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Cover Page Footnote

I would like to thank my wife, Anne Beatty, for many painstaking hours helping me through the thought process of deciding how to name our children, as well as her expertise in copy-editing. Without her help, this would be a poor product indeed.

The Surname Impossibility Theorem

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Synopsis

The Surname Impossibility Theorem offers solace to anyone who has struggled in the quagmire of choosing a surname for a child. I posit that it is impossible to find a method for giving a child a surname that satisfies the important criteria of being traditional, aesthetically pleasing, ancestor-respecting, non-sexist, gender-neutral and non-heterosexist. My mathematical approach defines what those criteria would mean and analyzes different naming systems to conclude that no method could satisfy all criteria. In the same way that Arrow's Impossibility Theorem proved that no voting method can satisfy all criteria for a fair election, I prove the impossibility of choosing a perfect surname.

Keywords: impossibility theorems, creativity.

1. Introduction

Perhaps the earliest childhood indication of my future as a mathematician was my sincere belief that everyone else was doing things the wrong way, and needed to listen to my instructions. Nowhere was this conviction more prominent than in regard to surnames; I have a hyphenated last name, and it did not make sense to me why other people had only one last name. Did their parents both just happen to have the same last name when they were married? And if not, where did that other last name go? How can you have one surname but two parents? As I grew older, I came to better understand the nuanced complications of marriage and children. Although my mathematical training lends itself to more rigid and well-defined constructs, that training also gave me a lens through which I view much of the world.

This paper is an examination of my creative process in using that lens to prove the inherent impossibility of a "perfect" naming system.

2. Creativity in Mathematics

When I think of creativity in mathematics, what first comes to mind is unusual applications of math. My training in theoretical math has profoundly changed the way I view the world, and I often apply concepts and techniques from my mathematical training in places where they may not initially appear to fit. However, in these unconventional applications, a conceptual idea from mathematics can still be illuminating, though perhaps less satisfyingly conclusive, than a typical mathematical proof. The creativity lies in using just enough of a mathematical concept or process to give some level of clarity to a real-world challenge.

For example, when studying evolution as an undergraduate, it helped me to "prove" it by creatively applying the concept of mathematical induction. If the geologic record shows that animals have changed in some way in the past (base step), and we can demonstrate that animals can evolve, currently, to adapt to their surroundings (inductive step), then mathematical induction verifies the validity of evolution. The proof is not precise, for a number of reasons: induction is discrete and evolution is continuous, and the geologic record does not guarantee any evolutionary steps, to name just two. Yet mathematical induction did help me conceptualize evolution. Math can become a creative vehicle for thinking through problems in all sorts of non-mathematical arenas. As an example of this approach, I will demonstrate how impossibility theorems provided me with clarity about the best way to name a child.

3. Impossibility Theorems

Heisenberg's uncertainty principle is arguably the most famous impossibility theorem in science and mathematics, and it was my first introduction to the idea of provable impossibility. Loosely speaking, the uncertainty principle states that, at any particular moment in time, it is impossible to precisely know both the position and momentum of a quantum particle. Abel's impossibility theorem states that there is no *quintic* formula, or anything higher: the quadratic formula is one of the most well-known (and well-memorized and -harmonized, for that matter) formulas in mathematics, and there is a similar (more complicated) formula for third-degree polynomials, and even an extremely convoluted formula for a general solution to fourth-degree polynomials, but at that point it stops. I was struck by the audacity of that impossibility when I first learned about it in high school, and became even more interested in it in graduate school once I was able to understand the proof.

What sold me on impossibility, however, was voting theory and Arrow's impossibility theorem. Precisely, Arrow's theorem can be stated as:

> "If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial." [2]

Loosely, Arrow's theorem states that, with three or more candidates in an election, it is impossible to have a fair voting method. What I found most striking about Arrow's theorem was twofold: both its method of proof and its applicability to real life. How does one prove something with terms like "satisfactory" that do not lend themselves to easy definition? Arrow's method was to find straightforward criteria that one would want in an ideal voting system, and then show that satisfying certain criteria necessarily resulted in failure of other criteria. Thus not all criteria could be met, so a perfect voting system is impossible. I now teach a course in which we investigate impossibility concepts involving voting theory [2], gerrymandering [1], and apportionment [3].

