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Fostering Mathematical Creativity While Impacting Beliefs and Anxiety in Mathematics

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Abstract

This quantitative study examined the notion of mathematical creativity and its relationship to epistemological beliefs of the nature of mathematics and mathematical anxiety. A counterbalanced design was employed, randomizing a class of elementary pre-service teachers into two groups and giving a pre- and post-test to determine if significant differences exist in the participants who are exposed to problem posing, divergent thought, and invented strategies, that is, a punctuated, intentional experience with mathematical creativity. This difference in mathematical anxiety, beliefs, and creativity was also gauged using repeated measures during the study. Furthermore, beliefs and anxiety were correlated with mathematical creativity employing pre- and post-test measures. The findings of this study suggest that mathematical creativity can be fostered and sustained under certain conditions. Also, the results indicated that mathematical beliefs and anxiety are significantly impacted by intentional experiences with mathematical creativity—alternative algorithms, divergent thought, invented strategies, and problem posing.

Keywords: mathematical creativity, beliefs, anxiety

1. Introduction

Two of the nation's strengths are its ability to problem solve creatively and thrive economically. To sustain any economic stability in a technological world; however, the Unites States must advance in the disciplines of science, technology, engineering, and mathematics. If ignored, the United States' prowess in mathematics may be in jeopardy.

The National Mathematics Advisory Panel's (NMAP) [1] concerns about the nation's mathematical prowess only heighten the mathematics education community's awareness of the need for mathematical problem solving. It is the opinion of the author that the kind of mathematical problem solving must be creative. One may ask, why? Because many of the problems that today's kindergartners will face when they are in the workplace have not yet been defined or identified. This means the next generation will have to create solutions to problems that do not already exist. Although more mathematics might not solve this dilemma, more of a certain, specific kind of mathematics might. That is to say, old ways will not necessarily produce new remedies for the next generation's unseen predicament. This situation calls for a revision of problems and solutions, so that old paradigms are replaced with fresh frameworks to unravel these unknown problems. In short, problem solving mathematically must be reconceived to meet the challenges of the future.

This re-conception is exactly what the National Council of Teachers of Mathematics (NCTM) [2] proposed in its vision for the state of school mathematics. NCTM's vision is to develop the kind of creative mathematics that will produce the changes necessary to meet the demands of the problems our children will face.

How will our nation produce mathematically creative problem solvers in classrooms as described by NCTM's vision? How can a nation sustain its mathematical prowess? One way is to examine the base and foundation of the nation's educational building blocks to maintain its mathematical might. Who are the key catalysts to creative mathematical classrooms? Are they not the classroom teachers of mathematics? These teachers must not be limited to the secondary mathematics setting, but they must include classroom teachers of elementary mathematics, as well.

Furthermore, to achieve mathematically creative classrooms, one must look at the pre-service teachers who will shape the students of tomorrow. Because they shape students during the crucial elementary years, elementary pre-service teachers are the catalyst to initiate change. The dilemma at hand, according to the relevant literature, is twofold. First, elementary pre-service teachers have formal or fixed beliefs of mathematics [3], [4]. This, in essence, is incongruent with NCTM's vision for the mathematics classroom. Second, pre-service teachers experience mathematical anxiety [5], [6], [7], [8].

There is little dispute that teachers make a difference in students' educational experiences. Teachers allow students the experiences to create, discover, and explore mathematical relationships in the classroom. They influence and inspire young learners to new wonders and curiosities about the mathematical universe. Schofield [9] argued that attitudes and beliefs of teachers are directly connected to students' attitudes and beliefs towards mathematics. When discussing mathematical dispositions, Vinson [10, page 90] noted that these attitudes and beliefs "influence how often mathematics is used, the willingness to pursue advanced work in mathematics, and even the choice of prospective occupations."

However, even with all of their potential assets, teachers can be devastating liabilities. For instance, teachers with mathematics anxiety are likely to produce students with mathematics anxiety [10]. Elementary pre-service teachers are reported to have high mathematics anxiety stemming from formal or traditional instructional practices [7]. According to their research, Swars, Daane, and Giesen [11, page 312] suggested that mathematics anxiety "has a negative relationship with a pre-service teacher's belief in his or her skills and abilities to be an effective mathematics teacher."

