

An Interdisciplinary Rendezvous Between Mathematics and Literature: Reflections on Beauty as a Perspective in Comparative Disciplinary Didactics and a Thematic Approach to Interdisciplinary Work in Upper Secondary School

Uffe Thomas Jankvist
Aarhus University

Helle Rørbech
Aarhus University

Jesper Bremholm
University College Copenhagen

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An Interdisciplinary Rendezvous
Between Mathematics and Literature:
Reflections on Beauty as a Perspective in Comparative
Disciplinary Didactics and a Thematic Approach to
Interdisciplinary Work in Upper Secondary School

Uffe Thomas Jankvist

Danish School of Education, Aarhus University, DENMARK

utj@edu.au.dk

Helle Rørbech

Danish School of Education, Aarhus University, DENMARK

hero@edu.au.dk

Jesper Bremholm

National Centre for Reading, DENMARK

jbre@kp.dk

Abstract

In this paper we propose a thematic focus on aesthetics in the context of an interdisciplinary collaboration between mathematics and literature (Language Arts) as a way to further students' reflections on and deeper understanding of what characterizes the two subjects. Furthermore, we argue that approaching aesthetics through the perspective of literacy can potentially strengthen students' understanding of ways of thinking particular to specific (academic) disciplines; ways of thinking that are otherwise often hidden when teaching focuses on more pragmatic aspects. G. H. Hardy's *A Mathematician's Apology* from 1940 serves as the recurring illustrative example in our discussions of the pedagogical potentials of an interdisciplinary rendezvous between mathematics and literature.

Keywords: aesthetics, interdisciplinarity, literacy, mathematics, literature.

1. Introduction

It is generally recognized, at least in the scientific community, that aesthetics is an essential dimension of the nature and epistemology of mathematics, see for example [9].¹ However, in primary and secondary educational settings, it is fair to say that the aesthetic dimension is rarely given much notice. In primary and secondary mathematics textbooks, for instance, it is highly unusual, if not unheard of, to find chapters or sections where the aesthetic dimension of mathematics is the thematic focus. Likewise, the aesthetic dimension is not part of international assessments such as PISA and TIMSS. In the Danish context, which is our primary reference in this paper, the aesthetic dimension is absent from the regulatory documents for the subject of mathematics in both primary and lower secondary school and upper secondary school [5, 6].

There might undoubtedly be some sensible reasons for prioritizing other elements and not emphasizing aesthetics in mathematics teaching, but this is not the issue we wish to address in this paper. Instead, the purpose of this paper is to argue for the status and pedagogical potential of the aesthetic dimension as a thematic focus in mathematics teaching in particular when realized in an interdisciplinary setting.² We will thus conduct our line of argument within an interdisciplinary framework consisting of collaboration between mathematics and Language Arts (LA), focusing in particular on the literature part of the LA curriculum.

There are several reasons for this approach. Firstly, it is our theoretically founded assumption that an interdisciplinary approach can strengthen the pedagogical potentials of working with aesthetics as a thematic focus. Secondly, we consider collaboration between mathematics and LA to be par-

¹ A precise definition of aesthetics is provided later.

² Other authors have also emphasized the importance of aesthetics in mathematics teaching, see for example [38]. Several of the articles in the special issue of the *Journal of Humanistic Mathematics* on the nature and experience of mathematical beauty are also on this topic; see <https://scholarship.claremont.edu/jhm/vol6/iss1/>. Our work here is in some ways parallel to these works, though our main goal is to argue for an interdisciplinary emphasis, tying aesthetics, mathematics, and literature to one another in productive ways.

ticularly advantageous with regard to the aesthetic focus and to allow for a different and more balanced form of interdisciplinary collaboration than is usually the case in interdisciplinary settings involving mathematics. Thirdly, there are interesting similarities and differences between mathematics and LA regarding the aesthetic dimension. Thus, it is interesting to study how the comparison between the ideas of beauty in mathematics and in literature and poetry can shed light on these similarities and differences between the two subjects and to explore the pedagogical potentials in this kind of comparisons.

Besides interdisciplinarity, the other main component in our line of argument is disciplinary literacy, which is an established concept within literacy research. In this paper, we will apply disciplinary literacy as the theoretical framework for our analysis and discussion of the aesthetic dimensions of mathematics and literature as we consider the concept of disciplinary literacy to be especially well suited for understanding and describing the pedagogical potentials of interdisciplinary collaboration between the two subjects. Using this theoretical framework, we wish to point to aesthetics as a way to further students' access to disciplinary content and content knowledge.

It is important to emphasize that what we lay forth in this paper is a theoretical contribution to the discussion of aesthetics as a thematic focus in mathematics (and LA) teaching. It is not an empirical study, and thus, the paper does not offer empirical data or experiences from classroom implementations that are analyzed using a specific methodology and producing a specific result. We acknowledge that the lack of empirical support might be considered a flaw by some, but, on the other hand, we maintain that theoretical reflections are essential both in scientific work in general and in comparative disciplinary didactics. As such, the arguments we develop in the paper can provide the theoretical basis for future empirical studies.

