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Recognizing Mathematics Students as Creative: Mathematical Creativity as Community-Based and Possibility-Expanding

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Abstract

Although much creativity research has suggested that creativity is influenced by cultural and social factors, these have been minimally explored in the context of mathematics and mathematics learning. This problematically limits who is seen as mathematically creative and who can enter the discipline of mathematics. This paper proposes a framework of creativity that is based in what it means to know or do mathematics and accepts that creativity is something that can be nurtured in all students. Prominent mathematical epistemologies held since the beginning of the twentieth century in the Western mathematics tradition have different implications for promoting creativity in the mathematics classroom, with fallibilist and social constructivist perspectives arguably being most conducive for conceiving of creativity as a type of action for all students. Thus, this paper proposes a framework of creative mathematical action that is based in these epistemologies and explains key aspects of the framework by drawing connections between it and research in the field of creativity.

Keywords. creativity, fallibilism, social constructivism

1. Introduction

There is a widespread belief that mathematical creativity is a talent only possessed by the most genius of people [49, 60, 76] or the most gifted of students [40, 46, 78]. This "genius view of creativity," titled thusly by Silver [72, page 75], portrays mathematical creativity as an inborn trait. In other fields, though, it has been shown that creativity is influenced by cultural norms, interpersonal interactions, and personal preferences [14, 31]. To date, cultural and social influences on creativity have only been explored minimally in the context of the mathematics classroom. By not recognizing cultural and social influences on mathematical creativity, we perpetuate the myth that some people have been gifted with an ability to create in mathematics and thereby make it difficult to embrace Silver's proposed alternate "contemporary view of creativity," [72, page 76] which portrays mathematical creativity as way of thinking in which any person can engage.

The perspective that mathematical creativity is a trait that is possessed by only some people has problematic implications for *whom* can become a professional mathematician. In this paper, *mathematician* refers to any individual who engages in doing mathematics. *Professional mathematician* refers specifically to individuals who are professional members of academic or commercial communities centered on mathematics. The image of mathematical creativity as an inherent ability is primarily based on the lives and work of prominent mathematicians from the 19$^{\text{th}}$ and 20$^{\text{th}}$ centuries, which includes almost exclusively white, male mathematicians born into relatively stable, privileged environments [30, 75]. The recognition of a limited form of creativity limits both the careers of students of non-male genders and non-white racial groups, and the future of mathematics itself. As of 2014, fewer than 30% of doctorate students in mathematics were women, and fewer than 5% and 8% of mathematics *bachelor's* degrees were earned by black and Hispanic students, respectively [54]. The professional discipline of mathematics grows by the ideas of the people who participate in it, so this lack of diversity in the people entering the discipline is troubling. When mathematical creativity is conceived of as a narrowly defined ability which is only identified in a relatively small group of humans, the potential for new ideas and approaches to enter mathematics is greatly reduced.

In order for a conception of creativity to act as a force that does not contribute to inequity within the discipline of mathematics, it must be reimagined. There is a need for mathematics educators to have an understanding of mathematical creativity as a way of doing mathematics that can be recognized and nurtured in all students. This need raises several questions. What is mathematical creativity? What role, exactly, does creativity play in doing mathematics? How can mathematical creativity be recognized and nurtured in the classroom, especially those classrooms with students who identify with cultures, races, and genders that are severely underrepresented in the discipline of mathematics?

These questions have become especially important over the last decade, as employers in the United States have increasingly called for mathematics educators to more centrally incorporate *creativity* in order to shape students into more productive employees [20, 91]. The government has matched these calls with substantial funding for Science, Technology, Engineering, and Mathematics educational programs that incorporate creativity. For example, the STEM 2026 report by the U.S. Department of Education [84] describes the significant role that government officials believe components such as activities that "encourage creative expression" (page *ii*) and learning spaces that "invite creativity, collaboration, co-discovery, and experimentation" (page *iii*) should play in preparing students for STEM careers. Since the report, the U.S. Department of Education has committed to spending approximately $200 million per year on STEM education [85]. There is a concern that this funding might go toward programs that reinforce exclusionary conceptions of creativity. As more and more educators are being asked to incorporate creativity into their practice, it becomes more important than ever to understand the meaning of mathematical creativity in the context of learning.

In this paper, I will propose a framework of creativity that is based in what it means to know or do mathematics and accepts that creativity is something that can be nurtured in all students. After summarizing trends in the research on mathematical creativity, I will trace the role and meaning of creativity according to prominent understandings of what it means to know and do mathematics held since the beginning of the twentieth century in the Western mathematics tradition, since these are most likely to have informed the mathematics standards taught in U.S. classrooms [25]. For each perspective of creativity suggested by various understandings of mathematics, I will consider its potential value for the mathematics classroom. Finally, I will explain key aspects of the proposed framework by drawing connections between it and research in the field of creativity.

2. Overview of Mathematical Creativity Research

A great deal of research on mathematical creativity presupposes that creativity is an inherent ability. Much of this work takes one of two related approaches: creativity as a four-stage process, or creativity as talent. Both approaches have associated common methodologies, which inform studies' findings about who can be mathematically creative and what mathematical creativity is like.

