped the 12 hour per week study requirement to 35 hours per week; handed in assignments minutes before the course’s weekly deadlines; and counted off the weeks as I slowly strove to meet the course’s completion date. I had ventured into a different field with its own vocabulary, way of thinking, and culture. My mathematical knowledge — linear algebra, real and complex analysis —

did keep me afloat, but I also noticed from histograms posted after each weekly exam that a large number of engineers were fading. Then it dawned on me. In addition to subject knowledge, it was my mathematical training that successfully carried me. Moreover, as I reflected on my pursuits outside the STEM fields, years studying, doing, and teaching mathematics prepared me for life in more ways than I could have imagined.

“Mathematics drives careers,” we tell our students, and a student might infer that it is mathematical knowledge that prepares one for success. In the STEM fields, a deep knowledge of mathematics does build a solid foundation, but mathematical training provides more than knowledge. When I had asked my daughter’s friend whether or not his mathematics courses were useful in medical research, he contended, “Yes! I’d advise someone to take as many mathematics courses as possible. It is not so much the knowledge, but the thinking methodology.” This methodology, although not exclusive to mathematics, is intrinsic to learning mathematics well, and it teaches valuable lifelong learning skills.

Mathematicians know how to learn. We contemplate ideas, meticulously examine definitions, search for patterns, logically derive conclusions, persevere in problem solving, converse with each other, and articulate precisely and succinctly. When we step into other branches of study, given time, we do well. Instinctively we know this; for this reason, in our classrooms, we pass on our methods with inquiry learning, active learning techniques, cooperative learning, and cross disciplinary collaboration. Books on problem solving and proof techniques aim to teach effective thinking strategies. We advise, provide classroom opportunities, and promise that these skills are useful, but do we cite specific examples? Do we share our stories?

This article is a concrete example of an abstract idea: how learning mathematics has influenced and helped me in my life; that skills acquired when learning mathematics transfer to other domains. I will share how my mathematical training served me in dissimilar endeavors — electrical engineering, art appreciation, and learning Italian. My experiences have convinced me that teaching about thinking and learning (thinking about thinking is metacognition) is crucial, especially in today’s rapidly ever-changing world with the accompanying all-too-easy access to sometimes-false information. Mathematicians have a good product to offer students. By passing on thinking and learning methods that our subject demands, we empower our students.

2. Electrical Engineering: Applying Deep Learning Skills

When I was hired by a large aerospace company, I stepped onto foreign territory, an alien culture with its own customs and language. People spoke, and nothing I heard made sense. Those first few months, panic gripped my entire being. In order to still waves of anxiety, I braced myself with the observation that students, who had struggled with calculus, were now employed engineers. But it was a specific memory from a colloquium long ago that provided my survival strategy. The invited speaker, an aerospace engineer, told the students, “My electrical engineering courses did not provide the topics I encountered in my present job. I just dug in and learned.” And so, I donned my mathematical armor, complete with knowledge and training, to dig in and learn.

Definitions, terms, notation! I told myself to begin with the fundamentals, as I had every time I attempted to learn a new branch of mathematics.

Adapt to new notation: The mathematician’s complex number i is the engineer’s j (they reserve i for current).

Probe for reasons: The engineer’s one-dimensional Fourier transform looked different than the transform I had seen in my functional analysis books. It is expressed as a function of frequency f , rather than of angular frequency $\omega = 2\pi f$. Electrical engineers find it easier to think in terms of “cycles per second”, or hertz. Note that the radio dial is calibrated in f , not in ω . Moreover, contrary to pure mathematicians, who prefer the Fourier transform to be a unitary operator, the engineers don’t care. The mathematician’s Fourier transform pair has the same coefficient, $\frac{1}{\sqrt{2\pi}}$, in each formula, whereas the engineer’s non-unitary transform pair has the coefficient, $\frac{1}{2\pi}$, only in the inverse transform. Furthermore, if I walked downstairs to chat with the image processing engineers, they worked with the non-unitary Fourier transform but written as a function of angular frequency!¹

Never assume because assumptions sprout errors: The conventional satellite’s azimuth and elevation coordinates are defined differently than the spherical coordinates that I had taught in calculus class. [Even within the engineering field, definitions differ. The NASA definition of left and right polarization is opposite to that the IEEE (Institute of Electrical and Electronics Engineers).]

¹Wikipedia has a list of other conventions for the Fourier Transform, available at https://en.wikipedia.org/wiki/Fourier_transform#Other_conventions, last accessed on July 11, 2019.