In order to provide a specific reference for our theoretical reflections, we center our line of argument on an illustrative example. For this purpose, we have selected the mathematician G. H. Hardy's famous essay *A Mathematician's Apology* from 1940 [14]. As the reader may well know, in this text, Hardy accords a prominent status to beauty and aesthetics as a core element of mathematics and mathematical activity. Furthermore, he uses numerous literary examples and comparisons as part of his reasoning. Using Hardy's *Apology* as the example for our argument, we focus on upper sec-

ondary teaching of mathematics and literature in LA, as we deem Hardy's text most appropriate for this grade level. It should also be mentioned that our main reason for looking into LA alongside mathematics is that aesthetics is a core topic in the LA curriculum in Danish upper secondary school. It features as an essential element in the general description of LA: "Through the intensive reading of texts in a broad sense of this notion, the discipline links linguistic, historical, and aesthetic perspectives and thus leads to interactions between experience, analysis and interpretation" [7, our translation].

We build our line of argument in three main steps. As the first step, we explain the theoretical jumping-off point for the ensuing reflections and discussions in the paper, more specifically the concepts of disciplinary literacy, aesthetics, and interdisciplinarity (§2). Likewise, as a backdrop for what is to come, we outline some of the challenges related to aesthetics and interdisciplinarity in the two school subjects of mathematics and LA (§3). As the second step, we turn to the illustrative example and present Hardy's idea of beauty in mathematics as laid out in the *Apology* (§4). This idea in turn serves as the point of reference for the following explication of the notion of beauty in poetry among Hardy's contemporaries (§5) as well as for the discussion of Hardy's comparisons of beauty in mathematics and literature (§6), which conclude the second step. As the final step, we address the pedagogical potential of an interdisciplinary collaboration between mathematics and literature (in LA) from the perspective of disciplinary literacy (§7). We close our argument by also taking into account the question of so-called disciplinary identity (§8), before drawing our conclusions (§9).

2. Disciplinary Literacy, Aesthetics, and Interdisciplinarity

Among scholars and researchers, the concept of literacy has been and still is the subject of much debate and dispute as to its proper meaning and use. While acknowledging the importance of this intricate theoretical discussion, we refrain from dwelling on it in this paper. When using the term *literacy*, we refer to the definition proposed by Skjælbred and Veum: "Literacy signifies the ability to interpret, produce, and reflect on texts" [39, page 18, our translation]. Following from this definition, disciplinary literacy designates the specialized literacies connected to different content areas, professions,

or domains, and it is characterized by distinct discursive conventions in the form of specialized ways of using verbal language, as well as other semiotic modalities, to shape knowledge and meaning [2, 26, 36, 37]. Several studies on disciplinary literacy point to this intrinsic interdependence between disciplinary content and discursive form as a particularly demanding challenge when students, or people in general, learn and try to master a specific subject or discipline:

The complexity of the process of learning to be literate in a content area lies in the fact that these skills are interdependent. That is, being able to access content knowledge depends at some level on one's understanding of the discursive conventions of the content area. In a similar manner, developing strong interpretive or rhetorical skill in a content area requires that one understands the relevant content concepts. [32, page 45]

In an educational context, the interplay between content knowledge and discursive conventions pointed to by Moje and colleagues [32] applies to all curricular areas of a given school subject. However, from the perspective of disciplinary literacy, we contend that the comparison of the aesthetic dimension of different subjects is interesting because this dimension, like disciplinary literacy, concerns the interaction between content and form within the subject in question. Focusing on the aesthetics of a subject is an invitation to contemplate and reflect on the specific relationship between content (content knowledge) and form (discursive conventions) that characterizes the aesthetics of this subject. In this sense, the aesthetics of a subject represents a direct way of addressing the disciplinary literacy of the subject and gaining insight into the interplay between content knowledge and discursive conventions that is distinctive to the subject in question. In the context of teaching, such insight might also influence and expand the students' perceptions of the subject at hand. Inspired by Gee [12, 13], we choose to rephrase the matter of students' perceptions of a given subject as a question of disciplinary identity. Here, we understand disciplinary identity as how a person is recognized and recognizes him- or herself as a member of a disciplinary community ([12, 13], also see [30, 35] for the related concept of a mathematical identity).

The concept of aesthetics covers a wide field: from its looser everyday use, referring to beautiful phenomena such as architecture, works of art, or experiences in nature, to philosophical and theoretical discussions from the 18th century and onwards. While this paper is by no means intended as a contribution to the general philosophical discussion of the concept, defining what we mean by mathematical and literary beauty in the following paragraphs requires a short definition of aesthetics.