If teachers directly impact students' creativity, then a teacher wishing to encourage mathematical creativity should presumably pose mathematical problems, ask higher order and reflective questions, encourage groups and whole class discussion, and provide opportunities to observe and explore mathematical relationships. To adopt this position of teaching mathematics one must believe that mathematics is not formal or static but informal and dynamic, according to Collier [3]. Collier [3, page 155] contended that this the opposite of traditional elementary school mathematics programs which, "have emphasized the formal content of mathematics, often at the expense of helping children see the creative and investigative nature of mathematics." In short, although, teachers (especially elementary pre-service teachers) possess great potential to impact young mathematical students, they also have the capability of endangering the nation's progress in mathematical creativity. Furthermore, studies in the area of mathematical creativity are sparse. This work will add to the body of knowledge and fill part of the gap in the current literature.

2. Purpose of Study

The goal of this initial study was to explore how a treatment of punctuated, intentional experiences with mathematical creativity and problem posing influenced mathematical anxiety, mathematical beliefs, and mathematical creativity. In addition, the study examined how these three variables might relate. That is to say, are these variables (mathematical creativity, beliefs, and anxiety) predictive of each other? If a punctuated, intentional experience with mathematical creativity fosters more mathematical creativity and influences mathematical beliefs and anxiety, this information would be valuable to curriculum designers, mathematics educators, and teachers of mathematics. Also, if relationships exist among these variables, the relationships would inform teachers and teacher educators what beliefs correspond to mathematical anxiety or mathematical creativity, or that mathematical creativity is associated with certain beliefs. Thus, those beliefs could be conveyed to their students with the intent to support mathematical creativity and the nation's mathematical prowess.

This quantitative study examined the notion of mathematical creativity and its relationship to epistemological beliefs of the nature of mathematics. The participants were assessed in this study using the following instruments: Creative Ability in Mathematics, Mathematics Belief Questionnaire, and the General Assessment Criteria. The guiding questions that were investigated in this study were the following:

1. What effect does a punctuated, intentional experience with mathematical creativity have on elementary pre-service teachers' mathematical creativity?

2. What effect does a punctuated, intentional experience with mathematical creativity have on elementary pre-service teachers' mathematical beliefs?

3. What relationship exists between elementary pre-service teachers' mathematical creativity and their mathematical beliefs?

4. What relationship exists between elementary pre-service teachers' mathematical creativity and mathematical anxiety?

3. Defining Mathematical Creativity, Beliefs, and Anxiety

3.1. Mathematical Creativity

No single definition exists for mathematical creativity. Often it is functionally defined and examined [12], [13], [14], [15], [16]. Mathematical creativity can be seen as the capacity to invent algorithms and strategies or even alternative approaches to a standard problem. Another way mathematical creativity has been defined is to overcome fixations and divergent products [14], [17]. Some have pointed out that mathematical creativity can be bifurcated into a process of thought or a product manifested in fluency, flexibility, and originality [12], [17], [18] defined these three products in the following manner: fluency—the number of different correct answers, methods of solutions, or new questions formulated; flexibility—the number of different categories of answers, methods, or questions; originality—solutions, methods, or questions that are unique and show insight.

In the area of mathematics education, some researchers and theorists have suggested several ways to foster creativity. However, could it be that creative teachers produce creative students? The teacher fostering creativity is one who poses problems, asks questions, encourages discussion, and provides opportunity to observe and explore in the learning environment [12]. Additionally, finding multiple methods, alternative algorithms, or unique solutions to problems increases the students' problem-solving ability and divergent thinking. Because problem solving and problem posing are central to the nature of mathematics (and for that matter mathematical thinking) Silver [19] proposed that mathematical creativity could and should be developed through inquiry-oriented mathematics instruction that employs ill-structured or open-ended problems during the problem-solving and problem posing process. By doing so, Silver [19, page 79] maintained, students will enhance "greater representational and strategic fluency and flexibility and more creative approaches to their mathematical activity." If mathematical creativity is to manifest itself in the classroom, Sriraman [20, page 32] conjectured that "students should be given the opportunity to tackle non-routine problems with complexity and structure—problems which require not only motivation and persistence but also considerable reflection." Elsewhere, Sriraman [21, page 27] suggested five overarching principles to maximize creativity at the K-12 level:

a. the Gestalt principle—freedom of time and movement,

b. the aesthetic principle—appreciating the beauty of unusual solution/connections to the arts and sciences,

c. the free market principle—encouraging risk taking and atypical thinking,

d. the scholarly principle—view creativity as contributing to, challenging known paradigms and extending the existing body of knowledge, and

e. the uncertainty principle—open-ended and/or ill-posed problems and tolerating ambiguity.

The traditional tragedy of school mathematics is the overemphasis on skill-and-drill or theorem-proof, theorem-proof routines [22], [23]. However, it appears that creativity is fostered through experiences and needs time to develop. This sentiment corresponds to Silver's [19, page 75] statement that creativity "is often associated with long periods of work and reflection rather than rapid, exceptional insight; and is susceptible to instructional and experiential influences." Furthermore, Haylock [17] expounds that overcoming fixations, both algorithmic and content, to produce divergent products strikes at the heart of mathematical creativity. McGannon [24, page 12] argues that mathematical "mechanization," "problem solving rigidity," and "functional fixity" are inhibiting factors that "militate against our [students] attacking new problems with an imaginative approach." Others have indicated that, in sharp contrast to these inhibiting factors, ill-structured, open-ended, or multiple-solution problems, along with problem posing support creativity in the mathematical classroom [25], [26], [17], [27], [28], [23], [19].

3.2. Mathematical Beliefs

Mewborn and Cross [29, page 12] contrasted two differing and prominent views of mathematics held by many. One sees mathematics as fixed and the other view considers mathematics to be fluid. The fixed view is associated with the traditional classroom. Furthermore, it is believed from this perspective that "mathematics is a collection of rules to be mastered, arithmetic computations, mysterious algebraic equations and geometric proofs." That is to say, mathematics is dead and to be examined like a corpse in an autopsy.

According to Van De Walle [30, page 12], the fixed position of mathematics believes that: "mathematics is a series of arbitrary rules, handed down by the teacher, who in turn got them from some very smart source." When mathematics is believed to be fixed, doing mathematics is following some rule, knowing mathematics is applying the correct rule, and determining the correct answer is held by the expert, i.e., some teacher or book.

In contrast to the fixed belief of mathematics, the fluid view, corresponds to the NCTM position. This fluid belief sees "mathematics as a science of pattern and order" [31, page 31]. Essentially, for this perspective, mathematics is ever living and expanding. It can be discovered and explored through predictions and conjectures. In one sense, mathematics is a noun, but it is more than that. It also includes the verbs (conjecture, discover, explore, investigate, predict, etc.) that conceive mathematics as well.

In summary, the nature of mathematics can be seen on a continuum with two polar extremes. One belief is that mathematics is static. That is to say, mathematics is a seemingly arbitrary set of rules that are unchanging and uncompromising. And the other belief, which is that mathematics is dynamic, views mathematics as an ever growing, ever changing body of flux. Imitation or regurgitation is how mathematics can be seen in the static camp. For those who see mathematics through the dynamic lens, however, it undertakes an active dimension of assimilation and creation.

3.3. Mathematical Anxiety

For several decades, anxiety has been studied within the content domain of mathematics. According to Hembree [32, page 34], mathematics anxiety is reported to be "no more than subject-specific test anxiety," although others have stated that it is basically, "a general dread of mathematics, and of tests in particular." Ma [33] simply defined it as dislikes, worries, and fears towards mathematics. Some researchers, however, have acknowledged the complex nature of describing mathematics anxiety, because as a construct it possesses both affective and cognitive aspects [34].

Mathematics anxiety has also been defined contextually. For instance, Hopko [35, page 336] illustrated it as "apprehension and arousal concerning the manipulation of numbers in academic, private, and social environments." Other researchers have stressed that mathematics anxiety produces avoidance behaviors to mathematics as a stimulus [36], [37]. In short, mathematics anxiety is an intricately complex and multidimensional construct [33], [34] that causes a "state of discomfort that occurs when an individual is required to perform mathematically, or the feeling of tension, helplessness, or mental disorganization an individual has when required to manipulate numbers and shapes" [11, page 312].