I first learned about instant runoff voting (a ranked-choice voting system) from a friend, a voting system which seemed to be a clear improvement over single, nontransferable vote systems common in the US. Soon after, I taught a math course in which we covered the mathematics of ranked voting systems, culminating in Arrow's theorem. An idea (ranked-choice voting) that at first seemed attractive now seemed much less so, as did every other voting system. After that first disappointment, however, I began to see Arrow's theorem, and impossibility theorems in general, as less of a limitation and more of an opportunity; impossibility frees one from trying to find a "perfect" option, and allows for discussion about which qualities are most important for a particular system. I have always appreciated mathematics for its logic and clarity, and been frustrated with how the real-world rarely conforms to the same kind of logic and clear-cut definitions–impossibility theorems, viewed correctly, are a bridge between the perfection of math and the muddiness of society.

Since I routinely look for mathematical constructs in everyday life, I began to wonder if undiscovered, unstated impossibility theorems lie at the heart of many conflicts. This initiated my creative endeavor to apply impossibility theorems to real-world issues–could political or interpersonal debates be reframed mathematically? In how many situations are our "values" really just the most desirable criteria for a system in which it is impossible to have all criteria satisfied? When my wife became pregnant with our first child, and our seemingly intractable surname negotiations began in earnest, I realized I could potentially use mathematics to put to rest my childhood qualms about last names. I began my search for a Surname Impossibility Theorem.

4. (Sur)naming systems

As with any new form of mathematics, one challenge is to create precise definitions. We first define what we mean by a *surname method*:

Definition 1. A **surname** (or *family* name) is the name passed down to a child from their parents.

In contrast to a given (or *first*) name, which a person typically goes by, or middle name, a surname represents "who you come from" as opposed to "who you are." In cultures of European heritage the surname is typically listed second, with the given name first. In many Asian cultures, however, the surname is listed first and the given name second.

Definition 2. A **surname method** (or surname *system*) is a rule that works, equally for each generation, to give a child a surname.

As a non-example, suppose two people with single surnames give their child both of their surnames, hyphenated. Those people are not necessarily following a surname *method*. The child cannot follow the same system as their parents, because the child now has two surnames instead of one.

My goal was to further define what criteria one would want in an ideal surname, definitions precise enough that one could prove whether a surnaming method satisfied or failed the criteria. I started by investigating two examples of common surnaming methods, illuminating the benefits and drawbacks of each system, and then using those benefits and drawbacks to define general criteria for an ideal surname. The two systems are described below. I should note that my focus is on surnames for *children*, as opposed to married-couple surnaming systems. Much of the discussion overlaps, but there are some key differences, which I will discuss later.

4.1. Anglophone surname method

This method is common in the United States and many other countries across the world. Children simply receive the same surname as their father. This method has some benefits—it is simple and aesthetically pleasing, since everyone has a single last name. It also has a long tradition of use and makes it easy to trace patrilineal lineages—for example, if you have the last name Hamilton, it is possible that you could trace your lineage back to one of the founding fathers of the United States. Lastly, if both parents have the same last name (it is common in the US, for example, for the wife to change her last name to her husband's last name when they are married) then everyone in the nuclear family has the same surname, which is beneficial for family identification.

On the other hand, the Anglophone method is patriarchal—the mother's (maiden) surname has no place in the system. If the wife does not change her name, then she will have a last name that does not match her children. Moreover, it is heterosexist—there is no clear way to apply the system for a family with two mothers or two fathers. While the Anglophone system does preserve *some* lineage, it only preserves one patrilineal line. Any surname attached to a mother, at any point, is lost.

4.2. Hispanic surname method

The method common in many Hispanic cultures is that every child receives two family names, one from their father and one from their mother [4]. The Spanish word for such a family name is *apellido*, and I will use that term when describing this kind of system, to distinguish from methods that only use a single family name. Every person has one surname comprised of two apellidos, a patrilineal apellido and a matirilineal apellido. Parents pass down their patrilineal apellido to children, with the patrilineal apellido listed first and the matrilineal second. This has a number of benefits: it represents both parents equally in the apellidos of the child, making it much easier to trace people's lineage. It is not heterosexist, as two men can both pass down their patrilineal apellidos to a child, as can two women.

The Hispanic system does have some drawbacks, however. Most importantly, the two apellidos do not always look or sound good together, and often lead to a very long surname. Because of this, in practice many people go by only one apellido (often the patrilineal). Similarly, it makes sense to connect the apellidos with a hyphen, but hyphenating last names is the fanny pack of naming systems—many people acknowledge its usefulness, but it is so aesthetically displeasing that few people opt for it. If the apellidos are not hyphenated, it can be confusing where the first and/or middle names end and where the last name begins [6]. While the system is certainly less sexist and heterosexist than the Anglophone system, it is always a patrilineal apellido that is passed down—so only *male* grandparents would have apellidos matching their grandchildren. In other words, the system is still patriarchal, it just takes two generations for the mother's (patrilineal) name to disappear. Similarly, while a homosexual couple would have no problem giving two apellidos to their children, those children would have either zero or two patrilineal apellidos, making it unclear which apellido to pass down to the next generation. The Hispanic system makes it easier to follow lineages, but some apellidos are still lost over time.

