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# College Mathematics Students' Perceptions of “Believing” Teacher Actions

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## Abstract

*Believing* and *doubting* – two methodological processes – deserve equal attention [7, 9]. When a teacher plays the *doubting game* in a mathematics classroom, her own mathematical thinking dominates, and she attempts to find flaws and errors and misconceptions in students' mathematical thinking. When a teacher plays the *believing game* in a mathematics classroom, she surrenders her own mathematical understanding and she attempts to find virtues and strengths and merits in students' mathematical understanding. Paradoxically, a teacher must believe her own mathematical understanding in order to doubt and a teacher must doubt her own mathematical understanding in order to believe. For this qualitative case study, the professor, Beth, purposefully played the believing game. She created a “believing” teacher action plan prior to teaching a *proofs* course and she wrote a teacher “believing” stance statement about midway through the course. After the course ended, Shelly interviewed students in order to answer the research question: How did students describe their classroom community, the teacher's actions and interactions with them, and their own learning? Students described some aspects of Beth's teacher action plan; however, their descriptions aligned more closely with her teacher “believing” stance statement. This implies that to move towards playing the believing game the teacher stance is critical.

**Key words:** believing game; teacher actions; teacher stance; mathematics

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“So she [the professor] really lets you put your thoughts into words, doesn’t rush you, and like listens and tries to understand.”

Student M

“Cleve ever to the sunnier side of doubt.”

Alfred Lloyd Tennyson

## 1. Belief and doubt

Much has been written about the topic of teacher beliefs and practices, how they impact each other [11, 14, 28] and how they impact student learning [14]. Mewborn and Cross [24] concluded that because teachers’ beliefs:

... directly affect the conceptions of mathematics that [their] students develop and the type of learners they become, we cannot bypass the influence of beliefs. Therefore, if our goal is to improve students’ learning of mathematics, we must begin the discussion with a focus on teachers since they will ultimately have the greatest impact on the development of future mathematicians, their understanding, and their subsequent achievement. (pages 267-268)

How teachers’ beliefs impact their practices and how their practices impact their beliefs about teaching and learning has been the topic of research for many years (see for example [11]). In fact, nearly thirty years ago, Wood, Cobb, and Yaker [31] advocated that the “processes by which teachers reorganize their beliefs and practices” must be examined (page 498). The two of us have written about times when our beliefs challenged us to reconsider our teaching practices and times when the results of our practices challenged us to reconsider our beliefs about teaching and learning [15].

We are convinced that teachers’ practices *can* honor and respect students’ mathematical thinking [17, 18] if teachers consciously balance believing and doubting them, while attempting to negotiate a shared understanding of mathematics. Therefore, we framed our research around Elbow’s [7, 9] notions of believing and doubting. Elbow is an expert in teaching writing and rhetoric, but we have found that his work applies to mathematics teaching as well.

Elbow [7, 9] asserts that we can improve our practice of understanding by *balancing* two disparate practices: methodological belief and methodological doubt. In terms of mathematics teaching, methodological belief (or believing or playing the believing game) is a means for finding virtues and strengths, no matter how unlikely students’ ideas, solutions or answers might seem. In contrast, methodological doubt (or doubting or playing the doubting game) is a means for finding flaws, contradictions, or misconceptions. Ideally, teachers should make conclusions only after considering the results of *both* believing and doubting when they hear students’ ideas, answers, or solutions [7]. However a paradox exists for teachers: In order to believe they must suspend or doubt their own mathematics and in order to doubt they must believe only their own mathematics. The second part of the paradox, believing one’s own mathematics in order to doubt, seems much easier to do; therefore, perhaps that is why, anecdotally, doubting predominates in many mathematics classrooms [17, 15].

Accordingly, teachers should make decisions only after judging the “hypothetical or conditional character of doubting and believing” [7, page 269]. In mathematics we typically respect teachers who find students’ mistakes and misconceptions, but we should also respect “midwife” teachers who listen to students and realize good ideas, no matter how poorly expressed, teachers “who help students give birth to nascent good ideas” [7, page 286] and “find fruitful implications” [7, page 288] in their students’ mathematics.

Importantly, methodological belief is more than merely seeing another person’s point of view; it is “a way to search for commonality and pieces of truth in another person’s ideas and viewpoint, and letting those new understandings push the listener to reconsider his or her own ideas” [26, page 98]. This search for commonality can have implications for the mathematical learning and understanding of the teacher as well as the student. These implications are critical to how we position believing as a way to strengthen our own understanding of mathematics.