In *Aesthetica* (The Aesthetics) from 1750 [1], the German philosopher Baumgarten makes a distinction between the beauty of objects (and materials) and the beauty of thoughts and experiences [41, page 18]. The Danish philosopher Jørgensen has developed Baumgarten's ideas, highlighting four different perspectives on aesthetics: Metaphysics of beauty, art theory, philosophy of art, and philosophical aesthetics [27, page 24]. According to Jørgensen, the metaphysics of beauty deals with the idea of beauty. Art theory, on the other hand, focuses on the expression of the artwork, while art philosophy deals with the nature of the artwork. Finally, philosophical aesthetics focuses on the aesthetic experience [27]. In our discussion of the potentials in an interdisciplinary collaboration focused on beauty in mathematics and LA, we primarily refer to Jørgensen's second category. However, when we discuss the learning potentials in an interdisciplinary collaboration focused on aesthetics in the two disciplines, and consider students' access to opportunities to learn, understand, and experience mathematics and poetry, we draw on a framework that somewhat resembles her fourth category. Jørgensen defines the notion of philosophical aesthetics as a way of thinking that has 'the character of feeling, perceiving, and sensing' [27, page 23, our translation], a definition that resembles the understanding of aesthetic reading within literature pedagogy.

In relation to interdisciplinarity, Jantsch's taxonomy for distinguishing between five different kinds of 'disciplinarity' [21], placed on a spectrum ranging from (1) multidisciplinary to (5) transdisciplinarity, is often used. While multidisciplinary indicates that the individual disciplines exist side-by-side in the educational system, transdisciplinarity indicates such a degree of interdisciplinary collaboration that the individual disciplines become invisible, such as when collaboration between biology and chemistry becomes biochemistry. Ulrichsen [43] argues that the three kinds of disciplinarity positioned

between these two poles are what may be considered ‘true’ forms of interdisciplinarity in an educational setting: (2) pluridisciplinarity, (3) crossdisciplinarity, and (4) interdisciplinarity proper. In approaches characterized by pluridisciplinarity, disciplines cooperate to address a common theme, for instance ‘obesity’, which can be addressed from the perspectives of both sociology and biomedicine. Crossdisciplinarity involves one or more disciplines acting in support of another discipline, such as when biology relies on mathematical methods to model a purely biological problem, possibly also drawing on philosophy to clarify certain ethical aspects. Interdisciplinarity proper, in Jantsch’s terminology, occurs when all involved disciplines are subject to a common external principle or problem. In such cases, the focus shifts from the individual disciplines to the connections between them. The situations for these three types of interdisciplinarity (2, 3, 4) are illustrated in Figure 1 (adapted from [23]).

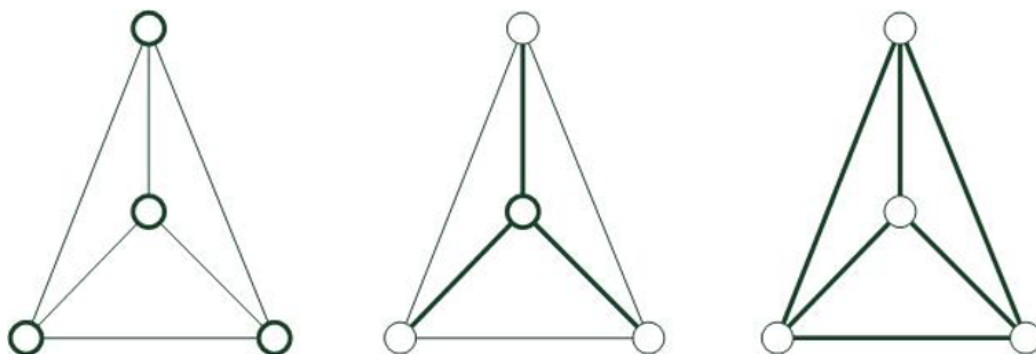


Figure 1: From left to right: Jantsch’s pluridisciplinarity, crossdisciplinarity, and interdisciplinarity proper illustrated as the emphasis (by dark coloring) on subjects (circles) and connections between subjects (lines) [19, 23].

Another perspective on interdisciplinarity in an educational context is provided by Lindenskov, who describes the possibility for the subjects (disciplines) to mirror one another based on a common theme or problématique [31]. Although such a perspective may at first seem similar to a pluridisciplinary approach, this need not necessarily be the case, as we will later illustrate with the common theme of aesthetics.