Relate ideas to mathematical counterparts: The space of finite energy signals is the Hilbert space L^2 .

All was fine for me if a book was available, but that was not always the case. Frequently in large corporations, critical information is internal which means that employees learn from colleagues or in-house documents that sometimes are indecipherable for the newcomer. The survival trick is to grab knowledgeable colleagues who value and speak with precision and accuracy. Then ask questions shamelessly, challenge perceived inconsistencies, and pin down ambiguities. Next, for self check, repeat back and ask for a critique, or write for evaluation. In academia we do exactly that when learning from professors, research advisors, and colleagues.

With time, in engineering, as in math, persistent study reaps knowledge. The longer one hangs around an unfamiliar field of knowledge, the more definitions become alive with meaning, equations unlock their mystery, concepts take shape, and ideas begin to connect into a structure of understanding. Believing this, my first half year I labored mightily to master engineering, physics, information technology, and my job. However, quality thinking extends beyond knowledge acquisition, and the full force of this lesson was brought home to me my first six months.

A grave system error surfaced two months prior to my employment. My team, spread throughout the country, each section with its specific role, met once a week, and as the error continued to plague the system, the discussions increased in heat and volatility. After five months on the job, I recognized that we needed someone with good abstraction skills and a deep knowledge of the system to resolve the error. Therefore, I asked a mathematician, with those qualifications, to help us. He questioned the different sectors, and five days later, after a phone conversation, he quickly sketched on scrap paper and blurted out, “I solved the problem!” Coordinate confusion errors! The resolution depended on his ability to see the big picture, connect the abstract mathematical theory that interwove with the engineer’s practical application, and mine the details.²

The task to re-write the interface documentation, succinctly and clearly, landed on my desk. In mathematics, every time we read, write, listen, ask questions, and converse with each other, we are perfecting our mathematical communication skills. Thankfully, I had many years of practice.

²I wrote a bit more about this problem as well as about using visual presentation tools skillfully in an earlier essay; see [2].

To write an interface document that would prevent future problems, I needed to understand the topic completely — every unspoken, unwritten gap. First, I studied the original document. Next, I extracted knowledge from the sharp, quick-thinking mathematician and decoded engineers' section reports. Besides gaining insight for my personal understanding, I, very importantly, needed to understand the source of confusion in my colleagues' heads. Years of teaching prepared me for that aspect. In a first linear algebra class, coordinate change problems are straightforward exercises. Here the coordinate change errors occurred in two places — ground horizontal-vertical coordinates to a satellite's azimuth-elevation, and linear to circular polarization. Subtleties abound in both cases. Therefore, when I re-wrote the document, I included the necessary mathematical details with brief explanations to prevent future possible errors. When done, I did what I was taught to do in my mathematical training. I handed it to several colleagues for proofreading. By the time I gave my final pitch for approval to replace the original with my updated, revised document, I had weathered my baptism of fire and acquired confidence.

This story and my continued time in aerospace proved to me that the mathematical thinking methodology — our obsession with precision, logic, analysis, problem solving, abstraction, and clear communication — transfers well and adapts successfully to STEM professions. Schools supply the required practice to build the expertise for the ever-changing technical fields, but later success hinges on that we know how to learn, acquire knowledge, apply knowledge in new ways, and communicate effectively. Engineers, scientists, and mathematicians, who learn, not the superficial for-the-test-learn but rather continuously push for that deep mastery, and who embrace the mathematical reasoning and thinking processes, drive advances in their field, creatively problem solve, and effectively communicate.

A psychologist remarked to my daughter at her college open house: "Philosophy keeps psychology honest." One might argue that "Mathematics keeps the sciences, engineering, and technology honest." With the appropriate spin, the mathematics community has an opportunity: we are the STEM traffic cops.

3. Art Appreciation: Unearthing Certainty Buried in Ambiguity

An equation, symbolically, reveals information, and an artwork visually communicates a message. At my first workshop in Art Goes to School (AGTS),

a volunteer art appreciation program that services the elementary schools,³ an Alexander Calder gouache stared at me from across the room.⁴ I stared back, and over the hour the picture metamorphosed from “just one of those abstract artworks” into a harmonious, balanced rendition of celestial objects.