However, methodological belief is often difficult to put into practice because of convictions that we are right and the other person is wrong [26]. For a mathematics teacher this might play out as a teacher searching for the right answers and the processes she deems important; she listens for points of disagreement in order to show weaknesses in the students’ mathematics. Additionally, teachers whose practice is more about doubt may not con-

sider how they can learn mathematics based on listening to their students' mathematics. In a previous article, we discussed how Beth, while teaching a mathematics course for preservice teachers, gained a deeper understanding of the definition of a polyhedron after playing the believing game with a student who was convinced that a particular figure was not a polyhedron [25].

We need both “doubting what looks right, and believing even what looks crazy or alien” [8, page 59]. For Elbow “. . . a crucial event must happen. I have to make a little act of letting go and give up full commitment to that point, to that effect. Not necessarily a large letting go . . . a temporary time-out from my rhetoric” [8, page 59]. Because suspending *all doubt* to believe might seem unnatural or unprofessional, “Assent [believing] is the crux, even though it’s only conditional and temporary” [7, page 279]. In another article, we described a practice which we named *reserved believing* [15]. When mathematics teachers practice *reserved believing* they listen to students and realize partial understanding or different understanding and classroom dialogue moves from persuasive rhetoric to shared meaning-making dialectic. It might feel like ritual [23] to say we have “some doubt” about students’ mathematics but it might feel strange to say we have “some belief” when we hear students’ mathematics that does not match our own. When teachers balance the practices of believing and doubting, students’ mathematical thinking is honored and respected [17, 18]

## 2. Research Narration

Within this section we use diegeses, or the telling of the research story through a chronological narration of important events or turning points.

Before the semester began, Beth created an action plan, a list of “believing” teacher actions. She intended to incorporate these “believing” teacher actions into her practice in the *Proofs* course (email correspondence, January 12, 2017):

- 1) Do problems in class that I haven’t done before. This creates an authentic space where I don’t have preconceived ideas about the problem.

- 2) Ask open-ended questions during the students’ exploration of the problem. Past experience has shown that I play the believing game more often when I ask open-ended questions.
- 3) When I don’t know how to respond to a student’s answer, say, “That’s interesting. Tell me more.” This allows the students to express more about their thinking, giving me the opportunity to find merit in that thinking.
- 4) Talk less. Let students present problems to each other. Allow students to ask each other questions about the presentations. (How do I actually encourage/facilitate students asking each other questions? Each classmate must prepare one question per presentation regarding the content? Is this playing the believing game? I think so. I think the instructor is believing that the presenter has the understanding and mathematical knowledge to answer the classmates’ questions.)
- 5) Say “I don’t know” referencing my own understanding. This could encourage students who think they know to speak up, giving me the opportunity to believe.

We also decided that at the end of the semester, we would interview students in order to explore the research question: How did students in the course describe the classroom community, the teacher’s actions and interactions with them, and their own learning?

However, about mid-semester, Shelly shared with Beth a research article written by Bondy, Ross, Hambacher, and Acosta [2] about a preservice teacher’s “warm demander” stance (see §§3.2, the subsection on stance). While discussing this paper and the notion of stance, we became convinced that “believing” teacher actions are specific practices. Yet, we grappled with the concept of teacher stance. This article made us wonder: Can a teacher purposefully use “believing” teacher actions without a “believing” belief? Succinctly, we decided, “Yes.” This is a tension that we, as teachers, have grappled with ourselves. In what ways might creating a “believing” stance help Beth build a bridge between her practice and her beliefs? We wanted to know more about teacher stance. Curious about the definition of stance, we found at least three interpretations:

- the way in which one *deliberately* stands or one's body position for sports such as baseball, ballet, or golf;
- the *purposeful* standpoint or perspective that one takes toward something;
- the *secure* ledge or foothold on which a belay can be fastened for rock or mountain climbing.

Our working definition for teacher stance combined these three interpretations as metaphors for stance:

A teacher's stance is a *deliberate* and *purposeful* statement about a teacher's beliefs, recorded and explicitly shared with others.

Hence, stance might be seen in a teacher's body position and observed in a teacher's actions or practice. Stance is thus a *secure* ledge or foothold between beliefs and practice. Recorded stance, in fact, *inspires and motivates* teacher actions.

After many conversations about the tension between beliefs, teacher actions, and stance, and about mid-semester of the *Proofs* course, Beth wrote the following "believing" stance statement and shared it with Shelly:

There is merit to students' thinking, and that thinking should be shared and valued in and out of the classroom. Students can, will, and must share their thinking for my benefit and the benefit of other students. I take responsibility for realizing this aim.