Studies that take up the creativity-as-process approach typically examine the creative processes of professional mathematicians who have successfully entered the discipline. These studies build on the model of creativity formalized by Wallas [89], based on Poincaré's [60] description of his own mathematical process: (1) Preparation (2) Incubation, (3) Illumination, and (4) Verification. Much creativity-as-process research includes interviews with professional mathematicians about their experiences doing mathematics, analyzed for similarities between the four-stage creativity model and the processes described in the interviews [39, 69, 77, 92]. Studies that take this approach often conclude that the four-stage process of creativity is a common feature of mathematical creativity. Therefore, they include recommendations that educators create opportunities for mathematics students to experience each of the four stages [69, 77, 92]. However, it is not clear that the mathematical creativity of elementary and secondary students will always occur in a process similar to Wallas' [89] four sequential stages. Is it possible that the four-stage process appears to be widespread because it is an approach that professional mathematicians learn from their teachers and mentors, rather than because it is the only form of mathematical creativity?

Creativity-as-talent researchers examine student work and typically assume that only some students can be creative [46, 53]. A common methodology within this research consists of quantifying the creativity of work produced by students during individual, timed problem-solving sessions and then testing statistical relationships between students' level of "creativity" and other characteristics, such as whether or not they had been labeled as "gifted" [21, 33, 45, 46, 52]. These studies build on the tradition of the Torrance Tests of Creative Thinking, which score participant responses to a wide variety of questions based on how uncommon they are as compared to the responses of other test-takers [83]. That is, a student receives a high score for using a strategy that few or none of their classmates use. Scoring responses based on their level of uniqueness guarantees that only a small number of students will be marked as creative. This approach cannot identify a form of creativity that can be enacted by most students.

Research in problem-posing, another area of mathematical creativity research, has done more to widen the scope of what counts as creativity for students, though it still does not offer a full picture of how creativity takes place in the classroom. Researchers in problem-posing explain that problem-posing, and especially the interplay between problem-posing and problem-solving, is an integral part of the creative process [72, 86]. The methodology of problem-posing studies often mirrors that of creativity-as-talent studies; students are asked to generate problems based on a given context or story in an individual, timed setting. Researchers judge student responses in part based on unique-ness, as in the creativity-as-talent approach, and often also attend to the level of complexity in students' work [22, 73, 86]. These studies demonstrate that the vast majority of students are capable of inventing problems, which suggests that they are capable of participating in mathematical creativity. These studies are not meant to provide a full image of student mathematical creativity, since participating students are typically only expected to do one task: pose problems.

3. Epistemological Perspectives on Creativity

In order for a definition of mathematical creativity to robustly inform creativity research in the classroom, it should not only position all students as having the potential to be creative, but also be grounded in what it means to know and do mathematics. Therefore, in this section I investigate the implications that three different mathematical epistemologies would have if used as a basis for a definition of mathematical creativity. These three epistemologies of mathematics are formalism, fallibilism, and social-constructivism.

3.1. Limitations of the Formalist View of Creativity

The formalist perspective of mathematics, popular among Western European professional mathematicians in the late 1800s and early 1900s, causes problems regarding acknowledging and developing creativity in most students. This role of creativity according to formalism is important to understand because this epistemology has had a great deal of influence on mathematics education in the United States [10, 25, 51], perhaps because of its prominence as public school was becoming more established in the country.

According to formalist mathematicians, mathematics is a logical system of knowledge [17, 48] that can be divided into two categories: self-evident axioms and provable ideas that logically follow from those axioms [36, 87]. The mathematical symbols and the logical rules that operate upon them are not thought of as symbolizing anything in the material world. Instead, concepts are judged to be mathematically true so long as they are consistent within a mathematical system [86]. This epistemology has been described as "absolutist" due to formalists' position that the truth of mathematical ideas is "absolutely valid and thus infallible" [25, page 9].

Since, according to this perspective, systems of mathematics exist outside of the human influence, formalists believe that humans can only uncover them, not create them [38]. Russell [67] describes mathematics as "a palace emerging from the autumn mist as the traveler [sic] ascends an Italian hill-side, the stately storeys [sic] of the mathematical edifice appear in their due order and proportion, with a new perfection in every part". This mathematical building is, somehow, not the work of human builders. The only relationship a human can have with this conception of mathematics it is to witness its beauty.

The formalist perspective that humans do not influence mathematics raises the question of what it is that mathematicians *do*. Formalists acknowledge two modes of thought that mathematicians may use to discover the palaces of mathematics: reason, which operates via logic or deduction and is central to doing mathematics, and insight, which is considered to be superfluous. Russell [67] describes insight as occurring suddenly and quickly, and having the power to "first arrive at what is new." Insight has the potential to lead to ideas that a thinker did not predict, which makes it the only creative act recognized by formalists.

Presenting insight and reason as completely disjointed ways of doing mathematics makes it possible for humans to decide to rank them. Strict formalists prize reason over insight, due to a belief that the ideas uncovered using insight are not necessarily true. In other words, insight is no better than a random guess; insight may uncover a piece of pre-existing mathematics, or it may not. Some formalists believe that all propositions of all mathematics could be discovered using only logic and deduction [48, 67].

According to early formalist epistemology, creativity is not needed, because deduction is sufficient to discover all of mathematics.