An intriguing question remains: who or what is responsible for producing mathematics anxiety? According to Trujillo and Hadfield [30], environmental, intellectual (or cognitive), and personality factors cause mathematics anxiety. Similarly, Cemen, as cited by Ma [33], proposed three causes to produce anxious reactions. First, there are environmental precursors, which tend to be negative experiences at home or in the classroom with mathematics. Next, there are dispositional precursors, which may entail negative attitudes towards mathematics or a lack of confidence in it. Finally, there are situational precursors, which are factors or formats of the classroom or its instruction. It is suggested by some that teachers who have high mathematics anxiety are likely to convey mathematics anxiety to their students [34]. Trujillo and Hadfield [38] have proposed a theoretical model for the causes of mathematics anxiety for pre-service teachers. They suggested that negative classroom experiences and unsupportive home environments produce mathematically anxious students [38]. Although teachers may not be the only catalyst for mathematics anxiety, it seems likely that they are an important factor.

4. Research Design

Thirty-two pre-service teachers were pre- and post-tested with four different instruments. To assess the teachers' epistemological beliefs about the nature of mathematics, the Mathematics Belief Questionnaire (MBQ) was given [3]. The Abbreviated Math Anxiety Scale (AMAS) instrument was used to measure the pre-service teachers' mathematical anxiety [35]. Using the following two instruments, mathematical creativity was measured: Balka's [39] Creativity Ability in Mathematics (CAMT) and Silver and Cai's [40], [41] General Assessment Criteria (GAC).

Next, two different groups were formed by randomly assigning the pre-service teachers. Using a random number table, pre-service teachers were either assigned to Group A or Group B. Group A first received the treatment (a

Pre-Assessment					Post-Assessment
Group A Randomize	MBQ AMAS CAMT GAC	Treatment	GAC	No Treatment	MBQ AMAS CAMT GAC
Group B Randomize	MBQ AMAS CAMT GAC	No Treatment	GAC	Treatment	MBQ AMAS CAMT GAC

Table 1: Counterbalanced design.

punctuated, intentional experience with problem posing, divergent thought and invented strategies) and Group B did not receive the treatment. Both groups were assessed using the GAC. Then, the treatment was switched. That is, Group A did not receive the treatment, but Group B received the treatment. Following the counterbalanced, the two groups were assessed with the GAC. As stated earlier, to establish a baseline measurement at the beginning of the semester the pre-service teachers were assessed with GAC, and then at the end of the semester the pre-service teachers were assessed with GAC again. (See Table 1 for the counterbalanced design with the pre- and post-test.)

5. Setting and Sample

This study employed a convenient sampling method. The participants who were selected as the sample for this study were juniors in college entering into the elementary education program at a research institution in the southeast region of the United States. These pre-service teachers met the University's course requirements for mathematics (a minimum of six hours to include college algebra or higher) and the College of Education's prerequisites as well (which comprises passing a general knowledge test and holding a minimum 2.5 grade point average). This sample of 32 students was studied in the students' first mathematics methods course. This sample was rather homogeneous overwhelmingly female with few minority students. Specifically, the sample was comprised of one African American female and thirty-one Caucasians pre-service teachers (of which one was a Caucasian male).

Although the sample was not randomly selected, the participants were randomly assigned to one of two groups. Using a random number table beginning with the fifth row and the seventh column, the first sixteen appropriate two-digit numbers in the table bifurcated the participants into these two groups.

6. Treatment

During the treatment, pre-service teachers participated in a ninety-minute class session. In a fifteen-week semester, the treatment was administered at the end of the second and the beginning of the third week of the semester to Group A and Group B, respectively. The following protocol was used to ensure the treatment was the same for both randomly assigned groups. The session cycled through a four-phase progression, which is the punctuated, intentional experience with mathematical creativity. The progression of the four phases was as follows:

1. expose to multiple perspectives,
2. pose an open-ended problem,
3. examine sample solutions, and
4. pose alternative problems.