The process by which Beth moved from a teacher action plan to her stance allowed her to "motivate her believing teacher actions" [direct quote from Beth]. Writing a stance statement and sharing it with Shelly moved Beth from playing the believing game because *research* said it was a good thing to do for her students' learning toward playing the believing game because *she trusted* it was a good thing to do for her students' learning.

At the end of the semester Shelly interviewed five students in order to explore the original research question: How did students in the course describe the classroom community, the teacher's actions and interactions with them, and their own learning?

### 3. Prior Work

Franke, Fennema, and Carpenter [11] helped us consider the connections between teachers’ beliefs and practice; we describe their work in §§3.1. We summarize depictions of teacher stance as described by the scant number of research articles we found in §§3.2.

#### 3.1. Beliefs and Practice

Franke, Fennema, and Carpenter [11] studied teachers’ beliefs and practices in the context of Cognitively Guided Instruction (CGI) professional development workshops. Twenty-one teachers participated in CGI over a four-year period. CGI professional development providers wanted teachers to begin to: offer students opportunities to solve problems in their own ways; listen to students’ mathematical thinking; and, use students’ mathematical thinking to make instructional choices. Briefly, they wanted teachers to use these practices rather than “move the lesson along” and show or tell “better strategies” (page 263).

Franke and colleagues [11] used four “Levels of Teacher Change” to describe changes in teacher beliefs as moving from Level 1 with a focus on teacher practices of telling or showing to other, higher levels focused on teacher beliefs about students’ mathematical thinking:

- Level 1 – The teacher believed students could not solve problems unless they had been taught specific strategies and the teacher provided no opportunities for students to solve problems using their own strategies.
- Level 2 – The teacher began to believe students could solve problems without being taught specific strategies.
- Level 3 – The teacher generally believed students could solve a variety of problems without being taught specific strategies.
- Level 4A – Level 3 *and* the teacher believed students’ mathematical thinking could help her make instructional decisions (for the group).
- Level 4B – Level 4A *and* the teacher consistently talked about individual students’ mathematical thinking.



The authors began with the premise that it is unclear how teacher beliefs and practices change (described in the context of their study as a linear progression from Level 1 to Levels 2-4B in “Levels of Teacher Change”) over time. They noted,

In some cases, teachers’ beliefs might be challenged by what they hear [in the CGI workshops] about the types of problems that children can solve and the strategies they will use to solve them. In other cases, teachers might pose problems [after the CGI workshops] based on the frameworks (with some expectation of the strategies the children might use) without necessarily changing their beliefs about teaching and learning of mathematics. (page 261)

Through the course of their study, Franke, Fennema, and Carpenter noted that out of the twenty-one teachers involved, eighteen teachers’ practice changed or progressed from Level 1 to Levels 2-4B; seventeen teachers’ beliefs and practice changed or progressed from Levels 1 or 2 to Levels 3-4B. Six teachers changed their beliefs first, five teachers changed their practices first, and six teachers changed their beliefs and practices concurrently. These results indicated no single pattern of change. However, the authors noted that for teachers to change or move beyond Level 3 in classroom practice, *belief changes were fundamental*. They attributed change or progression from Levels 1-3 to Levels 4A and 4B to teachers who regularly engaged in inquiry focused on students’ mathematical thinking.

Level 3, as described in [11], seemed to align with Beth’s list of “believing” teacher actions. For example, by saying, “I don’t know,” or “Tell me more,” Beth intended to give the students the authority to solve problems using their own strategies. When she planned to “talk less,” she would allow students to explore their own approaches to problems and to share those approaches with each other. By giving students open-ended problems, Beth envisioned her students would realize that there was not one right answer or approach to problems. Finally, by doing problems in class that she had not done herself, she would communicate to students that they were negotiating shared mathematical understanding along with her.

### 3.2. Stance

In order to further understand the connections between teacher beliefs and teacher practice, we explored an intermediate construct: teacher stance as a deliberate and purposeful statement that translates teacher beliefs into teacher actions. To this end, we conducted a search of literature using the terms: “teacher” and “stance”; and, “teacher stance”.

Scarino and Liddicoat [27] situated “stance” as professional and personal positions that teachers take towards their work. Accordingly, stance evolves over time in response to changing contexts. Wood, Cobb, and Yackel [31] used the phrase, “a teacher’s stance,” to describe a “constructivist” view of children’s mathematics:

Teachers should attempt to view children’s solutions from the perspective of the students and recognize that what seem like errors and confusions from an adult point of view are merely children’s expressions of their current understandings [20, 21] . . . In this way, a teacher’s stance toward mathematics will portray the subject as a creative activity that is not without uncertainty and that invites further dialogue [4]. This in turn involves the teacher and students in interactions characterized by a genuine commitment to communicate. Researchers who subscribe to this view have focused almost exclusively on the individual student and the processes by which the learner actively constructs mathematical knowledge . . . (page 498)

Other researchers have depicted teacher stance using diverse descriptors such as: “warm demander” [2], “inquiry” [5]; “passionate” [12]; “praxis” [13]; and, “Mind Games” [22]. More about each of these stance descriptors follows.