That day, I could not describe why the composition seemed balanced, I could only feel it. But I had had the same problem when introduced to abstract algebra eons ago. I could “see” the ideas, but could not write a proper mathematical proof to support my visualized conception. To articulate a convincing argument, I had to learn the vocabulary, the language, and algebraic reasoning. Similarly, over the years, as an AGTS volunteer, I came to learn the artist’s tools and methods: that color, value, line, shape, form, space, and texture build a composition using design principles to create an effect and convey intent. As I continued to look at art and learn the artist’s visual language, my visual acuity grew and so did my proficiency for “explaining” artwork. But what I find interesting is that while art invites personal interpretation, objective observation and analysis can unfold meaning.

When I teach an AGTS session, I try to encourage the children to actively engage their minds as well as their imaginations. An especially memorable teaching moment occurred in a fifth grade class during a discussion of Hughie Lee Smith’s “Boy with Tire” [3]. After the students described what they saw, not just one, two, or three children, but the majority claimed that the boy was sad and depressed. I probed further by asking them why they thought that. They replied, “Because he lives in a poor neighborhood; the houses need fixing.” I then asked the students to stand as if they were sad and depressed. Every student hung their head and slouched. Next, I requested that they stand like the boy in the picture. They all straightened their bodies and looked directly ahead. At that moment, one student could barely contain himself, “I know what the picture is about! You don’t need lots of toys to be happy.” From that point, the children focused on the boy’s toy, the tire. By encouraging the children to look closely and rethink, albeit kinesthetically, an assumption was challenged and a middle-class suburban boy’s perception shifted.

³See <http://artgoestoschool.org/> for more information on the AGTS program of Delaware Valley.

⁴Alexander Calder, *The Stars* (Les Astres), 1966 Musée National d’Art Moderne Paris, gouache on paper, available to view at <https://www.photo.rmnm.fr/C.aspx?VP3=SearchResult&IID=2C6NUOVNYTFH> last accessed July 11, 2019.

The difficulty when analyzing artwork is that fuzzy line between objective observation and opinion. For example, there is much that we do not know about the boy in the painting “Boy with Tire”, but my students eventually inferred that the artist did not intend to depict a boy who looked depressed.

Critical thinking and problem solving techniques that we apply in mathematics adapt well to art appreciation strategies. Why don’t you give it a try? Select an artwork of your choice, and take time to observe. Then play detective. Describe what you see. Ask questions. Which ones can you answer, which ones can you not? Make inferences and justify them using visual evidence. How do you feel when you look at this picture? Can you substantiate or validate your reactions to the picture by analyzing the picture’s subject, style, or artistic techniques? What do you think the artist is trying to communicate? Enjoy the process (this is not a test on art knowledge), but try to be cognizant of your thinking processes.

Note that the seeds of mathematical problem solving are present in the process outlined above: determine the observable facts, brainstorm for possible solutions, and logically test conjectures based on visual evidence. Next, grab a friend or colleague, and do what we do in mathematics: discuss, compare ideas, and brainstorm for new ideas. Play the “detective game” together. Because artwork evokes multiple meanings that stem from life experiences, this step can be especially helpful. When we collect and integrate the insights, viewpoints, and questions from other people, we gather more evidence to test and question our perceptions. For example, in “Boy with Tire”, my husband observed that the boy’s poised, upright stance emitted an air of confidence. This observation revealed information about the boy, and provided me the insight to use body language when teaching. But I also did my research, read about the artist, Hughie Lee-Smith, and got lucky. I discovered in the AGTS research files a letter that Lee-Smith had written to a volunteer about this painting: “When I was a little boy many, many years ago, proper toys were not easily obtained by youngsters in underprivileged neighborhoods. One of my favorite playthings was the cast-off automobile tire. These substitute toys were rolled down the street with great glee.”

When looking at art, some clues will reveal information; but still, the question remains: “How do we know for certain?” For example, when I looked at “Boy with Tire”, I noticed the fence and the boy’s direct gaze into the viewer’s eye, as if asking, “Are you going to cross the fence to meet me in my environment?” Of course my interpretation cannot be validated, and there are other possible, viable interpretations.

Ambiguity and uncertainty are inherent in the process of appreciating and finding meaning in art. The human brain filters perceptions through incomplete knowledge, flawed reasoning, erroneous memory, and inherent biases. Then, from faulty perceptions, we create stories. So how do we know when we have erred? The story of Euclid's Fifth Postulate offers us a model. Euclid, when he could not prove the Parallel Postulate from his four "self-evident" postulates, accepted it as a Fifth Postulate and then derived what we know as Euclidean Geometry. Subsequently, mathematicians, either by assuming variations of the Parallel Postulate or modifying Euclid's first four postulates, logically developed alternative geometries. In mathematics, when we work hard to prove a conjecture that we feel or want to be true, but logical reasoning does not yield results, we step back, pause, and free the intellect to drift, wonder, imagine, question, and re-conjecture. We also probe outside ourselves: search through published research and converse with colleagues. Ultimately, we know our assumed premises, rely on logic to accept a conclusion's validity, and use peer review for self-check. Otherwise if we cannot validate, we acknowledge uncertainty.⁵ This process adapts well to situations when ambiguity, uncertainty, and truth are intertwined.