3. Mathematics and Literature in Interdisciplinary Settings

Niss describes the discipline of mathematics as having a “five-fold nature” [33]. The first of these is as a scientific discipline of pure mathematics, seeking to find/create or describe and understand phenomena, relations, mechanisms, etc. belonging to a ‘particular universe’, usually referred to as mathematics. The second of the five is that of application, i.e. applied mathematics, by looking beyond this particular universe to answer questions regarding extra-mathematical phenomena, relations, mechanisms, etc. Thirdly, mathematics is a system of tools, comprising processes and products, which assist in decision making and actions within other practice areas. A given use of mathematics as a system of tools may have been the result of previous breakthroughs in either pure or applied mathematics. Fourthly, mathematics is a teaching subject or ‘school subject’ — at all educational levels. In teaching, ‘mathematics encounters people’s views, beliefs, thoughts, emotions, experiences, etc., which create particular problems and issues that are not to be found in the same manner in the other natures of mathematics’ [33, page 14, our translation]. Finally, the fifth nature concerns mathematics as a space for a particular kind of aesthetic experience. Most people who have had positive experiences with mathematics have had an ‘a-ha moment’, such as an epiphany regarding what previously appeared to be an unsolvable problem. Or they have experienced what previously seemed to be a mishmash of unrelated rules and procedures suddenly form neatly interrelated structures. Or they have experienced the coming into being of mathematical objects with fascinating — or even surprising — features or characteristics. Or they have discovered that something essentially complex may be formulated — or proved — in a very simple manner or by means of a very simple idea [33]. The examples are numerous, and in the following section we showcase an example provided by Hardy in the *Apology*.

As the reader might imagine, when mathematics enters into an interdisciplinary setting, its five natures may be valued and weighted rather differently. In many interdisciplinary educational settings, mathematics merely serves as a system of tools [20, 25], thus suppressing its other four natures, and in particular its first nature as a scientific discipline in its own right. This is especially common when mathematics collaborates with disciplines (subjects) within the natural sciences — in particular in a school context — since these

disciplines often rely on mathematical tools, applying mathematical theories, methods, etc. To some extent, this is also true of collaborations with the social sciences, not the least with economics [24]. Of course, in some cases the natural and social sciences may provide mathematics with questions and problems that serve as catalysts for developments within pure mathematics, as Hilbert has pointed out [17]. Such examples may bring the first three natures of mathematics to the forefront in a teaching situation. However, if a teacher wants to illustrate the fifth nature of mathematics to students, and perhaps link this aesthetic potential to its scientific nature, the arts seem to offer something different in terms of interdisciplinary collaborations in an educational setting. We certainly do not claim to be the first to point out potential synergies between mathematics and the arts, see e.g. [9, 10, 42];³ our contribution here is to explore the potentials of such a ‘rendezvous’ in an educational setting centered around the theme of aesthetics.

At the same time, meanwhile, it has been documented that it is difficult to convey the aesthetic dimension of literature and poetry to students [15, 18]. It is quite common to hear students state that their appreciation of literature has been ‘destroyed’ rather than enhanced by LA teaching, and that they cannot see the point in dwelling on the enigmatic phrasing in poems by Rimbaud, for example. ‘Why can’t he just tell it like it is’, they object. As such, the aesthetic dimension constitutes a pedagogical challenge in both LA and mathematics, although the main problem of this challenge differs: in mathematics, the aesthetic dimension is often neglected, whereas in LA (here Danish), it does not easily transmit to the students.

It is important to point out that the interdisciplinary collaboration between mathematics and LA we propose is not about reversing the interdisciplinary hierarchy to place mathematics as the main subject and LA in an auxiliary position, or in the opposite direction, to designate LA as supreme and mathematics as a subordinate. On the contrary, the collaboration we envisage is of a more balanced and integrative kind where both subjects will profit equally. Following Jantsch’s taxonomy mentioned above [21], the collaboration we propose would be characterized as interdisciplinarity proper, inasmuch as

³ There is even an annual international conference that explores the many dimensions of these synergies; see <https://www.bridgesmathart.org> for more information.

the collaborating disciplines share a common problem and the focus is on their connections, similarities, and differences; see also [19]. In other words, in the ideal case, the subjects come to mirror one another [31].

In the next three sections, we present an example illustrating a possible foundation for interdisciplinary work, for instance in upper secondary school, in the form of Hardy's aforementioned *Apology* and his views on beauty in mathematics, as well as various views on beauty in poetry.

4. Hardy's Perception of Beauty in Mathematics

Hardy was famous for his aphorisms, one of which was: "Sometimes one has to say difficult things, but one ought to say them as simply as one knows how" [40, page 47]. This idea of explaining complex matters and results in a simple — and aesthetic — way is deeply rooted in Hardy's perception of beauty in the subject of mathematics. In fact, when reading the *Apology*, it is easy to get the impression that Hardy saw mathematics as more closely related to the arts than to the sciences.

