My Last Fermat's Last Theorem Talk

Richard Delaware
University of Missouri - Kansas City

Follow this and additional works at: https://scholarship.claremont.edu/jhm

Part of the Arts and Humanities Commons, and the Mathematics Commons

My Last Fermat's Last Theorem Talk

Cover Page Footnote

My Last Fermat's Last Theorem Talk

Richard Delaware

Department of Mathematics & Statistics, University of Missouri - Kansas City, USA
`delawarer@umkc.edu`

Synopsis

From the dangerous snowfall preceding and nearly ending it, to the pleasant years-long fallout afterward, I tell the story of the events surrounding my final presentation of an expository talk on Fermat's Last Theorem in the same year but months before Andrew Wiles announced his proof.

Deep breath. My hands still gripped the steering wheel, preserving the illusion that I had been in control the last few seconds. Windshield wipers continued cracking rapidly back and forth, whipping away the falling snow, as my headlights illuminated the virgin snow field ahead. Stopped. At rest. Heart beating a fast rhythm. I twisted in my seat, looking back over my right shoulder to see two sharp tire tracks confirming my lightning fast sweep down the long embankment dividing the highway I had so recently been toiling along. Moments before on the road above, my hands had begun to sporadically cramp, so intently was I gripping the steering wheel in the stress of driving at highway speed into blizzard-like snowfall, and carefully maintaining a steady pace in the leftmost lane of that half of the divided road. But then, a towering semi-trailer truck began to pass me on the right, and in an involuntary reaction to the sudden rooster tail of snow arcing from its passage into my windshield, or because I lost sight of the road's edge, I inched slightly leftward, dropping my tires onto the narrow shoulder, after which gravity and physics swept me down the hill, ripping my breath away.

I and my new 1992 Honda Civic were undamaged, though marooned in that deep and spacious center divide of highway I 35 North somewhere northeast of Kansas City, Missouri, and still south of the Iowa border. Instantly I saw no hope of me driving myself back up the steep embankment to the road.

So, for a few breaths, as my heart calmed, I reviewed what had lead me here this Thursday night, February 25, 1993.

That fall, on the evening of October 16, 1992, my former student Doug emailed me with an invitation to give a colloquium talk for the mathematics club at the University of Iowa in Iowa City, where he was pursuing a Ph.D. in mathematics. Although Doug didn't know, this was my first invitation to give an expository mathematics talk outside the Kansas City area, so I was immediately excited to accept the offer. I responded four days later with a list of talks I could give, and other suggestions.

Eight years earlier, I'd met Doug in September 1984 when he was a Truman High School senior from Independence, Missouri, enrolled in the inaugural class of the so-called Mathematics and Physics Institute (MPI). The MPI was a new University of Missouri–Kansas City (UMKC) endeavor to be run out of the College of Arts and Sciences, led by the Physics Department, for gifted and talented high school seniors from, at that time, the public school districts of Independence, Fort Osage, and Kansas City, Missouri. The MPI was located in the UMKC Truman campus building just north of the Harry S. Truman Library and Museum in Independence, a thirty-minute drive east of the main UMKC campus. Students came to the MPI daily at 7:00 am, before their regular high school classes, for two periods of college calculus and college physics, for which they could earn both high school and free UMKC college credit. The local superintendents' monthly roundtable, in response to the Reagan National Commission on Excellence in Education 1983 report "A Nation at Risk," collaborated with UMKC to create and fund the MPI. I'd just been hired for the fall of 1984 by the Department of Mathematics to coordinate the MPI mathematics program, supervise the high school mathematics teachers working with us, as well as teach their UMKC Calculus I and Calculus II. I was grateful to have found this promising professional position, my first "real job," ending for me seven years of busy but undirected survival living after earning my Master's degree in Mathematics. I was there to meet Doug and the other seventy-two students at the Truman campus for the first day of MPI classes September 4, before our celebratory opening day September 13.

Doug was one of ten students that first year who took both Calculus I and Calculus II, finishing the year in May 1985 as the student with the highest MPI grade point average. After graduating, Doug initially studied engi-

neering at the University of Missouri–Rolla campus, but after a couple of years decided to move back to the Kansas City area and finish his degree in mathematics from our department. I was pleased to later write him a recommendation letter for admission to graduate study at the University of Iowa.