The description “warm demander” first appeared in Kleinfeld’s 1975 study [19] when he used it to portray the ways in which effective teachers of Eskimo and Alaskan Indian children interacted with their students. Teachers with a warm demander stance “embrace values and enact practices that are central to their students’ success” [2, page 420]. Bondy and Ross [3, page 58] encapsulated a warm demander teacher as one who treats “. . . students with unconditional positive regard, knowing students and their cultures well, and insisting the students perform to a high standard”. Ware [30], who influenced the work of Bondy *et al.* [2], used two terms to describe a warm demander: caregiver and authority figure.

Cochran-Smith and Lytle [5] adopted the term “inquiry” and defined “stance” as:

...the positions teachers and others who work together...take toward knowledge and its relationships to practice. We use the metaphor of stance to suggest both orientational and positional ideas, to carry allusions to the physical placing of the body as well as the intellectual activities and perspective over time...the metaphor is intended to capture the ways we stand, the ways we see, and the lenses we see through. Teaching is a complex activity that occurs within webs of social, historical, cultural, and political significance... [stance] provides a kind of grounding within the changing cultures of school reform and competing political agendas. (pages 288–289)

This metaphor of stance as both “orientational” and “positional” and as a physical act seems to situate it within both beliefs and practices. It also suggests directionality and movement.

Fried [12] used the moniker “passionate” to characterize teachers who work through the challenges they face related to a myriad of issues such as the amount of content to teach, the nature of assessment, and the most effective ways to motivate students. A passionate teacher establishes a stance: “...a philosophy, an attitude, a bearing, a way of encountering students based on a set of core values about kids and their learning potential” (page 139). A well-articulated stance that is grounded in genuine passion can facilitate respectful relationships between students and teachers, as well as among students and when this occurs “discipline comes down to a few simple rules” (page 181).

When teachers adopt a “praxis” stance towards mathematics teaching, they act by considering that their actions will be judged historically by the broader outcomes; they are aware of the learning that occurs in mathematics classrooms and the impact their actions have on their students’ mathematical identities [13]:

Teachers are always making decisions and acting in the moment-by-moment activity of the classroom, but if they are conscious of the moral implications of their actions for both the individual students and the world at large (humankind) that can only be judged historically, then they can be said to be engaged in praxis. (page 323)

A praxis stance is realized in teachers’ daily and in-the-moment decisions.

MacKenzie [22] coined the term “Mind Games” stance to denote interactional times when a middle school science teacher in her study moved from procedural instruction (rules and guidelines) to interactions when “students’ voices were more apparent, ideas were shared among class members, and the teacher was asking questions like, ‘How do you know? What was your evidence? What makes you think this was true?’” (page 145). The “Mind Games” teacher stance was one of curiosity and wonder.

### *3.3. From Beliefs to Practice Through Stance*

How do the myriad notions of stance explored in §§3.2 interact with the work of Franke, Fennema, and Carpenter [11] described in §§3.1? Recall the connections between “believing” teacher actions and Level 3 of “Levels of Teacher Change”. Similar connections can be made between Beth’s “believing” stance statement and Levels 4A and 4B.

By recording her stance, that student thinking should be shared and valued, Beth was conveying her conviction that student mathematical thinking can and should drive the classroom discourse. When this happens, that is, when student mathematical thinking drives the classroom discourse, it is the manifestation of an instructional decision made based on the students’ thinking. In her stance statement, Beth also wrote that students can, will, and must share their thinking for the benefit of everyone in the classroom. This aspect of her stance statement aligns with level 4B, which focuses on the teacher consistently talking about individual students’ understanding.

## **4. Methodology**

We chose a qualitative case study design to explore our research question. Our main target, or “unit of analysis” according to Baxter and Jack [1], was student perception. We chose qualitative case study because our research question was a “how” question; also we considered the contextual conditions relevant and wished to discuss them [32].

Stake [29] used the term “intrinsic” to describe the type of case study in which the researchers have a genuine interest to better understand the case. Intrinsic case studies are not used because the case is similar to other cases

but because the case is both particular in some aspects and ordinary in other aspects [29]. We had a genuine interest in our students' perceptions and sincerely wanted to understand them. More generally we believed strongly that research into teaching and learning should include the voice of the student.