First, the pre-service teachers were exposed to multiple perspectives given four numbers, shapes, or objects. (For an example see Figure 1.) They were asked questions like: Which one does not belong? Which one is different? What do they have in common? Which ones are the same? What is the pattern? Each question was followed by the question "Why?" pressing the pre-service teacher to justify the response. Not only were the pre-service teachers asked to formulate one rule and reason, but they were then asked to find at least two rules for each scenario. After each pre-service teacher responded to the prompt individually, the pre-service teachers as a whole were given the opportunity to share their responses in groups and finally with the whole class (replicating a "Think-Pair-Share" model).

In the second phase, an open-ended problem was posed. The pre-service teachers were then given the opportunity to approach each problem from several vantage points. Time was given for them to explore the problem and

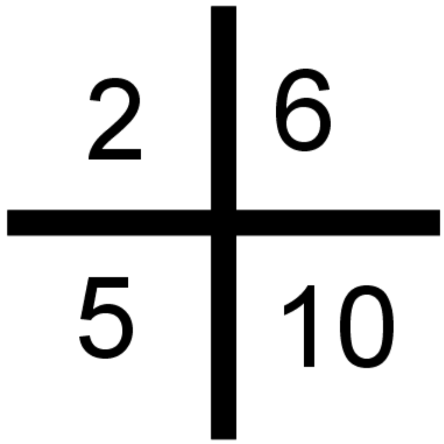

Figure 1: Example of a Multiple Perspective Task. Directions: Which One Does Not Belong? Why?

to work it using several different methods to find numerous solutions. For instance, the task, "Given a nine-dot unit-square grid draw as many shapes as possible with an area of 2 units" was adapted from Haylock [17, page 72].

Then, in the third phase, sample solutions of the open-ended problem were shown to the pre-service teachers. Time was given for the pre-service teachers to discuss and understand the variegated solutions. The solutions exposed the pre-service teachers to creative thought. As an example, consider the following open-ended problem. "Write down other results that can be deduced easily given result: $19\times35 = 665$" [17]. Common responses may include using some multiple of 10, like $190\times35 = 6650$, or utilizing the commutative property, $35\times19 = 665$. A resourceful response would be $665\div19 = 35$. However, the next responses, $17.5\times19 = 332.5$ and $35\times9.5 = 332.5$, were more innovative, along with $70\times19 = 1330$, $38\times35 = 1330$, and $1330\div70 = 19$. Among the more inventive responses, for the pre-service teachers which invoked the most discussion, were the following $(20-1)\times35 = 665$ and $(70\div2)\times19 = 665$.

Finally, in the last phase, the pre-service teachers problem posed. Given the previous open-ended problems, they were asked to pose alternative problems that stem from the original problem or its solution. Changing the parameters or conditions of the problem were suggested. For example, the open-ended problem "Using any combination of operations, make numbers 1 to 5 using four sevens at a time" was altered by the pre-service teachers [17]. Some suggested to expand the numbers that were to be made with the following: "Make numbers 10 to 25 using four sevens at a time." Others changed the number of sevens used, "Make numbers 1 to 5 using six sevens at a time." The previous posed problems were pedestrian compared to this one: "Make all the two-digit primes using four sevens at a time." This phase, at any rate, considered changing one or more characteristics of the initial problem by posing an alternative problem.

7. Instrumentation

The epistemological beliefs about the nature of mathematics of pre-service teachers were measured (at the beginning and at the end of the semester) to determine their beliefs toward four specific domains: "mathematics is a collection of rules, formulas, and procedures; mathematics is a creative endeavor; mathematical problem-solving allows for multiple approaches; and mathematics is best taught by direct instruction" [4, page 6]. This belief construct was measured with the Mathematics Belief Questionnaire (MBQ) developed by Collier [3] and used by Seaman [4] using a six-point Likert scale. Each item had a scale response that ranged from strongly disagree to strongly agree. The MBQ's was reliable. The scores reported ranged from .80 and .83 using "the proportion of total variance that is not due to error in measurement" [3, page 157]. This survey was comprised of forty questions. Half of the items were posed in a positive direction and the other half in a negative direction. The positively stated items aligned with a fluid view of the nature, teaching, and learning of mathematics. Conversely, the fixed view of the nature, teaching, and learning of mathematics were identified with the negatively stated items.