Meanwhile, I continued at the MPI. On my own initiative I started on the main campus a series of mathematics Expository Talks, and by the date in 1993 that I gave my talk in Iowa City, I'd organized seventy-seven talks for the series, of which sixteen were mine. These included talks explicating proofs or updating the status of unsolved problems and modern mathematical pursuits, all with titles I hoped would be interest-catching, such as, "En Route to Chaos: Sarkovskii's Theorem," "Pascal's Triangle Before Pascal: A Historical Picturebook," and so on. I had settled on a set of "rules" for speakers in the series, especially that the fifty-minute board talks must contain a mathematical proof. I would come to describe my vision as "Presenting stylish, commanding, and complete board expositions of proof, thereby offering an audience the chance to comprehend a mathematical argument in real time."

When Doug emailed me in October 1992, I was in my ninth year of teaching at the MPI. The University of Iowa students agreed to invite me as a speaker, offered to pay all travel expenses and provide me an honorarium. That January, through Doug, they asked me to come on Friday, February 26 to give an afternoon talk to undergraduate and graduate students, and specified the honorarium as one hundred dollars. I responded with an enthusiastic "yes" the next day. They chose my "Fermat's Last Theorem" talk, which I had first given about two years earlier on March 22, 1991, in my Expository Talks series at UMKC. This would be my second (and as you will read, my *last*) presentation of it. During the next two weeks further details were settled, and they asked me for a title and abstract.

> **Title**: Fermat's "Last Theorem" c.1667-1993: The most famous marginal note in mathematical history
>
> **Abstract**: A historical survey of the primary results (subject to the speaker's predilections) toward solution of the conjecture, from about 1637 to the present, including proofs for the $n = 3$ and $n = 4$ cases, as well as for "almost all" n. A handout with a copy of the actual marginal note (in Latin) and other information will be provided.

On the morning of Thursday, February 25, Doug warned me by email that two to five inches of snow were expected in Iowa City. Around noon, I laughingly responded in turn that in Kansas City we had been hit by nearly twelve inches of snow. (The official record I learned later was ten and a half inches, making it the fourth highest snowfall to that date in Kansas City history.) Both the MPI and UMKC closed that Thursday. I remained at home through the afternoon, and since I would not have to teach my Finite Mathematics course at UMKC that evening, I could leave for Iowa earlier than I'd planned. I left home at 4:20 pm, my still new Honda Civic odometer reading 5,772. Though the streets were mostly cleared, a dense snow pack encrusted the roads. I started my trip traveling the same fifteen-mile route I took each weekday morning to the MPI in Independence, passing by the Truman building ninety minutes later, three times the usual trip time. Stopping briefly across the street, I topped off my gas tank.

Hours later I found myself stranded in that snow field off highway I 35 North.

After my deep breath, I turned the headlights off, popped open the car door, climbed out wrapped in the warmth of my coat, pulling my navy-blue toque tight over my head, and trudged upward through the steep snowbank toward the road. A half-mile back I had glimpsed an overpass leading to what seemed like a brightly lit gas station and other stores. As I was hiking up the embankment I mentally prepared myself for the cold walk. But, when I reached the roadside, to my surprise a highway patrol car pulled up. The officer cracked open his window, said he saw me careen off the road, and asked if I would like a ride to the lighted overpass. Of course, I said yes. I got in. We continued in the direction I had been driving, turned off onto the next overpass, crossed over to the other side of the divided highway, headed back in the direction of Kansas City, and again left the highway rising onto the lighted overpass I had been planning to walk to. As we crossed over the highway, a tow-truck passed us driving in the opposite direction. The officer asked if I would like to contact it. Again, I said yes, and he radioed the truck immediately, directing the driver to follow his patrol car to the roadside above my stranded Honda.

He and I then continued across the overpass, re-entered highway traffic and drove to where he had picked me up. After a few minutes the tow-truck arrived. I climbed down the embankment with the driver who lugged behind him a tow cable unwinding from a winch on his truck.