Just as we emphasize to our students the importance of logically proving theorems from fundamental axioms, Amy Herman, an art historian and lawyer, uses art to teach these very principles. In her book *Visual Intelligence* [1], she discusses techniques that she developed for her course, "The Art of Perception," where she uses art to hone skills needed for effective job performance. Herman exploits the ambiguity in the process of looking at and finding meaning in art, in order to train people to better distinguish between observable fact, inference, opinion, and bias. For this reason, FBI agents, police officers, educators, and medical professionals flock to enroll. She argues that looking at art cultivates observation, critical thinking, and communication skills. In her class, collaboration is crucial because it brings together different eyes, experiences, and perspectives, which then offers the opportunity to adjust mindsets and modify actions. By strengthening the ability to look closely, assess, analyze, articulate, collaborate and challenge, Herman maintains that we can better make informed decisions and alter how we perceive the world. Interestingly, a Google search reveals that many medical schools across the country now include "looking at art" sessions that employ Herman's techniques in order to cultivate highly desired skills needed for quality medical practice.

⁵In physics of course, we have The Uncertainty Principle, which might serve as an exceptionally apt metaphor, too.

Their objective parallels what we hope to accomplish in our mathematics classes.

Mathematics relies on logical and analytic deductive reasoning. So do the other STEM fields but when these fields gather empirical evidence to support hypotheses, uncertainty creeps in and conclusions are often derived inductively. As we cross over to the social sciences and humanities, a higher degree of uncertainty infuses into disciplinary methodologies. Probability and statistics are our link to these fields.

Probability and statistics classes provide our students the opportunity to learn both the concepts and the powerful constructs of these disciplines, as well as the limitations of probabilistic and statistical thinking when testing hypotheses. Students learn not to be lulled into supporting a hypothesis based on insufficient or faulty data and to become more aware of the necessity to search for facts and evidence that might debunk a favored, biased presumption. These courses are great also for another reason. Opportunities for writing assignments abound, on the misuse and abuse of statistics, inductive versus deductive reasoning, how correlation is not causation, etc. Such writing activities encourage students to reflect on basic concepts and applications as well as to cement learning.

Years ago, my students handed me some interesting critiques of Michael Moore's statistical methods in his film *Bowling for Columbine*. I have subsequently thought it might be worthwhile to assign a semester-long project with the objective to explore a question/topic of interest that necessitates statistical techniques. Then in order to foster thinking about learning, have the students write weekly on their ideas, doubts, missteps, and evaluations of processes. Statistics provides a great opportunity to tease out the implications and limitations of trying to unearth certainty buried in ambiguity.

4. Learning a Foreign Language: Humanizing Persistence

How does one persevere in a field when faced with a lack of innate ability? Even the best of the best and the most motivated can be beaten down if the struggle seems futile. For me, working toward fluency in Italian, a fairly easy language for most English speakers, tested my resolve. When my classmates and teacher chattered and conversed, after fifteen years of dedicated study in Italian, I still heard babble interspersed with a few recognizable words. Reading and writing gave me no problem because mathematical study techniques adapt easily: search for patterns, understand grammar concepts and how they build the language's structure, and of course, exercise disciplined study.

But for the oral and aural precision, my ear undermined my efforts. With this personal struggle, my mathematical training came to rescue me in surprising ways, and in turn, enabled me to help my students.

The conventional advice to improve a skill is to keep practicing, but progress happens with productive practice. Years of industrious practice did nothing to improve my ear's ability to distinguish sound. In mathematics when we beat our heads on a problem without results, we return to the problem and ask, "Do we really understand the problem?" I finally pinpointed that simple things like the unaspirated Italian "p" were tripping me up (my brain heard a "b"). My problem: distinguishing phonemes, the basic units of sound in language! I needed to return to that first fifteen-minute lecture in my first Italian class. My mantra in mathematics "Hone in on the problem's root, learn the basics, and practice them effectively and often" took on new meaning in Italian. Thereafter, with focused, repetitive exercises, I have been training my mouth to pronounce the Italian phonemes and my ear to hear subtle differences.