#### 4.1. Course and Students

The students in this study were enrolled in *Proofs*, a course designed to introduce students to essentials of logic, methods of proof, and set theory. This course served as a prerequisite to upper-level mathematics courses: number theory, geometry, abstract algebra, real variables, and complex variables. *Proofs* was a sixteen-week course taught in the spring semester at a Midwestern university with a student population of approximately 15,000. It met for 50 minutes, three days a week. The course was taught in the Department of Mathematics and Statistics, which had approximately 120 mathematics and/or statistics majors. *Proofs* was a required course for the mathematics majors in two of the three possible tracks, the pure mathematics track and the general track. Students in the pure mathematics track often considered graduate school while students in the general track often chose secondary mathematics education. Of the ten students in the class, seven were mathematics majors, two were computer science majors, and one was a high school student.

#### 4.2. Data Collection and Analysis

After Institutional Board Approval and before we began to collect data for this study, we created an Interview Protocol (see Appendix A), based loosely on Beth's teacher action plan (as previously described in §2) and on our research question.

Shelly interviewed five students who were in the course. We used convenience sampling because these five students were available, none had graduated, and they were amenable to being interviewed. Convenience sampling may not be representative of the population [6], but the population of all students in *Proofs* was quite small. Additionally, the course had ended, grades had been posted, students interviewed had earned a variety of grades in *Proofs*. These aspects, together with the fact that Shelly, not their instructor Beth, interviewed them, helped ensure students were honest and frank when responding to the interview questions.

While the use of retrospective interviews is sometimes criticized as being one of the least likely types of interviews “...to provide accurate, reliable data for the researcher” [10, page 510], we trusted the students’ memories to be temporal but accurate based on their perceptions of their experiences.

After transcribing the interviews, we organized the students’ responses in spreadsheets—one spreadsheet for each student—and listed each interview question and the student’s response. We then used their quotes from interview questions #2 and #4 in order to capture their perceptions of the classroom community. We used their quotes from interview questions #3 and #6 to elucidate their perceptions of the teacher’s actions and interactions. Finally, we used their quotes from interview question #5 and all other questions to understand their perceptions of their own learning.

## 5. Overview of Student Responses

Here, we first share demographic data about the students in §5.1. Then, we explore the students’ interview responses in the context of the three themes from the research question: the classroom community (in §5.2); the teacher’s actions and interactions (in §5.3); and the students’ learning (in §5.4).

### 5.1. Demographic Data

As mentioned previously, students who were interviewed earned a variety of grades in *Proofs*. All were majoring in mathematics or mathematics was one of their majors yet their career goals were varied. For four of the five students *Proofs* was a required course. We recorded the following data about each student participant: 1) A letter identifying the student, 2) the interview date, 3) gender, 4) college standing, 5) major, 6) the grade received in *Proofs*, 7) career goals, and 8) the answer to the question: Why did you take *Proofs*?

Below we list relevant information on each participating student.

- Student C, interviewed 09-09-17, male, Junior with major in Applied Mathematics, received B in the course.
  - Career goals: “Maybe a job at an observatory or NASA.”
  - Why take course: Elective -“... the concept of proofs kind of interested me.”

- Student D, interviewed 09-09-17, male, Junior with major in Mathematics (and maybe Statistics), received A in the course
  - Career goals: “Math doctorate and teach at the college level . . . or high school teacher . . . or business.”
  - Why take course: Requirement “I probably would have taken it anyway . . . it wasn’t number theory but kind of a precursor to that.”
- Student K, interviewed 10-30-17, female, Junior with dual major in Mathematics Education and Mathematics, received C- in the course
  - Career goals: “Teach secondary but originally music education (percussion)”
  - Why take course: Requirement—but chose to take when Dr. Noblitt was teaching it.
- Student M, interviewed 09-09-17, female, Junior (but with enough credit hours to qualify as a Senior) with dual major in Mathematics and Business Informatics, received A in the course.
  - Career goals: An internship in business “kind of turned me [away] from that” and considering law school.
  - Why take course: A “core class” and prereq for some other advanced classes. She is currently taking *Geometry* because Dr. Noblitt is teaching it.
- Student S, interviewed 10-17-17, female, Junior with dual major in Mathematics and Statistics, received A in the course.
  - Career goals: Sports (to use stats)
  - Why take course: Requirement—“ . . . but some interest in it, because I don’t know, I liked proving things.”

## 5.2. The Classroom Community

We focused on students’ responses to interview questions #2 and #4 in order to capture their perceptions of the *Proofs* classroom community:

Question #2: Describe the classroom community in *Proofs*.