At the bottom, I sat briefly in my driver's seat to shift my manual transmission Honda into neutral, while he attached the cable to the rear of the car. We both stamped back up the hill, and once there he powered the winch that slowly tugged my car up the embankment onto the narrow roadside above. He unhooked the cable, I paid him with my AAA credit card, shook hands with both men, and within minutes was alone again.

The entire astonishing sequence of events had taken probably no more than forty minutes. After folding myself again into my driver's seat, I eased onto the highway, crossing immediately to the rightmost lane, where, still a bit dazed, I would persist for the rest of my trip. Just beyond the Iowa border I turned onto I 80 East and finished the final leg of the trip without stopping. I arrived at the Iowa City downtown Holiday Inn at around 12:30 am, a trip of 347 miles, and one of over eight hours.

After the harrowing Thursday drive, I don't recall how Friday began. I doubt I woke early. My Visa bill tells me I ate at Givanni's Italian restaurant that day, but not when. I met up with Doug at The Great Midwestern Ice Cream Company as he had suggested and made some copies at a nearby Kinko's. In any case, we eventually walked over to the university. My talk was scheduled for 3:30 pm in room 105 of Maclean Hall, seating around forty. I planned to speak for fifty minutes and provided the audience with an eight-page handout. I began with a translation of the Latin note Fermat jotted sometime around 1637 in the margin of his edition of Diophantus' ancient book Arithmetica. We only know this because after Fermat died in 1665, his son reprinted that Diophantus text in 1670 and included Fermat's jotting, here translated from the Latin:

> "It is impossible for a cube to be written as a sum of two cubes or a fourth power to be written as the sum of two fourth powers or, in general, for any number which is a power greater than the second to be written as a sum of two like powers; I have a truly marvelous demonstration of this proposition which this margin is too narrow to contain."

In symbols his statement says:

> *If $n > 2$ is a natural number (like 3, 4, ...), then $x^n + y^n = z^n$ has no natural number solutions for x, y, and z.*

Fermat never again referred to this dramatic and intriguing claim. To the date of this talk, about 356 years after Fermat's private claim, I reminded the audience that no proof was known. And, although this statement had been informally dubbed "Fermat's Last Theorem," in fact it still stood as only a conjecture.

I then launched into a collection of quotations, some sober, some amusing, about the halting progress, surprising pitfalls, and reams of failed attempts over the years to prove this statement. (In 1983, Howard Eves wrote: "Fermat's Last 'Theorem' has the peculiar distinction of being the mathematical problem for which the greatest number of incorrect 'proofs' have been published.") The story of the "theorem" persists even in popular culture, appearing for instance in the March 27, 1989, second season "Star Trek: The Next Generation" episode "The Royale," which opens with a scene of the character Captain Jean-Luc Picard working on the proof in his spare time, four hundred years in the future.

I mentioned at the board the 1770 Euler proof of the $n = 3$ case and demonstrated Fermat's own proof of the $n = 4$ case. By 1993, a handful of such cases had been proven, more comprehensive results by Kummer (1857) and others had been established, and a proof was known that any counterexample would necessarily require an exponent n larger than the huge number 10^{17}. I ended with the wonderful 1985 proof by Granville and Heath-Brown that in a well-defined sense Fermat's Last 'Theorem' was in fact a theorem for "almost all" n.

After the talk, I answered a few questions, and when I began to mop sweat from my forehead, I heard a faculty member say that I had definitely been "working" up there at the board.

I then gratefully joined Doug and his fellow graduate students at the Deadwood Tavern for food and cold beer. The company was excellent, though I was a dull guest.

Following a night's sleep and filling up my gas tank, my drive home on Saturday morning was thankfully uneventful. (The final odometer reading was 6,443, a total of 671 miles traveled.)

———o———0———o———
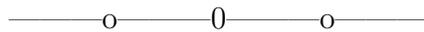
Yet, why, you may ask, was this the last time I gave this talk? The answer is epic in scope.

That summer, about four months after my Iowa visit, at around eleven am on Wednesday June 23, 1993, having worked in isolation for nearly seven years, and finishing part III of his three-day presentation during a conference at the Isaac Newton Institute for Mathematical Sciences at Cambridge University in Cambridge, England, mathematician Andrew Wiles put down his chalk. Over those three days he had outlined his masterful proof of the narrowly technical Shimura-Taniyama Conjecture for so-called semi-stable elliptic curves, ending with a corollary that was in fact Fermat's Last Theorem. The front-page headline of the New York Times the next morning announced: "At Last, Shout of 'Eureka!' In Age-Old Math Mystery." Excitement within the mathematical world was palpable.