4.3. Other naming methods

There are many other methods of choosing surnames, of course. My goal is not to discuss every possible surnaming system, but merely to motivate the construction of good criteria for a surnaming system. With that said, below is a (non-exhaustive) list of potential surnaming systems, some of which are more commonly used than others. I will refer to these later when I create criteria for our ideal surnaming system:

- Anglophone system: all children take father's surname.

- Hispanic system: all children have two apellidos, one from each parent. Parents always pass down their patrilineal apellido, which is always listed first. For example, Rishi Garvey-Shah and Elizabeth Clayton-Ayres would have children with the surname Garvey-Clayton.

- Modified Hispanic system: identical to traditional Hispanic system, *except* females pass down a matrilineal apellido and males pass down a patrilineal apellido. Rishi Garvey-Shah and Elizabeth Clayton-Ayres would have children with the surname Garvey-Ayres.

- Portmanteau system: Parents blend their surnames into a new surname for their children. For example, Wendy Lilliedoll and Alex Nord could have children with last name Lillienord or Nordiedoll.

- Alternating surnames: children's surnames alternate between the surnames of the parents. For example, all daughters could take the mother's surname and sons could take the father's surname.

- New surname: Parents create a new surname that all of their children would have. Thus Grace Stephens and Josh Olsen could give their children the surname Namaste.

- Surname of given names: Children are given two apellidos, which are the given (first) names of their parents. So Annie Lange and Susan Dickerson would have children with the surname Annie-Susan or Susan-Annie.

5. Definitions of criteria for an ideal surname method

Armed with the benefits and drawbacks above, I attempted to come up with precise definitions for criteria which would exemplify an ideal surnaming system. As the example surnaming methods demonstrate, though, many benefits and drawbacks do not fall into a black-and-white category, but instead a large grey area. For example, both Anglophone and Hispanic systems are sexist, but at varying levels, with the Hispanic system much less so. Similarly, a shorter name is likely to be more aesthetically pleasing and practical than a long combination of apellidos, but how does one quantify such things as aesthetics and practicality? Below are the definitions I created, with examples to help illustrate each one. These criteria are informed in part by research found in [5].

An ideal surname method would satisfy all of the following criteria:

- **Traditional**: The naming system is consistent with traditional practices for a particular culture. It should be noted, though, that if a large number of people begin to use a *non*traditional surnaming method, that method would become more traditional. Both the Anglophone and Hispanic systems would be considered traditional in most parts of the world, and other methods nontraditional.

- **Aesthetically pleasing and practical**: The naming system produces surnames that are practical and aesthetically pleasing, both visually and vocally. In general, shorter names are considered more aesthetically pleasing than longer ones, and in particular a single surname is better than multiple apellidos. Similarly, having options is preferable to having a prescribed surname. There is no way to "prove" aesthetics or practicality. However, in names, as in design, something is considered more aesthetically pleasing if it is more efficient, and reducing length is the main way to be efficient with language. My impression is that society wants to shorten my hyphenated surname, whether it be through dropping one of my apellidos or not providing enough space on an internet form field for a longer surname.

- **Ancestor-respecting**: The naming system indicates the heritage from which the child comes. In other words, a child's first name is something chosen by the parents, but the surname is more of a linguistic manifestation of one's DNA—it represents who you are, not who you choose to be. Choosing a completely new surname would generally not be ancestor-respecting, whereas apellido systems respect the lineage of both parents as well as some grandparents and beyond.

- **Sibling Matching**: All siblings who are offsprings of the same set of parents have the same surname. Most surnaming methods satisfy this, with the Alternating Surname method being an exception.

- **Egalitarian**: The naming system works equally well with, and gives no favoritism to, any particular type of person or relationship. Thus the system should treat both parents equally, and more specifically should be:

  - *Non-sexist*: The naming system equally represents both mother and father's surnames. The Anglophone system would fail this criterion, whereas the Hispanic system would satisfy it somewhat—the Modified Hispanic system would satisfy it entirely.

– *Gender-neutral*: The naming system works the same regardless of gender, allowing for transgender/non-binary children. The Surname of Given Names method would satisfy this criterion, whereas Alternating surnames would fail it, because the surname given depends on the gender of the child.