At least one formalist, Russell [67] uses this ranking (i.e., deductive reason is good and creative insight is not) to reinforce racist beliefs. He attributes reason to members of "civilized society" and insight to "savages," terms Russell used throughout his career to express white supremacist assumptions [63]. Of course, Russell is far from alone in presenting the formalist notion of mathematics as an a-human construct as part of an argument for the superiority of Western cultures [38]. A hierarchical binary of reason and insight enables the problematic belief that there are groups of people who can do mathematics, and groups who cannot, and that those latter groups are less "civilized" because of it.

Another limitation of the formalist characterization of creative insight as unimportant is that it does not represent many mathematicians' most valued experiences in the discipline. Many professional mathematicians have recalled guesses or realizations that led to an important mathematical development [7, 77]. Henri Poincaré [60] even described such moments as the most defining parts of his work in mathematics. A description of doing mathematics as only testing or applying logical rules does not allow for the mathematical experiences that many mathematicians, professional or otherwise, most value. Since many consider creativity to be a defining aspect of their work in mathematics, problems are likely to arise if an educator positions creativity as superfluous. Holding this perspective in the classroom might lead to creativity being siloed within extraneous projects that are not integral to the learning process, thereby limiting students' opportunities to develop their mathematical creativity.

Is it enough, then, to simply reassess the hierarchy of reason and insight, and portray creativity as useful and important in mathematics, without reconsidering the formalist assumption that mathematics exists outside of humans? A shift toward celebrating insight seems like it could do some work toward upending Russell's racist portrayal [67] of insight as inferior to deduction and as the domain of the "savage." However, continuing to conceive of creative insight and deductive reasoning as completely dichotomous ways of thinking still allows for a hierarchical ordering of ways of doing mathematics, and therefore a hierarchical ordering of the people who do mathematics in those ways.

Indeed, those formalists who consider insight to be useful can simply flip the hierarchy: Creative insight is now more important than deductive reasoning. According to this hierarchy, creative insight is no longer thought of as an undesirable mode of thought relegated to an imaginary group of "savages," but as a rarified trait possessed that "cannot be possessed by everyone" [60, page 324]. By simultaneously redefining creative insight as an exclusive trait and as the most important part of doing mathematics, formalist mathematicians have repurposed the concept in such a way that it can be used to exclude most people from legitimately participating in mathematics.

Finally, the formalist definition of creativity as insight wholly separate from reason makes it difficult to position creativity as something that can be supported or nurtured. If creativity cannot employ any deduction or logic, then useful ideas must occur to mathematicians essentially out of the blue [30, 60], which does not allow for any mindful creative acts. The implication for the mathematics classroom is that students can only prepare for creativity by examining a given mathematical context and then waiting, hoping that they have been gifted with mathematical intuition. Formalist epistemology is incompatible with a definition of creativity that can be used to recognize mathematical creativity in the work of any given student. In order to develop an understanding of mathematical creativity for the classroom, there is a need for a more deep-seated epistemological change.

4. Fallibilist Innovations' Implications for Creativity

Fallibilism, an epistemology of mathematics popularized in the 1970s, differs from formalism in large part due to the basic assumption that mathematical knowledge is fully created by human thought and action, rather than existing separately from humans. According to fallibilists, mathematics begins "from a problem and a conjecture" [17, page 347], rather than an initial system of axioms, and then grows via human actions, such as "a process of successive criticism and refinement of theories and the advancement of new and competing theories" [17, page 349]. Mathematics does not guarantee truth; any concept can be disproven or modified [44]. I argue that, unlike formalism, fallibilism is consistent with a concept of creativity that can be supported and nurtured in students.

Since there is no pre-existing body of fallibilist mathematics, Russell's "insight" and "deduction" have no place in a fallibilist understanding of doing mathematics. How could a mathematician gain insight into something that is not there? How could they deduce something that does not yet exist? Instead, in order for mathematics to exist, mathematicians must engage in modes of thought that *create* mathematics.

Furthermore, fallibilism does away with the formalist rigid separation between creativity and reason. In his fictional classroom dialogue *Proofs and Refutations* [44] a pivotal work in fallibilism, Lakatos instead introduces "conscious guessing" (page 30), a mode of thought that *simultaneously* borrows from the mindful nature of deduction or reason and the ability of insight to arrive at new ideas. Conscious guessing is not a random shot in the dark, like Russell's creative insight, but it is also not a guarantee of truth, like Poincaré's and Hadamard's creative intuition. A mathematician can be aware of and in control of conscious guessing, rather than being resigned to hoping that creativity will occur. Furthermore, removing the creativity/reason dichotomy also makes it more difficult to claim either type of thinking as the domain of one group of people, as Russell [67] did by claiming creative insight was an undesirable mode of thinking used by "savages" or as Poincaré [60] did by claiming that creative intuition was an exclusive trait of "mathematicians," a term he used to refer to a restricted, small group of people.

The fallibilist perspective has two main distinctions from formalism that have implications for the role of creativity in mathematics education. First, fallibilism positions mathematics as a totally human invention, which means that it provides a coherent rationale for creativity being a part of mathematics education. Second, fallibilist creativity does not rely on the existence of an unknowable and uncontrollable mathematical intuition that some students may lack, which means that the fallibilist perspective does not position any students as incapable of mathematical creativity.

However, fallibilist creativity falls short in explaining why the people who have had success thus far in being creative in mathematics have primarily tended to be white and male. Fallibilism on its own offers no way in which to understand social and cultural forces that impact mathematical creativity and its acknowledgement.