And the already high drama was not yet over. In a disturbing echo of the hundreds of previous failed proof attempts over those 356 years since Fermat penned his marginal claim, some attempts from supremely talented mathematicians, a few months later a gap was found in Wiles' proof. The mathematical world collectively held its breath. Happily, in a final dramatic twist, about a year later, on September 19, 1994, struck by a flash of inspiration during a quiet moment at his desk after working for months in collaboration with his former student Richard Taylor, Wiles closed that gap, and the proof of Fermat's Last Theorem was at last complete.

When such a longstanding question is settled, simultaneously excitement and a moment of sadness follow. New frontiers open as an era of uncertainty ends, as does the tantalizing draw of a solution yet to be found. Doors close, and interests may move elsewhere. Yet overall the lasting effect is renewal.

——————o———0———o——————
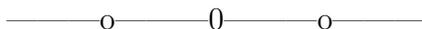
Of course, my little talk was now obsolete, so I permanently retired it. However, the door was not entirely shut on my last presentation of this talk. I would later learn from Doug the response to my performance, help someone who'd attended my talk land a job, and eventually invite Doug in turn to speak at UMKC.

Nearly two years later I was helping to assemble my nomination packet for the UMKC College of Arts and Sciences Dean's Award for Outstanding Teaching (which I would win in 1995). At my request, Doug wrote for me a thoughtful letter of recommendation to include in that packet, in which he recalled my Iowa talk, and from which I learned for the first time its positive impact:

"Richard is a gifted speaker in any setting. His talents as an expositor and an organizer have helped him to sustain a regular series of colloquium talks at UMKC and surrounding schools. In keeping with his philosophy of teaching, these talks are designed to open the world of mathematics to a wider audience. As chairman of the graduate student colloquium committee at Iowa, I had the pleasure of inviting him to speak here. He was by far the most well received colloquium speaker I have seen in six years at Iowa. His talk on Fermat's Last Theorem drew a standing room only audience. This famous conjecture has motivated three centuries of mathematics. Richard presented the history of the problem, describing the people who have thought about it, the ideas it spawned, and the questions it has provoked. The entire talk was easily accessible to undergraduates, and still informative to professional researchers."

———o———0———o———
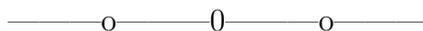
Another pleasant though unexpected outcome of my talk was the email I received on February 9, 1995, from Bernadette, an expansive, red-maned fellow graduate student of Doug's I'd met in Iowa City at the Deadwood Tavern where we gathered afterward. She had earned her Ph.D. from Iowa and was now preparing a job talk for an interview. Remembering my presentation, she asked me to confirm, or at least provide the correct details about a story that in the 1850s the well-known mathematician Kummer once produced a flawed proof of Fermat's Last Theorem. I responded by email four days later with a detailed historical outline including the statement that the story had been checked in 1975 against contemporary documents and found to be probably untrue. Bernadette responded three days later:

> "What you sent was exactly what I wanted to put into my job interview talk. Thanks a million! And it worked . . . I got a call from the chair of the department . . . offering me the job! I'm going to accept."

———o———0———o———
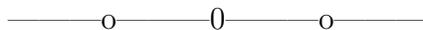
Finally, I closely followed Doug's subsequent career as he completed his Ph.D., made a name for himself in low-dimensional topology, and soon afterward landed permanently at Boise State University in Boise, Idaho.

About six years later, during a prestigious two-year National Science Foundation fellowship, while he held a postdoctoral position at the University of Maryland–College Park, I returned the favor he afforded me in my Iowa City talk by inviting him to speak at UMKC on December 4, 1998. He presented several talks on knot theory: one for MPI students (as he had once been), one for UMKC undergraduates (as he had also been), and a third for graduate students and faculty. I'm pleased to report that it did not snow that day. His talks were well-received, and as far as I know, not the last time he gave them. So, in a burst of success, ended the final happy consequence of my last Fermat's Last Theorem talk.