– *Non-heterosexist*: The naming system works equally well for both heterosexual and homosexual parents. The Anglophone system fails this criterion, as homosexual couples would have zero or two patrilineal surnames. Apellido systems satisfy it in the first generation, but children will end up with zero or two matri/patrilineal apellidos, making it unclear what apellido the child should pass down. The Portmanteau system would satisfy this criterion completely, as there are no rules for how the surnames should be blended.

6. Statement and Proof of Theorem

Theorem. *(Surname Impossibility Theorem) It is impossible for a surname system to be ancestor-respecting, egalitarian, and aesthetically pleasing.*

Proof. I will demonstrate that completely satisfying two of the criteria necessarily causes a failure of the third criterion.

- (ancestor-respecting $\wedge$ egalitarian) $\implies \neg$ (aesthetically pleasing):

  In order to be ancestor-respecting, a surname must include full surnames of parents (and grandparents, ideally). To be egalitarian, a surname system must equally include the apellidos of both parents. By including multiple apellidos, the surname becomes longer and less aesthetically pleasing (for example, the Modified Hispanic system). Thus satisfying the criteria of being ancestor-respecting and egalitarian forces a surname method to fail aesthetically.

- (aesthetically pleasing $\wedge$ ancestor-respecting) $\implies \neg$ (egalitarian):

A surname can be kept short (aesthetically pleasing) and respect ancestors to a certain degree. In order to remain short, the surname method must leave out some portion of the child's lineage and thus would fail to be egalitarian (the Anglophone system is an example of this).

- (egalitarian $\wedge$ aesthetically pleasing) $\Longrightarrow$ $\neg$ (ancestor-respecting):

  An egalitarian and aesthetically pleasing (that is, short) surnaming system is possible, but in order to keep the surname relatively short, the surname cannot equally include apellidos from both parents. This means that the surname must change each generation, and thus cannot be ancestor-respecting (the New Surname system is a good example of this).

Satisfying any two of the three criteria will result in failure of the third criterion, thus no surname method can be aesthetically pleasing, ancestor-respecting, and egalitarian. It follows that no naming system can satisfy all ideal surname criteria. □

7. Comparison of Surname Methods

For some surname criteria, it is clear whether a particular method fails or satisfies the criteria. For example, a method either treats both males and females equally, or it does not. Thus a method can clearly be found to satisfy or fail the criterion of being non-sexist. Other criteria, though, encompass a large grey area. For example, unless one intends to append every apellido of every ancestor to the surname of a particular child[1], no method can be completely ancestor-respecting. But there is a range as to how much a surname connects a child to parents, grandparents, and beyond. Thus for many criteria it is not about "fail" or "satisfy," but a question of the degree to which a given method satisfies the criteria. Especially since we have proved that no surname system can satisfy all criteria, the question changes from "what is the ideal surname method" to "which method does best amongst all the criteria?"

[1] Just ask Pablo Diego Jose Francisco de Paula Juan Nepomuceno María de los Remedios Cipriano de la Santisima Trinidad Ruiz y Picasso how that works out.

To answer this question, it makes sense to give each method a score for each criterion. I chose to give integer scores between 0 and 3, with zero for a method that completely fails a criterion, three for a method that completely satisfies a criterion, and one or two for something in between. For example, in reference to the aesthetically pleasing criterion, the New Surname method scores a perfect 3 because parents can choose whatever name they want, and one would assume that they would choose a surname that is aesthetically pleasing to them. The Anglophone and Alternating methods both score a 2 because the resulting surnames would be relatively short, but there is also less choice involved in the surname, so they do not get the full score. Portmanteau and Given Name methods both score a 1, because they tend to be longer surnames, but not as long as the apellido methods (Hispanic and Modified Hispanic) which both score a 0 (I acknowledge that this scoring system is unnecessarily arbitrary and discrete, as well as somewhat subjective).

The table below summarizes my scores for the surname methods discussed in this paper:

	Traditional	Aesthetic	Ancestor	Matching	Egalitarian	Total
Anglophone	3	2	1	3	0	9
Hispanic	3	0	2	3	1	9
Mod. Hispanic	2	0	2	3	2	9
Portmanteau	0	1	1	3	3	8
Alternating	1	2	1	0	2	6
New Surname	0	3	0	3	3	9
Given Names	0	1	1	3	3	8

A few comments about the table comparing surname methods:

- The table reinforces the conclusions of the Surname Impossibility Theorem, as every method has at least one criterion in which it scores a 0, and almost all methods have multiple criteria in which they score a 0 or 1 (the Modified Hispanic system being the only exception—it fails aesthetically, but scores a 2 or 3 on all other criteria).