5. Social and Cultural Influences on Creativity

Given that humans create mathematics, it stands to reason that specific communities of humans create different systems of mathematics [26, 35, 43, 90]. The individuals involved in doing mathematics have an "epistemological significance" [43, page 7] in that their mathematical actions have the potential to change the course of how mathematics develops. Mathematicians' personal characteristics, relationships to one another, the relationship of any given group of mathematicians to the wider mathematical community all have implications for how mathematical creativity takes place and what mathematics can be created.

In the discipline of professional mathematics, Ernest [26] explains that social individuals impact the accepted set of mathematical concepts and practices in two ways: generating new claims and either accepting or rejecting these claims. The ideas that are generated within any given society are likely influenced by shared cultural values (whether the new ideas embrace or rebel against them). Those ideas then become part of the discipline at the discretion of a group of gatekeepers from the academic community [14, 26].

These descriptions of the social construction of mathematics make it possible to explain how systems of privilege and oppression can influence *who* is in the position to be creative and to evaluate creativity. These individuals are likely to be those who have been put in a privileged position that enables them to generate ideas [14]. At the very least, these individuals must be given access to the domain of mathematics [27]. In the United States, this access is systemically inequitably distributed across race, class, and gender lines.

It stands to reason that the social construction of mathematics would play out at both larger and smaller scales. At the larger scale, consider the legacy of the famous mathematicians of Ancient Greece, members of a society whose image is often used as a tool for western colonialism [5]. Their favorite topics of study, such as two-dimensional geometry and the distinction between rational and irrational numbers, are now very well-represented in modern secondary mathematics curricula, unlike the mathematical products of societies that have been colonized [15]. This has direct implications for any potential student creativity; a great deal of the creativity in which students engage in school will likely be connected to the mathematical history of Ancient

Greece.

The social process of generating and accepting or rejecting mathematical ideas also occurs at the level of individuals doing mathematics together in real time. Some individuals generate ideas, and some subset of the members of the group may or may not decide to accept them according to some set of criteria or argumentation. For the purposes of the current paper, I will consider the implications that this has for doing mathematics in the classroom.

Whether students are in the position to be creative or evaluate creativity depends on the status that individual students have in their class or group [23] which are often influenced by perceptions of and assumptions about students' gender, race, and social class. To date, research has primarily explored the impact of students' gender on whether they are perceived as being creative. Unfortunately, creativity is often thought of as a male trait, which leads to creativity being most frequently recognized in male students [88]. This may be the result of two factors: male students are more empowered by their experiences to act creatively and to express their ideas to others; and the creativity of male students is more likely to be recognized and accepted than the creativity of female students [34, 81]. It seems that many of the qualities that are often named as markers of creative people, such as risk-taking, are characteristics most typical of white males, who have limited experience with adverse experiences as compared to other individuals [50].

The impact of race on perception of mathematical creativity has not been as well-researched as the impact of gender, but it seems likely that the "white male math myth" [80] with which students must contend in mathematics education extends to mathematical creativity. Gate-keepers to students' potential mathematical careers, as well as students themselves, may even – knowingly or not – consider white-ness and male-ness themselves to be common characteristics of mathematical creativity due to repeated exposure to the image of the creative male professional mathematician.

6. Framework of Mathematical Creativity

As of yet, the role that oppression and privilege have on mathematical creativity have been severely underexplored in empirical studies, especially as it relates to the classroom. There is a need for a framework that grounds creativity deeply in its community and enables one to probe how the organization and practices of that community influence all stages of creativity, including its acceptance or rejection.

According to a fallibilist and social-constructivist understanding of what it means to do mathematics, I define a *creative mathematical action* as one that transitions a given mathematical context into a new version of mathematics by creating ways of doing or thinking about mathematics that were previously not possible for a particular community of mathematicians. The premise that creative actions are integrally grounded in a mathematical context reflects the perspective that creativity is not the sudden, random work of an individual alone, but is necessarily shaped by the existing mathematical concepts and practices of a community. Because creativity leads to new mathematical possibilities, the set of mathematical concepts and practices of any community is not pre-ordained; rather, it will be different depending on which creative acts are done. Defining creativity as being particular to a community of mathematicians reflects the implications that who the mathematicians are, their relationships with one another, and their relative participation in privilege and oppression, will have for how and what they create. Furthermore, because actions are grounded in their own community, actions that are creative within one community may or may not lead to new mathematical possibilities in another. The Creative Mathematical Action Framework (CMAF) in Figure 1 highlights these four components of creativity (i.e., action, context, new possibilities, and community) and the relationships between them as a model that can be used to identify and trace creativity in mathematical contexts, especially the mathematics classroom.

How could the CMAF be used to model student creativity in the mathematics classroom? In the remainder of this section, I will describe each component of the CMAF in more detail.