My quest for fluency, with maximum effort and minimal results, caused me to ponder, "What kept me studying when I was struggling and can I pass on these tips to my struggling students?" Mastering a foreign language rivals that of acquiring mathematical expertise. Both require multiple complex brain activities, diligence, and persistence. Neither endeavor is a spectator sport. The tactics that refreshed my enthusiasm and re-fueled my perseverance are many, and here I will relate a few.

Cheerleading — the kind smile and pat-on-the back, encouragement revives the spirit —together with focused guidance paves the way to improvement. In mathematics my proof-writing skills improved substantially thanks to personal help from my professors. Correspondingly, in Italian I had to concentrate on improving my auditory and oral precision: demonstrations of lip and tongue position when making phonemes [5], along with games and practice with native Italian speakers. Therefore, in order to better help my students, I took full advantage of my one-on-one office hours to diagnose the source of their problems. For example, when I realized that a student fixated on detailed study of examples without understanding the concept, I suggested that she try to understand the concept behind the example and then showed her how that concept could be applied to many examples.

When I returned to graduate school after being away for several years, what kept me from sinking forever into the swamp of despair, were study sessions with friends. Their guidance helped me, but I also discovered that I had something to contribute (usually connecting an example to a concept).

Many times, we are unaware of our innate strengths, often because they are as natural to us as breathing. Therefore, in my Italian class, when I could resolve a classmate's confusion over the grammatical construction, I showcased that, even if I were a dummy in conversation, I was not a total dummy, and I also became more welcomed in group activities. Taking this idea to heart, when teaching, I aimed to ferret out my students' strengths. To the student who intuitively grasped concepts but was weak on calculations, I'd say, "You are able to understand the idea, something others find difficult, now take time to practice the basics." To a diligent student who was strong in calculations, I'd say, "You are really good with logically calculating steps in an algorithm, something many students find difficult; now let's take some time to understand the concept." Moreover, I was forever encouraging them to work together so that in addition to helping each other, they might discover their own strengths. Feeding and nurturing the positive builds self-confidence.

Taking this idea a step further, I began to think about the good strategies that my students used when encountering difficulties in their preferred endeavors. The last time I taught Mathematics for Liberal Arts, on the first day, I assigned a reflective essay: "Write about something you like to do, perhaps a sport, music, a subject, a hobby, or a passion of any type, and then discuss the methods you use to overcome difficulties and obstacles." The following class, we analyzed their varied strategies and discussed how these strategies apply when studying mathematics.

My mathematics classes were serious (and I've been very serious in this article) but it was a light-hearted Italian teacher who reminded me that humor, if aptly applied, transforms stress into healthy perspective. One evening after listening to my laments, my Italian teacher disclosed that when he first came to the USA, he could not hear the difference between the three words, "hustle," "hassle," and "asshole." He had volunteered to assist a soccer coach for young boys, and when he returned home after the first practice, he remarked to his wife that the coach was really tough on those little kids. When the coach was yelling, "Hustle, Hustle, Hustle!" he had heard "Asshole, Asshole, Asshole!" This story, in addition to making me laugh, reassured me that others share the same struggles. And so, when teaching, I lightened up. I joked and showed my silly side.

I conclude this section with a basic truth: Learning requires time. Years, many years, of focused, intense mathematical study cultivated and sharpened my skills and habits, and I am still learning. Although my progress in Italian lags behind my peers, I am slowly improving. Therefore, why should I expect

my students to demonstrate marked improvement or change over a semester? Rather, I told myself, I am handing my students a map that they might one day unfold as a guide to better navigate their thinking and learning methodologies.

5. Teaching Mathematical Thinking: Stepping Down from Abstraction

Mathematicians are adept abstract thinkers, who connect the dots, fill in the space between the dots, and predict the next dot's location. We are aware that competent thinking builds a strong foundation in STEM disciplines. We are aware that scientific research and the rapidly changing tech industry hunger for employees who are creative problem solvers, who reason effectively and communicate clearly. In our daily lives, if we are not to become prey to the data, images, articles, and videos designed to sell, convince, or propagandize, we know to step back and objectively, logically analyze. When we undertake a new topic, we apply effective learning techniques. I consider myself an abstract thinker; yet I came to fully appreciate the power of my mathematical training when I ventured outside my safe realm of pure mathematics. In unfamiliar territory, my experiences tested my learning skills. So, how do we reach our students, many of whom are very different from you and me? My two guiding principles are: Create opportunities that merge practice with reflection, and foster the personal connection.