- While the Anglophone and Hispanic systems score relatively well (tied for first place in total points), that is largely due to their high scores for being traditional. If any of the less traditional methods came to be in more common use, then their scores would increase.

- If the New Surname method were to become a more common method (and thus score a 3 for traditional), it would have the highest score of all methods. On the other hand, it is the only method that completely fails to be ancestor-respecting, which is one of the primary purposes of a surname.
- No method scores the maximum of 3 for ancestor-respecting, as no method preserves all ancestral apellidos.
- Although the Modified Hispanic system scores a zero for aesthetics, it should be noted that it does allow for some aesthetic variety that other methods may lack. Specifically, the order of the apellidos is not prescribed, so there is a choice of, for example, Finn-Johnson or Johnson-Finn. Similarly, one can choose to hyphenate, or not hyphenate and leave a space, or to combine the two apellidos without any space at all, but still capitalizing each apellido[2].

8. Married couple surnames

My focus has been on surnames for children, but similar surname issues arise for married couples—should each partner keep their own name, or should one or both partners change their surname so that they match? Generally speaking, the issues that arise with married-couple surnames are similar to those mentioned above, with the following caveats: certain surnaming methods force, or at least encourage, certain married-couple surnaming methods, and choosing to keep (or change) one's own surname can exacerbate issues created by surnaming methods.

For example, if a woman chooses to keep her own surname in the Anglophone system, she will have a different surname from her children, which many women would find undesirable. This encourages women to change their surname when married, exacerbating the sexism of the Anglophone system. Similarly, couples using the Alternating method must keep their surnames, otherwise the alternation of surnames makes no sense. In contrast, under the modified Hispanic system, couples can choose to keep their own surnames or change to the (future) surname of their children—both options work just fine;

[2] If MacGregor can do it, why not FinnJohnson?

it is just a question of the value the couple places on identifying with one's own name (and ancestors) versus having a nuclear family where everyone has the same surname.

9. Conclusion

Impossibility theorems, by nature, inevitably eliminate the hope of an ideal option in a given situation. As such, one could view an impossibility theorem as depressing—but I do not. By removing the ideal as a possibility, we are free to discuss what qualities we hold most dear. Instead of being a dead-end, the Surname Impossibility theorem opens up a potentially productive values-driven discussion, wherein each couple gets to decide what is most important to them. This may lead to some disagreement, but compromise is an important tool to learn, and never more important than when preparing to have a child. I have seen many versions of this play out amongst my friends and family. Many couples choose a single surname for their children, but then make an effort to link their child's lineage to the other side of the family tree through given and middle names. Other families I know were unable to agree on a surnaming method, and used it as a bargaining chip—all children took the surname of one parent, but they all follow the religion of the other parent. Thus while the heritage of one parent does not show up in their children's surname, it is present in their lives in other important ways. As children grow, they also get to choose for themselves how they would like to present themselves. Whether it be Robert Zimmerman changing his name to Bob Dylan, or the many aesthetic benefits of being referred to as Prince, Madonna, or Snoop Dogg, as children age they can choose for themselves rather than be locked into decisions made by their parents.

My seven-year-old self would have scoffed at the surnaming decisions made by some of my friends, but armed with the Surname Impossibility Theorem (and hopefully a bit more humility than I had as a child) I can recognize their choices as simply different ways of expressing their values, which I respect. By taking a creative mathematical approach to a real-world dilemma, I was also able to come to terms with the necessarily unsatisfactory options for giving my children their surname. My wife and I opted for the modified Hispanic method when our children were born. For us, both equality and respecting ancestors were of paramount importance. My brother, on the other hand, could not abide losing any lineage connecting his children to their grandparents. In particular, his wife is the last child in her lineage, and her surname would be lost if not passed on. He and his wife chose to give their children her last name as a single surname, and have his hyphenated surname as the children's middle names. The Surname Impossibility Theorem puts these decisions in their proper context—not as differences to be argued about, but as a means to express our values and pass them on to the next generation as best we can.

This procedure is by no means limited to surnames, of course. Mathematical ideas and processes can creatively model many situations in the real world, if one is willing to give up a bit of mathematical precision to potentially elucidate useful truths. If nothing else, this kind of creative application of math reminds me of the many ambiguities inherent in human existence, and the wide variety of valid conclusions people make due to those ambiguities. One can only hope that recognizing the impossibility of the ideal can make us all more accepting of others' choices, which is to say, their values, in surnames and beyond.

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