In the *Apology*, Hardy spends quite a lot of space explaining the role of mathematical beauty in the work of the pure mathematician:

The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics. [14, page 14]

He addresses the beauty of both mathematical theorems and mathematical proofs:

The beauty of a mathematical theorem depends a great deal on its seriousness, as even in poetry the beauty of a line may depend to some extent on the significance of the ideas which it contains. [14, page 17]

As an example of one of 'the most beautiful theorems of the theory of numbers', Hardy mentions 'Fermat's two square theorem on the law of quadratic

reciprocity' [14, pages 17–18]. However, since he deems the proof of this theorem too difficult an example for non-mathematicians to follow (Euler's proof draws on quite a bit of elementary number theory), he instead turns to Euclid's theorem and proof for the existence of infinitely many primes, and Pythagoras's theorem and proof for the square root of 2 being an irrational number. To illustrate Hardy's perception of mathematical beauty, we provide his paraphrasing of the former of the two.

The prime numbers or primes are the numbers

$$(A) \quad 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \dots$$

which cannot be resolved into smaller factors. Thus 37 and 317 are prime. The primes are the material out of which all numbers are built up by multiplication: thus $666 = 2 \cdot 3 \cdot 3 \cdot 37$. Every number which is not prime itself is divisible by at least one prime (usually, of course, by several). We have to prove that there are infinitely many primes, i.e. that the series (A) never comes to an end.

Let us suppose that it does, and that

$$2, 3, 5, \dots, P$$

is the complete series (so that P is the largest prime); and let us, on this hypothesis, consider the number Q defined by the formula

$$Q = (2 \cdot 3 \cdot 5 \cdot \dots \cdot P) + 1.$$

It is plain that Q is not divisible by and of $2, 3, 5, \dots, P$; for it leaves the remainder 1 when divided by any one of these numbers. But, if not itself prime, it is divisible by some prime, and therefore there is a prime (which may be Q itself) greater than any of them.

This contradicts our hypothesis, that there is no prime greater than P ; and therefore this hypothesis is false. The proof is by *reductio ad absurdum*, and *reductio ad absurdum*, which Euclid loved so much, is one of a mathematician's finest weapons. [14, pages 18–19]

According to Hardy, a ‘mathematical proof should resemble a simple and clear-cut constellation, not a scattered cluster in the Milky Way’ [14, page 29]. Euclid’s proof surely fulfills this criterion; it is a proof by contradiction, and the contradiction is established by means of construction, i.e. of a prime number that is larger than the assumed largest prime number. Hardy remarks:

I said that a mathematician was a maker of patterns of ideas, and that beauty and seriousness were the criteria by which his patterns should be judged. I can hardly believe that anyone who has understood the two theorems will dispute that they pass these tests. [14, page 21]

In regard to seriousness, Hardy’s argument is that primes are the very ‘raw material’ of which we must build arithmetic, and Euclid’s proof ensures us that we will never run out of ‘raw material’. He then goes on to argue that neither of the two theorems displayed has the slightest practical importance. Based on what we know today, for example concerning the use of prime numbers in cryptography, we must of course disagree, keeping in mind that Hardy at the time had some three thousand years to support his observation. Still, this is not the primary issue of interest in the present paper, where we are concerned with the perception of mathematical beauty. Yet, Hardy’s insistence on the ‘uselessness’ of the two theorems supports an observation that Hardy sees mathematical beauty as closely connected to pure mathematics (Niss’ first and fifth nature of mathematics [33]).

5. Beauty in Poetry From the Perspective of Hardy’s Contemporaries

Hardy’s comparison of the beauty of mathematical theorems and of poetry (and literature) is not unique. A number of poets and literary critics have compared the poetic style, the accuracy of the metaphor, and the abstraction of a work of art with mathematical precision and ‘the severe logic’ of a mathematical problem ([11, page 35]; see [4] for an even earlier comparative study of mathematics and poetry, and [28] for a more recent exposition on the same theme.). In the following, we shift our focus to literature studies.

The context of our discussion is mid-20th century literary criticism; in other words, perceptions of and thinking about poetry contemporaneous with Hardy's *Apology*. This allows us to reflect on similarities and differences in the conceptions of beauty in the two disciplines, to challenge Hardy's arguments on literature from the perspective of literature theory, and to point at the pedagogical potentials in this comparison

In the 1940s and 1950s, New Criticism emerged as a new trend in literary research and criticism in the United Kingdom and the United States [22]. Two interrelated assumptions about the literary work of art were at the heart of the movement. While the *theory of autonomy* expressed the idea of the poem as an independent entity, freed from the intentions of its author as well as from history and other contextual circumstances, the *theory of structure* embraced the aesthetic unity of the poem's form and content. Well-known metaphors such as a 'crystalline structure' [11] and a 'well wrought urn' [3] bear witness to the new critics' understanding of poetry as an autonomous aesthetic totality.