These items can be understood using Seaman's [4] categorization into four general themes or domains: mathematics is a collection of rules, formulas, and procedures (items 1, 3, 9, 14, and 28); mathematics is a creative endeavor (items 2, 5, 6,12, 18, 20, 23, 24, 33, 35, 37, and 39); mathematical problem solving allows for multiple approaches (items 4, 7, 8, 10, 13, 15, 19, 25, and 30); mathematics is best taught by direct instruction (items 21, 22, 26, 29, 31, 32, 34, 36, 38, 40).

To measure mathematical anxiety, the pre-service teachers were pre- and post-tested using the Abbreviated Math Anxiety Scale (AMAS) at the beginning and the end of the semester. The survey contained nine items. Each item was on a 6-point Likert scale, ranging from 1 (low anxiety) to 5 (high anxiety). The scores from the instrument were reliable. The internal consistency was reported to possess Cronbach's alpha of .90 with a mean and standard deviation of 21.1 and 7.0 respectively [35].

Mathematical creativity was measured using two different instruments. At the beginning and at the end of the semester they were tested using Balka's [39] Creative Ability in Mathematics Test (CAMT). Balka [39] reported a Cronbach's alpha of .72 and a standard error of measurement of 7.24 for CAMT reliability. The problems solved in class were assessed using Silver and Cai's [41] General Assessment Criteria (GAC). At four punctuated times, this scoring rubric was used to assess the pre-service teachers' creativity.

8. Data Collection and Analysis

Various methods of data collecting were employed in this study of mathematical anxiety, mathematical beliefs, and mathematical creativity. To collect data on mathematical anxiety and mathematical beliefs, two different surveys were distributed electronically. As for mathematical creativity, data were collected via the traditional means—paper and pencil.

The participants were asked to complete the MBQ and the AMSA on the internet. During the first week of class, participants were asked to log on to the online course management system and complete the forty-item questionnaire for beliefs and the nine-item survey for anxiety. The internet survey permitted the participants extra time to complete the surveys and gave them the convenience of choosing when and where to complete them as well.

Data on mathematical creativity was collected in class using the Creative Ability in Mathematics Test (CAMT) and the General Assessment Criteria (GAC). The participants were allowed the whole class period to respond to the CAMT at the beginning and the end of the semester. At punctuated times during the semester, participants were given four different items scored by the GAC. Each time, fifteen minutes was given to the participant to complete the item.

9. Research Findings

What effect does a punctuated, intentional experience with mathematical creativity have on elementary pre-service teachers' mathematical creativity?

A two-way within-subjects analysis of variance was conducted to evaluate the effect of a punctuated, intentional experience with mathematical creativity on elementary pre-service teachers' mathematical creativity. The dependent variable was mathematical creativity. The within-subjects factors were treatment groups. The mathematical creativity main effect and mathematical creativity x treatment groups effect were tested using the multivariate criterion of Wilk's lambda (Λ). The mathematical creativity main effect and treatment group interaction effect, was significant, $\Lambda = 0.21$, $F(2, 6) = 51.09$, $p < 0.01$, as well as the mathematical creativity-treatment groups interaction effect, $\Lambda = 0.60$, $F(2, 6) = 9.06$, $p < 0.01$. Once again, the results support the conclusion that a punctuated, intentional experience with mathematical creativity develops elementary pre-service teachers' mathematical creativity as seen in Figure 2 and Table 2.

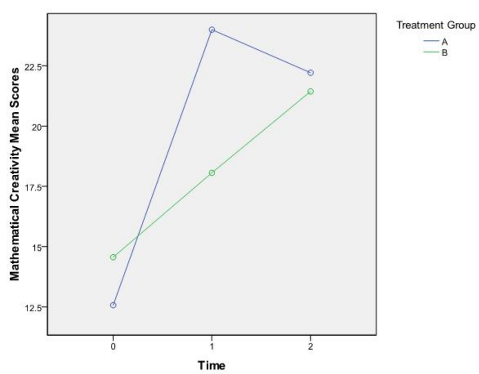

Figure 2: Mathematical Creativity Scores During the Intervention.