7. Creative Action

In creativity literature, creativity is typically conceived of as a trait of a person, product, or process. This framework portrays it as a type of action. In this section, I will explain the characteristics of actions that have the potential to be creative according to the CMAF and give examples of actions

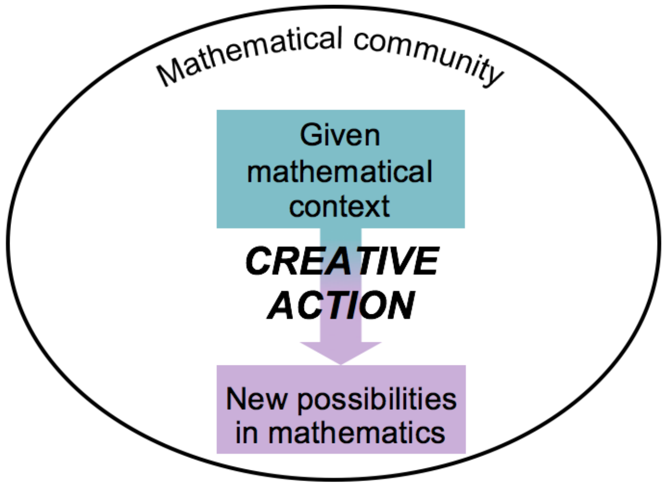

Figure 1: The Creative Mathematical Action Framework.

that can be creative. I will also explain how this conception of creativity is related to conceptions of creativity as a type of person, product, and process.

Characteristics of creative actions.. A creative action may take many forms. The actions that have creative potential are those that students decide to take of their own accord, using their own agency. Pickering describes *human agency* as active and hallmarked by "choice and discretion" [57, page 117]. These actions may occur as single actions within a set process, such as when a student selects a formula and then applies it. It is also possible that many actions with creative potential may occur together, such as when students create wholly original methods that they use to solve novel problems.

Alternatively, actions that students take as a result of closely following directions or standard ways of doing mathematics in their classroom community are not be considered to have creative potential. When engaging in actions that do not involve human agency, students rely on Pickering's *disciplinary agency*, meaning that the discipline (or, in the case of schooling, the classroom's version of mathematics) "leads them through a series of manipulations within an established conceptual system" [57, page 115]. The series of manipulations may exist at different grain sizes. For example, a student may repeat the steps or directions provided in a textbook example, or a student may follow a learned problem-solving heuristic. In some cases, disciplinary agency is implicit. For example, a student might be given a quadratic equation and told to solve the equation. Perhaps the instructions do not specify how students should solve the equation, but the students have just spent an hour in class solving similar equations by using the quadratic formula. Thus, a student using the quadratic formula would be following the usual mathematical practice of the students in the class. Since solving quadratic equations using the quadratic formula is a standard aspect of doing mathematics in this hypothetical classroom community, the action lacks the potential to create new ways of doing or thinking about mathematics.

Types of actions that are often considered to have the potential to be considered creative are common in creativity literature. Two actions frequently highlighted are *combining* and *selecting* [32, 60, 79]. Combining is typically considered to be potentially creative when the items being combined are not initially considered to be similar, or related, to one another. Similarly, selection is considered most potentially creative in cases in which an item is selected for a purpose that is different from its usual use. In both cases, it is important that the person who combines or selects makes the agentic choice to do so; if they combine two items because they were told to do so, then according to the CMAF, the action would not have creative potential.

Beyond combination and selection, there is a great deal of variety in the actions researchers have identified as being potentially creative. For example, posing problems, which refers to generating a mathematical question regarding a given context, has received extended attention in mathematical creativity research [66, 71, 86]. Additionally, Hanson (2015) argues for valuing the creativity of mental actions besides those that generate initial ideas, such as "choosing, supporting, interpreting and refining ideas" as well as "selecting, emphasizing, and powerfully presenting ideas" [32, page 372]. Though literature regarding creative actions refers almost exclusively to *mental* actions, research identifying imagination within embodied mathematical thinking suggests that embodied cognition is fertile ground for creative mathematical activity [56].

Creative action as related to other conceptions.. Traditionally, creativity is thought of as a quality of either a person, a process, or a product. The emphasis on action makes the CMAF bear some resemblance to process-based conceptions of creativity. Consider the four-stage creative process conception of creativity, which typically explains creativity as following a standard path: preparation, incubation, illumination, and verification [60, 89]. Although the CMAF does not require that creativity follows a particular sequence of stages, both conceptions portray creativity as something that a person enacts that exists at a particular time. Both also point to the importance of what happens after the moment of creativity, though the four-stage process' final step of verification is usually explained as simply checking whether the new idea is accurate, rather than whether or not the action will go on to create new ways of doing mathematics in the community.

Conceiving of creativity as an action that people may take could be fitted to the conception of creativity as a personal trait by measuring each individual's frequency of taking creative action. However, this would have to be done sensitively in order to not undercut the importance of the circumstances in which the creative acts take place. Not positioning creativity as a feature of an individual has the potential to avoid problematic consequences such as the extreme overrepresentation of white males in the group of mathematicians who are identified as creative [30, 75].

Finally, creativity as a type of action can be thought of as being related to the conception of creativity as an attribute of a product, the perspective evident in many studies that analyze students' solution to math problems, such as those described earlier in this paper [21, 33, 45, 46, 52, 58, 64]. Another methodology concerned with creative products is the Consensual Assessment Technique, in which domain experts judge final products across a wide range of factors, including creativity [1]. Versions of this methodology have been used in mathematics and in other disciplines, often in order to understand which contexts are most likely to bring about creative products [1] or to determine how groups of judges conceive of creativity [19, 42]. It is possible that so many studies focus on products rather than actions because products are more easily accessible to researchers than processes or actions, which may be internal or not recorded concisely [75].