In this notion of autonomy and in the new critics' interest in the solitary poem on the page of a book, it is possible to find analogies to Hardy's *Apology* for pure mathematics. Just as Hardy saw mathematical beauty as a harmonious relationship between a simple form and 'severe ideas', the new critics understood the relationship between the parts and the whole of the poem as a pattern of resolved tensions [22]. The graphic expression, the sound of the poem when read aloud, the significance of metaphors and other tropes, and the layers of meanings and ideas were seen as parts of an ambiguous and paradoxical whole. For this reason, poetry, modern as well as older, contains experiences and ideas of ambiguity and complexity inaccessible through the meanings and logics of everyday language. Or as Brooks pointed out:

The conventional theories of communication offer no easy solution to our problem of meanings: We emerge with nothing more enlightening than this graceless bit of tautology: the poem says what the poem says. [3, page 74]

The only way to approach the meaning of a poem is to read its ambiguous, (ironic) and paradoxical utterances through a close reading strategy, working

one's way through the complex relations between its layers to gain an understanding of the unique meaning of its specific unity of content and form [44]. Reader-response theories criticised this approach to literature, stressing diverse readers' different readings. In continuation of the new critics' understanding of the aesthetics of literature, Brooks called literature 'the language of paradox' and stressed that simple literary forms like metaphors, symbols and paradoxes hold a complexity of thoughts (not unlike Hardy's point concerning Euclid's proof):

I have said that even the apparently simple and straightforward poet is forced into paradoxes by the nature of his instrument. Seeing this, we should not be surprised to find poets who consciously employ it to gain a compression and precision otherwise unobtainable. [3, page 10]

As the critics stress, the beauty of poetry (and literature) is not only a question of form and rhetoric, but also of a complexity of ideas that lie beyond everyday language, logic, and reality. However, in his reflection on the difficulty of poetry, Brooks does not blame the poet, but the reader's ability to read such texts:

[...] a great deal of modern poetry is difficult for the reader simply because so few people, relatively speaking, are accustomed to reading poetry as poetry. [3, page 76]

This is a challenge which literary pedagogy still tries to address through reflections on how to teach students to read and understand poetry and literature [29, 34]. The ideas of New Criticism have had a major impact on literature teaching in Danish schools from the 1960s and onwards, in particular in connection with the teaching of modernist poetry. Although later literary theories break with some of these ideas, such as the theory of autonomy, other ideas developed by New Criticism have become a basis for LA literature education.

6. Hardy's Perception of Beauty in Literature

In the *Apology*, Hardy refers to harmonic patterns in literature to highlight the nature of mathematical beauty, and literary critics from the same period refer to mathematics to describe the clear-cut form of modern poetry (and to some extent of literature in general). However, Hardy also points out the different ways in which patterns of ideas shape mathematical theorems and literary texts. To demonstrate these differences, he quotes two verses from Shakespeare's *The Life and Death of Richard the Second*:

Not all the water in the rough rude sea/Can wash the balm off
from an anointed King (*The Life and Death of Richard the Second*, Act 3, scene 2)

Hardy makes the following comment:

Could lines be better, and could ideas be at once more trite and more false? The poverty of the ideas seems hardly to affect the beauty of the verbal pattern. [14, page 14]

When reading literature and poetry, the perception and saying of the words may overshadow the weakness of the ideas presented, Hardy stresses. This does not mean that ideas do not matter in poetry, but it shows, according to Hardy, that mathematical ideas hold a different gravity and are less affected by time and history than literary ideas. He uses a quotation from another Shakespearean drama to illustrate this point:

After life's fitful fever he sleeps well (*Macbeth*, Act 3, scene 2)

Hardy finds this verse more beautiful than the previous ones, because:

The pattern is just as fine, and in this case the ideas have significance and the thesis is sound, so that our emotions are stirred much more deeply. The ideas do matter to the pattern, even in poetry, and much more, naturally, in mathematics. [14, page 17]

Yet, looking at the first passage through the eyes of the literary critics quoted in the previous section, we must disagree with Hardy's judgement. When the verses from *The Life and Death of Richard the Second* are judged by their use of 'the language of paradox' and of 'an indirect message' [3], they form a beautiful unity of a complex statement and a simple form. What is expressed as a paradox in the compressed form of the verses would require a much longer and less aesthetic explanation, if expressed in everyday language.

We would like to make two points here. Firstly, we agree, of course, with Hardy's point, that ideas in mathematics and in poetry, in spite of some similarities, take very different forms. Secondly, we believe that ideas do matter to the patterns of the expression, both in poetry and in mathematics. The new critics' point was, however, that ideas in poetry and literature transcend established linguistic conventions and thereby open potentials for new ways of thinking. Yet, to capture the similarities and differences of the beauty of the two phenomena demands different modes of thinking and reading. To study the beauty of the verses of a Shakespearean drama or the beauty of modern poetry, one needs to read 'poetry as poetry' [3], just as one needs to read (and think) mathematics as mathematics in order to understand the beauty of theorems and proofs of Euclid, Pythagoras, and Fermat.

In this way, an interdisciplinary rendezvous between mathematics and literature in an upper secondary school setting has the potential to make students reflect on disciplinary ways of thinking.