Question #4: Compare this course to other mathematics courses you have had at [this University]. How was *Proofs* unique?

Students used descriptors such as: “close”, “bizarre in the best possible way”, “conversational”, and “tight-knit”. They also talked about the structures that Beth shaped: “open classroom”, “discussion-based”; “everybody participated”, “group work”, and “hands-on”. They described classmates as “friends” and “friendly faces”.

In that regard, M said: “Um, I think we all kind of became friends in that class which I think is unique.” M felt that everyone was “encouraged” to participate in the classroom conversations. D elaborated,

... we all had to speak up ... she constantly split us up into groups ... maybe it was just that perfect mix of personalities. Like it definitely seems like something that was intentional but I’ve had classes before where teachers, professors, tried to make it to where it’s a more tightly knit class and people in the class just weren’t having it. So maybe some luck but it was definitely intentional. (Interview, 09-09-17)

K offered praise, “I really liked the dynamic of it. It was a good dynamic.” However, she added mild criticism,

Um, in some cases, though, it would get a little difficult for some people to get opinions or things like said and heard just because there, you know, it would start with some hand-raising but the further along we got the more people were just shouting over each other sometimes. So sometimes it was a little hard to get words in... (Interview, 10-30-17)

C noted, “... I would say I was closer to people in that class than probably any other class I’ve taken here at [university name].” However, he went on to say this closeness was the result of a smaller class size and the fact that it met early afternoon, “... people weren’t tired all the time.”

S laughed when asked about the classroom community and then said, “I’ve never been in a class that was kind of like that before so it’s kind of hard to describe.” S used expressions such as “conversational” and “comfortable to



express my opinion and kind of like say what confused me . . .” and “very explorative” to describe the classroom community in *Proofs*.

When asked to compare *Proofs* with other mathematics courses at the university, C talked about listening to his classmates “different perspectives” even though mathematics is “pretty black and white”. He noted,

There’s [sic] only so many ways you can take the derivative of a function but there are multiple different, I think there were six ways of proofs . . . Each person liked their [sic] own way . . . I really like the proof by contradiction . . . (Interview, 09-09-17)

As a result of the *Proofs* course, C seemed to think there were six ways to prove something, each taught during the semester as separate methods.

D recalled that there were fewer homework problems in *Proofs*, about “six as compared to 15-20 in other mathematics courses” and most other courses were “lecture where we would take notes for the duration of the class.” Both K and S reiterated what D said about lecture in other courses. K said,

. . . the professor will stand up at the board and, you know, they’ll [sic] write stuff and they’ll [sic] do a bunch of examples and stuff like that. But that’s really the extent of it. Like it tells you ‘why’ but it doesn’t get you the *actual* [emphasis] ‘why’ . . . Like this is what you do to get the answer. (Interview, 10-30-17)

However K continued by calling attention to the fact that she gained a, “. . . deeper understanding and deeper comprehension of ‘why’ certain things work and ‘why’ certain things don’t.”

M talked about the uniqueness of Beth’s listening:

She always lets you have a chance to finish your thoughts. She doesn’t interrupt you with her thoughts and she really tried to understand what you were saying . . . So she lets you put your thoughts into words, doesn’t rush you . . . I felt like I belonged. (Interview, 10-17-17)

This sense of belonging, expressed by M seemed to resonate throughout the students’ perceptions of the classroom community. It was as if their classmates were their “friends” (M) and they were exploring the mathematics together in *Proofs*.

### 5.3. The Teacher’s Actions and Interactions

Students’ responses to interview questions #3 and #6 helped us elucidate their perceptions of Beth’s “believing” teacher actions and interactions:

Question #3: Describe Dr. Noblitt’s “believing” teacher actions during class.

Question #6: Think about interactions with Dr. Noblitt and yourself. Generally speaking, how would you describe them? Is there a particular interaction that you remember well? If so, why do you think it has stayed in your memory?

Students used descriptors such as: “facilitator”; “really pleasant”; “nice”; “very positive”; and, “very encouraging”. When describing teacher actions all students referred to Beth’s focus on facilitating discussions. C said Beth would start the discussion but then she would let students “make the conclusions . . . on our own or as a group.” D made a point of saying that Beth wanted input from everyone. M liked the “big class discussions” better than the discussions in her small groups but she also noted that she liked the mix of small group and whole class discussions. M also said her classmates in *Proofs* were more willing to participate “than I’ve seen in a lot of [other] classes.”