It is difficult to fully disentangle actions from their products, such as the act of connecting two ideas and the resulting connection between them. Consider the action of proving a theorem. The act of proving and the resulting proof are closely linked, and it seems unnecessary to argue that a proof itself is not creative if the act of making that proof is. I argue for an emphasis on the action rather than the product in large part because studying the creative actions, rather than their static output, may enable researchers to more deliberately probe *how* creativity takes place, rather than being resigned to making assumptions about why products take the forms that they do [21].

Conceiving of creativity within action raises many questions that can be asked of students learning mathematics. What creative actions do students engage in within classroom contexts? Do they mirror the creative actions of professional mathematicians, or are there creative acts that are unique to students? What kinds of creative acts are students capable of, given some ideal circumstances?

8. Creativity as Informed

In order to emphasize that creativity is not a random phenomenon, one of the primary components of the CMAF is that it is grounded in the existing mathematical context. In this section, I explain the implications that this has for the nature of creative actions and then describe connections that contextual grounding has to the concept of preparation, popular in research about the creative process.

8.0.1. Creativity as shaped by context.

Creative actions link the known mathematical present and the yet-to-be-known mathematical future. This means that they are shaped by current mathematical concepts and by mathematical practices that have been enacted so far. Each concept retains traces of the mathematician's experience with the object, which influences the new possibilities that the mathematician may imagine for that concept [18].

For example, in order to combine mathematical concepts, one necessarily chooses concepts with which they have some experience, even if that experience is scant. At the very least, the mathematician is limited to combining those concepts of which they are aware. A mathematician's decision to select multiple concepts to combine is informed by their experience with these concepts. That is, the mathematician does not combine two concepts at random – unless, of course, they employ randomness on purpose [70], in which case the mathematician uses their experience with the concepts and knowledge of mathematical values in their community to guide their decision to select or reject randomly created combinations.

Connection to creative preparation.. Creativity-as-process researchers also speak of the influence of mathematical knowledge on creative moments. The first stage of Wallas' [89] creative process is preparation, which the mathematician is said to do consciously. This conscious preparation is said to influence the creative moment that follows, even according to those who believe that the creative moment itself is subconscious [30]. The CMAF does not contradict these researchers' depiction of the influence of conscious preparation. The primary difference here is that according to the four-stage model, any conscious activity happens well before the moment of creativity, which is performed by the subconscious. The CMAF suggests that mathematicians' use of knowledge in the creative moment can be self-aware and deliberate, rather than occurring only in the ungovernable subconscious.

There are many questions to ask about the informed nature of creativity within the context of learning mathematics. How should students be informed in order to be able to enact creative acts? What kinds of knowledge contribute to student creativity? And how can the informed nature of creativity be observed in students' actions?

9. Creating New Possibilities

Another component of the framework is the new possibilities that are created by creative actions. In this section, I will describe the implications that this has for the development of mathematics. I also briefly mention references to possibility in other mathematical creativity literature.

New mathematical possibilities.. In the CMAF, an act is not creative unless it leads to an expanded form of mathematics. That is, after an action takes place, somebody must be able to have thoughts or ask questions that would not have been possible before. Perhaps a known procedure can now be used in new mathematical situations, or perhaps a newly constructed structure enables mathematicians to notice previously unknown things about a known mathematical concept. Until these new possibilities are realized, an action only has the *potential* to be creative. The expectation that creative acts lead to new possibilities is related to the common expectation that creativity be novel, but it is also deeply connected to a common perspective that creativity be appropriate or useful in some way [59]: An act that leads to new possibilities proves its appropriateness when it is taken up by other members of the given mathematical community.

Because creative actions create new mathematical possibility, they change the future of mathematics. Just as creative actions themselves are informed by the given mathematical context, the actions go on to inform the mathematics that is done in a community. To illustrate the way in which creative actions create new possibilities in doing mathematics and thereby impact the mathematics developed in a community, I draw on a fictional example from Lakatos' dialogue between a teacher and his students in *Proofs and Refutations* [44]. Early in the dialogue, Lakatos' Teacher character crafts a proof about a phenomenon related to polyhedron. (The teacher character is not named in Lakatos' dialogue. Therefore, I refer to him as *Teacher* here.) In explaining his proof, he asks his students to imagine a polyhedron as being formed out of thin rubber, removing a face of the polyhedron, and then stretching out the shape to lay flat. His act of proving allows his students to engage in thoughts that would have been highly improbable had they not heard the proof. Some of his students invent polyhedra that they present as counterexamples to specific parts of the Teacher's proof. For example, a student named Alpha imagines the polyhedron that he describes as "a picture-frame" [44, page 19], which looks like a rectangular prism with a square-sided hole punched through its middle (see Figure 2). Alpha likely would not have imagined this shape if he had not been trying to think of a polyhedron that could not be stretched out flat when thought of as being made of a thin rubber. The Teacher's proof created the possibility for Alpha to invent this polyhedron in particular.

By creating new possible ways for students to think, the Teacher's proof impacts the mathematics that the class creates. Later in the dialogue, the Teacher presents a modified version of his proof that addresses Alpha's picture-frame counterexample by requiring that the polyhedron in question must be one that can be stretched out flat, unlike the picture-frame. Had the Teacher presented a proof that did not reference stretching, Alpha probably would not have invented the picture-frame polygon, and the Teacher would not have modified the proof in the way that he did.