7. Interdisciplinary Potential in a Literacy Perspective

If we now contemplate the question of mathematical and literary beauty and aesthetics from the perspective of disciplinary literacy, the potential of an interdisciplinary approach becomes apparent. This is the case because the aesthetic dimensions of mathematics and literature explicitly illustrate how discursive form and subject-related content are interwoven in specific ways in the two subjects, and thus constitute a direct and easily accessible manifestation of their disciplinary literacy. In this respect, Hardy's *Apology* represents a suitable text for at least two reasons. Firstly, we witness, in this text, a mathematician's explicit reflections on the beauty of pure mathematics and

on how this beauty is the result of a specific relationship between mathematical content and form. In this sense, you might even say that Hardy's *Apology* constitutes a metatext on disciplinary literacy (of mathematics). Secondly, Hardy's essay explicitly invites the interdisciplinary perspective by comparing mathematical and literary beauty as we pointed out and exemplified in the preceding section.

In regard to the first reason, we saw earlier, using Euclid's proof as our example, how the relationship in question is characterized by 'a pattern of ideas' fitting 'together in a harmonious way' or in 'a simple and clear-cut constellation', thereby showing us pure mathematical reality. It is interesting to note that Hardy, when speaking about the beauty of mathematics, uses the term 'ideas' and not 'form' or 'language', which indicates that he adheres to an essentialist idea of mathematics that does not explicitly recognize the discursive, the rhetorical, or what we here refer to as the literacy aspect of mathematics. Nevertheless, his metaphors give him away, so to speak. When Hardy describes the arrangement of ideas in Fermat's, Euclid's, and Pythagoras's theorems and proofs as patterns and constellations, it is a roundabout way of admitting the importance of the semiotic or discursive arrangement of these ideas, thereby, following the explanation above, pointing to an aspect of the specific disciplinary literacy of mathematics. Furthermore, as the explanation of Euclid's theorem presented earlier quite clearly demonstrates, one of the ways mathematical thinking (constituting patterns and constellations) manifests itself is in the use of a specialized language and mathematical notation, i.e. disciplinary literacy (see e.g. [24]).

As to the second reason, we saw in the paragraphs concerning the aesthetics of poetry that, even though Hardy's comparative approach to mathematics and poetry helps bring to the fore the aesthetic dimension of mathematics, it may represent what literary critics contemporaneous with Hardy would have considered a simplistic view of poetic beauty. Hardy's view, as displayed in the *Apology*, is not sufficiently sensitive to the important differences between mathematics and poetry. Hence, within the perspective of disciplinary literacy, Hardy's *Apology* is interesting because its comparative approach helps elucidate what constitutes mathematical understanding and the discourse of beauty. Furthermore, it shows us the difficulty, or maybe even impossibility, of a given disciplinary literacy (in this case mathematics) representing

and grasping a phenomenon outside its disciplinary boundaries (in this case poetry).

In an educational setting in upper secondary school, therefore, it could be beneficial to let Hardy's *Apology* form the textual foundation for the interdisciplinary collaboration between mathematics and LA (in the sense of interdisciplinarity proper). Having students read Hardy's text in both mathematics and LA — preferably in combination with selected excerpts on literary aesthetics (e.g. Hugo Friedrich [11]) or, perhaps even better, segments from essays on poetry by modernist poets (e.g. T.S. Eliot, Charles Baudelaire or Ezra Pound) — would provide them with a rare opportunity to reflect upon and discuss aesthetics. This discussion would include aesthetics both as a general phenomenon (the interconnectedness of form and content), and, more importantly, as a specific aspect of the disciplinary literacy of the two subjects.

Through the interdisciplinary collaboration, with its double perspective, the two subjects might mirror each other's aesthetic literacy [31], thereby helping students grasp the specific nature of both mathematics and LA by allowing them to reflect upon both their shared characteristics and their differences. Thus, when students contemplate mathematics using the mirror of literature, it might help them to recognize and better understand what constitutes aesthetics in relation to mathematics. It may allow them to appreciate that aesthetics is an essential dimension of (pure) mathematics — i.e. the necessary and harmonious interconnection between (mathematical) content and form (semiotic or discursive representation), whereby a unique and perfect form is given to content that exists as mathematical reality prior to its representation. Similarly, studying poetry and poetics as reflected in Hardy's mathematical aesthetics might function as a pedagogical opening that supports students' appreciation and understanding of poetic language and aesthetics as an intrinsic part of literature itself. In other words, discussing and comparing poetry with examples of the necessary interconnection between mathematical content and discursive form provided by Hardy might help students realize that the modernist poet cannot 'just tell it like it is', because if he tried, 'it' would no longer be the same.