C said he could joke with Beth and he described two specific incidences when he joked with her. He went on, “. . . it was such a fun class with a really good teacher. And up until that point [in his college career] I had been having some trouble with my math classes . . . But this one helped . . . so my grades started improving a lot more last semester.” D also said Beth was “one of the fun ones” and described the time that Beth met with each student in the class individually to talk about a test.

K described Beth as “. . . very positive and very encouraging. . . There was a lot of, um, kind of genuine care.” M said, “Um, I always feel good after I have an interaction.” She elaborated on why:

Cause I don’t like to go to professors and just say, ‘Hey, I don’t understand this.’ And they say, ‘Oh, I just wanted you to say this, this, and this.’ Cause it feels like I’m just going for the answer and they’re just giving it to me. . . . I mean I go to people’s offices a lot but sometimes I’m more reluctant to go. (Interview, 09-09-17)

S said Beth liked to interact in ways that showed concern and that made connections to S's outside life. S participated on the university track team and Beth made a point of asking about her training and about track events. She was taking another class from Beth at the time of the interview and said, "...I was excited that she was teaching [the other class] and not somebody else. Just because of my relationship with Dr. Noblitt." Like K, S used the words "genuine care" to describe Beth's interactions with her.

This sense of "relationship" (S) between Beth and her students seemed apparent throughout their perceptions of the teacher's actions and interactions with them. This relationship was two-way in the sense that they could joke with Beth and she was interested in their outside lives. Interestingly, students had much more to say about their interactions (Question #6) with Beth than about her teacher actions (Question #3). Perhaps this was because they were participants in the interactions rather than merely observers of the teacher actions. They formed a relationship because they were negotiating a shared understanding of mathematics as a classroom community.

#### 5.4. Students' Learning

Students' responses to Question #5 helped us understand how they viewed their learning in *Proofs*.

Question #5: Recall a proof that you found particularly interesting or intriguing. What made it interesting or intriguing?

When asked to recall a proof they found particularly interesting or intriguing four of the five students recalled a proof or vaguely described some aspect of a proof. C "vaguely" [his word] remembered the proof of infinitely many prime numbers by contradiction. D said it was going to be hard to remember but, "It was where you assume  $k = 0$ , um, no, you prove that  $k = 0$ , you assume  $k = n$ , and then you use that to prove that  $k = n + 1$ . Proof by induction. Or strong induction." D went on to say he would remember if he had his notebook with him. K jumped right on this question and said, "...the finger trick." This was the finger algorithm proof. She went on to accurately describe how to use her fingers to show multiplication of seven times six and also explained why this finger algorithm worked. M was less clear about recalling a proof but she said there was one she remembered with factorials. S said that "...proving the square root of two is an irrational number.

That stuck with me probably . . . and I’ve used it since then in another class.” She recalled that it was a proof by contradiction.

Looking at other interview questions also gave us some insight into students’ learning. D, in response to Question #3 said he learned to think [emphasis with voice]. S talked about learning from her “failures” in response to Question #3. D, in response to Question #4 and the homework, said, “. . . learn how, like the process as opposed to how to do this one specific question.” K was most adamant about how *Proofs* allowed her to gain a “deeper understanding” and a “deeper comprehension” of “‘why’ certain things work . . .” as compared to other mathematics courses. S talked about *Proofs* as geared toward “hands-on” learners:

It’s not as like, I think in some of my other classes it’s just lecture and you take notes. It’s kind of hard to learn sometimes, and like going home and trying to think like what do I do with this now that I have it. Whereas like in 302 [the *Proofs* course] we always like were moving and we like knew what we were going to do next and like had things to practice on it. And I just think that like it was more geared, it helped me learn better with how it was set up. (Interview, 09-09-17)

Students’ perceptions of their own learning in *Proofs* focused on the notion that they gained a deeper understanding of the mathematics because they were not merely learning a procedure in order to pass an exam and this was because of the classroom community along with the teacher actions and interactions.

## 6. Discussion

Recall our research question: How did students in *Proofs* describe their classroom community, the teacher’s actions and interactions with them, and their own learning? Here we analyze the student responses in relation to this question and the construct of stance we described in §§3.2.

Students felt a part of a classroom community in which their mathematics was honored and respected. This loosely related to Beth’s teacher action 4 from her plan: “Talk less. Let students present problems to each other. Allow students to ask each other questions about the presentations.” They also to

some extent talked about “believing” teacher actions 3 and 5. Students did not say Beth used specific phrases like “That’s interesting. Tell me more. I don’t know.” However, they described her interest in their mathematical thinking.

Yet, when we considered Beth’s “believing” stance statement, it became clearer to us that the students’ perceptions mirrored her stance. Recall from §2 that Beth recorded her stance as follows:

There is merit to students’ thinking, and that thinking should be shared and valued in and out of the classroom. Students can, will, and must share their thinking for my benefit and the benefit of other students. I take responsibility for realizing this aim.