Listening plays a major role in building the personal connection. When I listened to my students, I learned who they were, and I learned their goals, interests, values. But if I probed further, asked questions, studied how they learned, and analyzed their study habits; I was better able to short-circuit erroneous thinking and faulty study skills. I could then counsel and address their challenges with tailored methodology. For example, in a pre-calculus class, during one-on-one office hours, I encountered several students who checked the correctness of their work by whether or not they had solved a problem the same way as in the book or as I had in class. Because this perception of problem solving needed to be addressed, I threw out my pre-planned lecture and devoted class time to a problem solving game. I wrote an algebraic-trig problem on the front board. Their job was to figure out the unknown's identity using any legal method available. On the side board, we compiled our toolbox — techniques for solving equations. Then, logically stringing together these techniques, we solved for the unknown in as many ways as possible.

Sometimes revising curriculum or a course is simply not feasible, but we still could pass on our learning techniques. A few minutes dedicated to an illuminating explanation or a compelling narrative could validate the strengths of mathematics. For instance, we deliver students a resource when we explain that precise definitions remove ambiguities; that concepts unite applications; that examples illustrate and give substance to concepts; that clear, logical arguments convince (depending on the recipient); that succinct, focused exposition communicates; that conversation could illuminate and sharpen ideas. A story that reveals your trials and tribulations, your “a-ha” moments or your learning strategies not only teaches the students, but also humanizes the learning process. Invite a colloquium speaker from another field, and after the talk, gather students for an informal chat over tea or dinner. It was during one such dinner that I packed away a tip that sustained me when I walked into aerospace.⁶

Consider avenues that allow students to transfer and adapt learning strategies that enlighten and hone lifelong learning skills and demonstrate that our methodology is not unique to our field. Essay assignments that evaluate, analyze, and apply a mathematical concept to another field provide opportunities to cultivate higher order thinking skills. Team teaching with someone in another discipline cross-pollinates and enriches both fields, especially if the students are made cognizant of how mathematical techniques lend something valuable to the subject. It might be fun to co-teach linear algebra with an electrical engineer, differential equations with a physicist, statistics with a social scientist, geometry with an artist. Encourage students who undertake projects and internships, and past math majors in new careers, to share a specific situation in which their mathematical background aided them.

Personal disclosure about a specific experience grounds the message; an abstraction connects to the concrete, and, in turn, a practical example supports an idea’s authenticity.

6. Conclusion

In January 1991, when the United States initiated the Gulf War Air Campaign, my professor devoted time at the beginning of class to discuss that momentous world event. He closed the discussion with this question, “During these circumstances, does mathematics help us?” I raised my hand and tried

⁶This is the tip I mentioned above at the beginning of Section 2.

to express something like, “Mathematics helps us with critical thinking skills so that we can better evaluate and make good decisions” The class was silent. After class my friend turned to me and remarked, “Mathematics courses do not help us in these types of situations.” Obviously my response was not as clear as my faulty memory wanted to believe. At that time, I was a thirty-eight-year-old mother of three, my first year back to graduate school to earn a PhD, and my time outside academia convinced me that yes, mathematics is useful beyond knowledge content. Twenty-eight years later in this article (I am a slow writer), I’ve attempted to better respond to my professor’s inquiry.

Because competence in mathematics demands fruitful learning, we have an opportunity to promote the importance of our field and to pass on our good learning methods: our relentless commitment to precision and rigor in logic, analytic and creative problem solving, communication skills, collaboration techniques, and dogged persistence. By doing so, we hand our students a powerful tool to use when they leave our classroom doors and make their way in this rapidly changing world with its easy access to all sorts of information. When students become cognizant of and are able to analyze and apply mathematical learning techniques to new situations and domains, when they know how to learn, they reap benefits beyond the classroom. The fundamental techniques we employ when learning and doing mathematics hold tremendous potential to have an impact.

I leave these reflections by offering an invitation. Immerse yourself in an unfamiliar domain (that is, if you have not done so); even better if you do not have a natural talent in this new domain. Then as you learn, pay attention. Examine your processes. Notice your emotions and reactions. Assess how you overcome limitations and what encourages you. Then compare and contrast this process to your mathematical endeavors. Afterwards, reflect on this experiment and think how you could narrate it to your students so as to compel them to continue striving in their mathematical training. I look forward to reading about your experiences and learning from you.

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References

My quest to hear correct Italian phonemes led me to [4] and [5]. The former gives excellent advice on acquiring language fluency, much of which transfers to acquiring mathematical expertise.

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