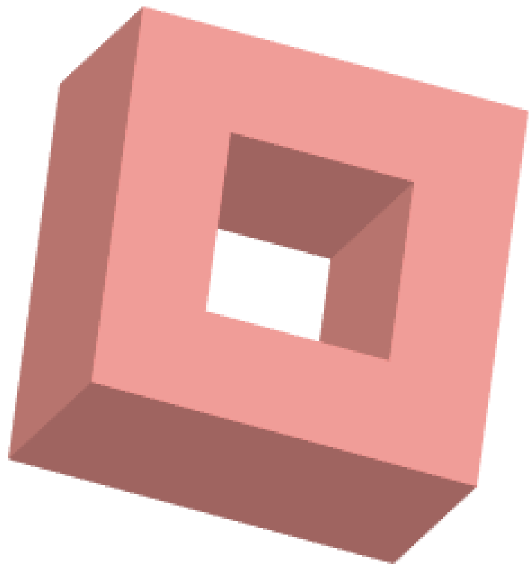

Figure 2: Alpha's picture-frame polyhedron, which cannot be stretched flat when a single face is removed [44].

Possibility in the literature. Other research in creativity has also recognized possibility as a critical feature of creativity. For example, Liljedahl & Sriraman suggest that one type of mathematical creativity is the act of forming possibilities that "allow an old problem to be regarded from a new angle" [47, page 19]. Another related strand in creativity research is "Possibility Thinking" (PT), which focuses on elementary education including but not limited to mathematics. PT investigates moments in which students ask the question "What if?" and act or think "as if" something were true [9, 12]. By analyzing instances in which students "shift from 'what is this and what does it do?' to 'what can I or we do with this?'" [12, page 4], PT trains its lens on those actions that have potential to be creative. Several questions arise due to the understanding of creativity as a quality of those actions that lead to new possibilities in mathematics. How do students take up one another's ideas in the classroom? And if students do not get the chance do to make progress on their own or one another's ideas, due to the nature of their classroom environment, is it still possible to recognize the potential for creativity in student work and discourse?

10. Creativity as Based in a Community

The community in which an action takes place influences all other components of the framework: who takes the action itself, what informs that action, and how to consider the new possibilities created by the action. In this section, I will describe the implications that the community has for each of those components. I will also explain how the framework's portrayal of creativity as being defined within its community relates to other research on creativity of individuals who are not necessarily professionals.

Community influences on other components of the framework.. Each of the previous three components of the framework (i.e., the action, context, and new possibilities) exist within a particular community of individuals. A community has features such as social structures and values, which necessarily influence each of those components.

Societal systemic factors, including social constructions of gender and race [3, 13, 37, 41], and social dynamics within small groups or a workplace [82, 93] have the potential to influence who takes creative actions. Those in more privileged positions, often due to their race, gender, and/or class, may have had more access to information to generate new ideas [14]. Some individuals may be more likely to have the confidence to take action, or be better situated to communicate about that action to others in their community.

Furthermore, community members likely will not respond to all individuals' actions in the same way. Engle *et al.* [23] explain that some students have more influence in the development of ideas in the classroom than others, which cannot always be explained by the quality of their arguments, which has direct implications for the mathematics that can be created in a classroom community. Some individuals may be met with more skepticism or acceptance than others. Again, the judgment of the ideas proposed by an individual is likely to be mediated by others' perceptions of the individual's race, gender, and class; in classrooms in the United States, at least, white, male students are likely to benefit the most [62, 88]. In more egalitarian communities, these dynamics may play out differently, perhaps resulting in more community members participating in creativity.

The values of a community also serve to inform creative actions. For example, individuals who are members of communities that highly value elegance and simplicity might be more likely to work toward developing proofs that are concise, whereas those who belong to communities that more highly value transparency and communication might be more driven to craft proofs that are easy for others to understand. These different drives might lead individuals to create different lines of mathematical reasoning as they explore the same concepts, which could influence how other members of the community interact with those concepts going forward. Cultural values might even shape which questions arise and therefore invite potential creativity in the first place; consider that in the United States, many professional mathematicians are tasked with developing algorithms that would increase their employers' profits or efficiency. In some cases, this has had the effect of mathematicians creating strategies that reinforce racist and sexist biases held in the society to which the mathematicians belong (Angwin et al., 2016; Dastin, 2018). Of course, it is also through creativity that the values of these cultures have the potential to change (Chappell, 2012), especially if mathematicians mindfully consider relevant aspects of their community's culture. Consider a recent proposal to prevent professional mathematicians from continuing to reinforce bias by way of a mathematical version of the Hippocratic oath (Sample, 2019). This oath would operate by influencing what mathematical actions are taken due to explicitly adjusting the values of the community of professional mathematicians.

In order to be creative, an act must lead to an expanded version of mathematics for a given community, but it does not need to create new mathematical possibilities for every individual who has ever participated in mathematics. That community is the place where the new possibilities afforded by a creative action come to fruition. Consider a student who proposes a technique that enables her classmates to solve problems that they previously thought to be unsolvable, although professional mathematicians might have had no problem solving them. This student's action might not be creative for the historical discipline of mathematics, but the student was creative for her own class. A community may be even smaller than a class, perhaps consisting of a group of students completing classwork together. I propose that a community may even be one individual working alone, since an individual has a set of mathematical concepts and ways of doing mathematics that guide their practice. Consider the student who, for example, notices a feature of a geometric shape that she then relies upon to solve several homework problems. The discovery of this feature may not be creative in the larger community of her classmates if the student does not share it, or even if she does share it, but her classmates do not find the feature to be useful, or were already taking advantage of it.