Reviewing student responses to the interview questions, we feel confident that Beth’s stance, and not necessarily her specific “believing” teacher actions and interactions with students, was more evident in the students’ perceptions; this seems evident given their descriptions of the classroom community and their own learning. Therefore, what did we each learn from this research?

### *6.1. Shelly Reflects*

After Beth wrote her “believing” stance statement, I decided to write a stance statement:

I believe I must prepare my preservice students for teaching all children and for schools as they should/could be rather than maintaining schools as they currently exist.

I have this statement posted on the front cover of the mathematics methods textbook that I use. I have shared this recorded statement with my students so that they know what inspires or motivates my actions as a mathematics teacher educator.

Although this statement does not focus on the practice of methodological belief [7, 9], the believing game is still a crucial component of my pedagogy: when I model mathematics lessons and activities in my class I focus on belief rather than doubt as I attempt to find virtues and strengths in preservice teachers’ mathematics, no matter how unlikely their ideas, solutions or answers might seem. It is important to honor and respect preservice teachers’ mathematical thinking and to model belief rather than doubt.

I demonstrate my belief about the importance of negotiating shared mathematical understanding. Preservice teachers can learn mathematics from each other and I can learn mathematics from preservice teachers.

I felt honored that Beth opened her classroom door and made me feel like a member of the classroom community. When I visited *Proofs*, I watched and listened as her students proved mathematical concepts. I learned mathematics along with them. I also learned that my stance statement motivates me to align my teacher actions and beliefs and that sharing my stance statement with students helps them understand how and why my teacher actions affirm my beliefs.

### 6.2. Beth Reflects

Writing a “believing” stance statement and sharing it with Shelly helped me to dig deeper into my motivation to play the believing game as well as to strengthen my belief that playing the believing game was beneficial to students. It motivated me to employ the teacher actions that prompted me to play the believing game in the classroom. It made my beliefs more *real*.

By spending a semester purposefully playing the believing game, I learned a lot about myself as a teacher and a lot about students as learners. I learned that I enjoy the classroom more when I worry less about the content covered and more about the quality of classroom discourse and student interactions. I also learned mathematics. When I really listened to students in order to understand the merit in their thinking, I gained mathematical insight that I did not have before. I also learned that students respond to their thinking being valued in ways that enrich classroom discussions. When I played the believing game with them, they were willing to take authority over their own learning and drive the classroom discussion themselves.

By putting my stance into action, this research facilitated my own growth as a teacher and a mathematician. I am grateful to have had the opportunity to participate in research that prompted me to formalize my stance. Perhaps my stance will change in the future, but part of it will remain true; “I take responsibility for this aim.” Regardless of what my stance is or how it changes, it is my responsibility to see it to realization.

## 7. Concluding Remarks

Consider the way in which we conceptualized stance previously in §2:

A teacher's stance is a *deliberate* and *purposeful* statement about a teacher's beliefs, recorded and explicitly shared with others.

Thus a teacher's stance is a deliberate and purposeful statement that operationalizes beliefs into teacher actions. Stance might be seen in a teacher's body position and a teacher's actions. It is a secure link between beliefs and practice. In our research we saw that students' perceptions of teacher beliefs and practices did not focus on the "believing" teacher actions. Rather their perceptions described her "believing" stance. Bondy *et al.* [2, page 445] note that, "...it is surely more difficult to develop a stance than a set of strategies, just as it is more difficult to show students you care than it is to say the words". Yet, like a secure ledge or foothold on which a belay can be fastened for rock or mountain climbing, Beth's stance, perhaps, allowed her to "cleve ever to the sunnier side of doubt".

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## A. Appendix

The Interview Protocol consisted of collection of Demographic Data and answers to six interview questions.

### A.1. Demographic Data

- Gender identification
- College standing (Freshman, Sophomore, Junior, Senior, Post-Bac)
- College Major
- Career goal(s)
- Final grade in *Proofs*

### A.2. Interview Questions

1. Why did you take *Proofs*? Was it a requirement or elective for you?
2. Describe the classroom community in *Proofs*.
3. Describe Dr. Noblitts’s teacher actions during class.
4. Compare this course to other mathematics courses you have had at [this University]. How was *Proofs* unique?

5. Recall a proof that you found particularly interesting or intriguing. What made it interesting or intriguing?
6. Think about interactions with Dr. Noblitt and yourself. Generally speaking, how would you describe them? Is there a particular interaction that you remember well? If so, why do you think it has stayed in your memory?