At the same time, the interdisciplinary comparison might also encourage students to reflect upon the differences between mathematical and literary

aesthetics. For instance, they could discuss how the latter, unlike the former — at least as implied by Hardy — may be characterized by a complex and paradoxical parity between content and form, where the content does not exist prior to the discursive form. Instead, the poem constitutes a new reality inasmuch as its unique unity of content and form brings forth an aspect of reality that did not exist before the poet created the poem by putting the words on paper.⁴

8. A Question of Disciplinary Identity

We are fully aware that the interdisciplinary collaboration we propose above is quite an ambitious and challenging approach, not least with regard to the demands it places on the teachers involved in terms of knowledge and competencies. That being said, in this paper, we will not examine in further detail the didactic details regarding the interdisciplinary design, such as questions about the specific organization of the collaboration between mathematics and LA or the actual learning activities connected to the students' work with Hardy's *Apology*. Obviously, questions like these will need to be explored and specific designs will need to be tested and developed in empirical settings. This is a task for future research.

In this final part of the paper we will instead dwell on a problématique that is of a theoretical nature, but nevertheless closely connected to what happens when Hardy is introduced in interdisciplinary educational settings. This problématique regards the question of disciplinary identity. As explained earlier, disciplinary identity signifies how a person is recognized and recognizes him- or herself as a member of a disciplinary community. It is a discursive-performative or 'Meadian' concept of identity that differs strongly from an essentialist or 'Eriksonian' perspective on identity as a 'core' entity of a person [8] and, as mentioned, our conceptualization of it here is primarily inspired by Gee [12, 13].

We have argued that the interdisciplinary collaboration enabled by Hardy's

⁴ We are aware that this touches upon a long-standing debate between a Platonic and a constructivist notion of mathematics as a phenomenon, see e.g. [16], but in this paper, we will not go further into this intricate philosophical discussion.

Apology, with its twin perspectives on the aesthetic literacy of mathematics and literature, allows students to approach and gain insight into the specific nature of the two disciplines. In other words, we argue that the interdisciplinary collaboration has the potential to let students grasp what it actually means to be a mathematician, a poet, or a literary critic — what it means ‘to think mathematics as mathematics’ and ‘to read poetry as poetry’. Gee stresses the discursive-performative nature of his notion of identity by using the composite ‘being-doing’ when pointing out that identity is a question of being-doing a recognizable social person in a given context [13]. Furthermore, he points out that being-doing a social identity manifests itself by integrating ways of “talking, listening, writing, reading, acting, interacting, believing, valuing, and feeling (and using various objects, symbols, images, tools, and technologies)” [12, page 719]. In this sense, allowing students ‘to think mathematics as mathematics’ and ‘to read poetry as poetry’ through the interdisciplinary work with Hardy’s *Apology* is a way of giving them the opportunity to be-do the disciplinary identity of a mathematician and of a poet or a literary critic. In addition, it allows students to discuss and reflect on the characteristics of these disciplinary identities, and it could thereby potentially provide students with a valuable meta-perspective on mathematics and literature as school subjects, which, in turn, may help them gain a better understanding of the relationship between mathematics and literature as school subjects and as professional disciplines.

As such, the interdisciplinary approach to Hardy’s work might stimulate a broader understanding of mathematics among students, helping them to identify mathematics as something more than a school subject and a system of tools (cf. Niss’s description of the five-fold nature of mathematics [33] mentioned above). Likewise, regarding literature, the approach could enable students to move beyond a view of the subject as a scholastic and analytical exercise, and instead to see and recognize literature as a unique form of expression that allows humans to interpret themselves and the world around them in a free and artistic format.

9. Concluding Reflections

In this paper, we have drawn attention to the theme of aesthetics as a path to a deeper understanding of both mathematics and literature within the (upper

secondary) school subjects of mathematics and Language Arts — and, not least, as an opportunity to reflect on differences and similarities between the two subjects from the perspective of comparative disciplinary didactics. From a theoretical standpoint, we have argued — and attempted to illustrate through a detailed example — that interdisciplinary collaboration between mathematics and literature (in LA) with aesthetics as a focal point may foster students' disciplinary literacies.

We believe we have illustrated a potential way of providing students with an opportunity to understand disciplinary ways of thinking from positions inside and outside the two subjects of mathematics and LA by establishing a mirroring between the two subjects through interdisciplinary collaboration. More precisely, the interdisciplinary collaboration regarding Hardy's essay presents students with an opportunity to see and understand beauty and aesthetics within and between the thinking of the two different subjects, to understand disciplinary problématiques (also in historical and epistemological contexts), and thus to widen the scope of school-based literacy traditionally connected to the two subjects. Although aesthetics forms a pedagogical challenge in teaching literature, as explained above, and even though it is often not present in mathematics teaching, we argue that aesthetics approached through a literacy perspective possesses potentials to further students' understanding of the subjects' basic ways of thinking that are usually hidden in school settings that are often focused on teaching procedures and methods.

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