Connections to creativity levels.. Defining creativity as being defined within a community bears some relation to a common practice in creativity research of acknowledging different levels of creativity, especially *little c creativity* and *big C Creativity*. Craft defines little c creativity as purposeful actions one takes while "coping with everyday challenges" [11, page 51], whereas big C Creativity changes a domain in a historically monumental way [1]. Recognition of little c creativity marks an important departure from the creativity research that searches for common features of the lives and work of famous domain experts [1, 74]. Beghetto and Kaufman [4] further expanded the levels of creativity by introducing mini-c creativity, a form of creativity for students; and pro-c creativity, which includes the creativity of experienced professionals that is not quite domain-changing.

Similar to the frameworks put forth by Craft [11] and Beghetto and Kaufman [4], the CMAF acknowledges that actions do not need to be domain-shifting, monumental acts to be considered to be creative. The CMAF extends this line of thinking further by removing the separate categories of creativity (e.g., little-c versus Big-C), instead positioning each act of creativity as deeply connected to the community in which it occurs. Instead of defining an action based on whether it was taken by a student or a professional, the CMAF asks how much the action does to transform mathematics in the given community. Creating new possibilities for a community is a complex and future-expanding act no matter who makes up that community. In the context of mathematics education, students join different mathematical communities as they join new classes from year to year, and also as they transition between full-class discussions, small group work, and individual work. Do students engage in different creative behavior in these different circumstances? What evidence is there, in any of those circumstances, of students opening one another up to new possibilities? And what implications does the community-grounded perspective of creativity have for the role of the teacher, who is a member of the classroom community, but also has membership in other mathematical communities through their own studies?

11. Concluding Thoughts

My initial questions were about the role and meaning of creativity in mathematics and how it can be recognized and nurtured in all students in the mathematics classroom. First, conceptualizing creativity as existing within a particular community leads me to acknowledge the potential of all students to engage in creativity that leads to new mathematics. Students do not have to wait to be mathematically creative at a later date; given the right circumstances, they can create mathematics at any moment. Furthermore, I conclude that recognizing the role of values and social structure within a community is central to understanding who gets to be creative and how creativity happens.

A clear next step is to use the CMAF to analyze the work of students collaborating in the classroom. It would be particularly important to trace how social dynamics influence who gets to be creative and whose creativity is recognized. I am interested in using the framework to analyze student discourse in classrooms with students from backgrounds that are well-represented in the discipline of mathematics (i.e., white, male, and economically advantaged) and those who are not. This could provide information about particular mechanisms by which creativity is recognized and encouraged in the cases of some students, and ignored or curtailed for others. It could also be informative to identify classrooms in which these mechanisms do not operate differently for different students; what makes that possible? Findings from this type of work could be useful to classroom teachers who aim to support creativity in all students. Another group who might find this to be useful is curriculum writers who wish to design activities that allow for student creativity and to provide effective support to teachers who implement their curriculum.

As the CMAF is used, I believe it would be important to think of the framework as something that has the potential to evolve. De Freitas and Sinclair explain that mathematical concepts are constantly "becoming" [18, page 80] as they are used by humans. Similarly, every use of the CMAF would inherently serve to add nuance and connections to the framework, so that it is "never identical to itself" [18, page 80]. I believe that any framework has the most potential to be powerful in terms of detecting student creativity and the factors that influence it if the framework is mindfully re-considered and potentially adapted, when appropriate, as it is used.

One thing to note is that the CMAF has been developed in the context of decontextualized mathematical concepts and practices, whereas some mathematics educators who promote creativity study lessons and projects in which students explore or take action on topics relevant to their families or local communities [6, 24, 29, 65]. Many social justice-oriented and socio-political mathematics educators recognize the human-*created* nature of mathematics and identify *creativity* as a valuable student learning goal, often conceived of as students producing heterogeneous ideas [28, 29, 61]. In many ways, this perspective is in line with the CMAF. Both allow for each individual and community to be empowered as a generator of mathematics, potentially expanding the discipline itself and potentially decentralizing power within mathematics. However, the CMAF was developed within the context of decontextualized mathematics, whereas many social-justice oriented mathematics educators study community and space-based projects [6, 24, 29, 65]. How could using the CMAF to analyze student mathematical creativity in learning environments like these lead to further evolution of the CMAF?

One potential concern about the CMAF is that it is based in a fallibilist and social-constructivist epistemology, whereas many mathematics classrooms in the U.S. have formalist underpinnings. Is a framework like the CMAF incompatible with these contexts? Perhaps it would be difficult for some educators to adopt without support. Or, perhaps adopting a definition of creativity like the CMAF would push upon other elements of the formalist classroom, such as an absolutist view of mathematics or the separation of creativity and logic. I am hopeful that a framework like this one could enable educators to recognize the creative work in which their students are already engaging, a small but powerful move towards nurturing student mathematical creativity and expanding what counts as mathematics.

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