A paired-samples t test was conducted to evaluate mathematical creativity as to whether the means of the pre-test was significantly different from the post-test. The results indicated that the pre-test sample mean for mathematical creativity (M = 35.13, SD = 10.56) was significantly different from the post-test sample (M = 40.24, SD = 11.42), $t(31) = 19.99$, $p < 0.01$.

The effect size of d was 3.53. The 99% confidence interval for mathematical creativity mean ranged from 30.01 to 40.24 on the pre-test and 34.81 to 45.88 on the post-test. Figure 3 shows the distribution of the mathematical creativity scores. The results support the conclusion that a punctuated, intentional experience with mathematical creativity increases or fosters elementary pre-service teachers' mathematical creativity.

	Time 0	Time 1	Time 2
Group A	12.57 (2.62)	24.00 (6.10)	22.21 (5.52)
Group B	14.56 (3.79)	18.06 (3.49)	21.44 (6.10)

Table 2: Effect of Mathematical Creativity on Pre-service teachers. Entries given as mean (standard deviation).

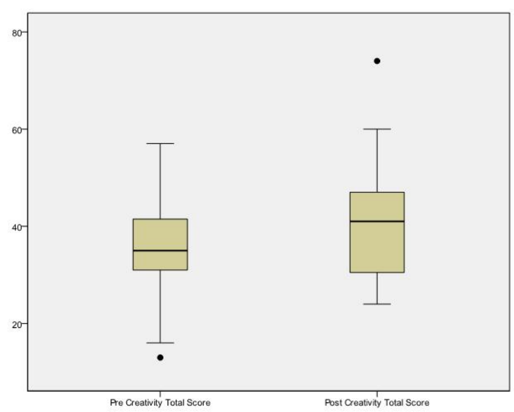

Figure 3: Boxplots of Pre- and Post-Creativity Scores for CAMT.

What effect does a punctuated, intentional experience with mathematical creativity have on elementary pre-service teachers' mathematical beliefs?

A paired-sampled t test was conducted on the mathematical beliefs scores to evaluate whether the means of the pre-test was significantly different from the post-test. The pre-test sample mean 150.19 (SD = 15.77) was significantly different from the post-test sample mean 185.63 (SD = 21.14), $t(31) = 49.67$, $p = 0.01$. The 99% confidence interval for mathematical creativity mean ranged from 142.54 to 157.84 on the pre-test and 175.37 to 195.88 on the post-test. The effect size of d was 8.78. Figure 4 shows the distribution of the mathematical beliefs scores.

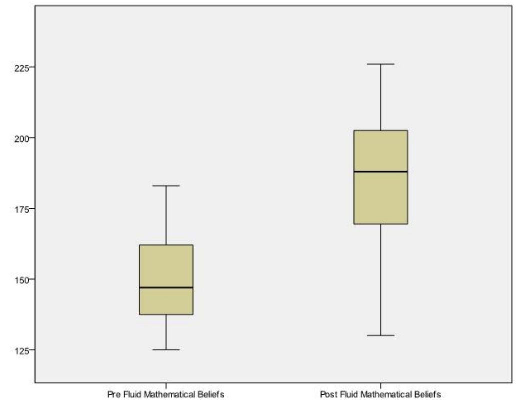

Figure 4: Boxplots of Pre- and Post-Mathematical Beliefs Scores.

The results support the conclusion that a punctuated, intentional experience with mathematical creativity increases elementary pre-service teachers' beliefs that mathematics is fluid.

9.0.1. *What relationship exists between elementary pre-service teachers' mathematical creativity and their mathematical beliefs?*

Correlation coefficients were computed among the following two variables: mathematical creativity and mathematical beliefs. Although a medium correlation coefficient appeared in the pre-test correlation between mathematical creativity and mathematical beliefs scales, it was not significant $r = 0.256$, $p = 0.078$. At the same time, the correlation between mathematical creativity and mathematical beliefs scales for the post-test was not significant either $r = -0.084$, $p = 0.324$. In general, the results suggest that no relationship exists between elementary pre-service teachers' mathematical creativity and their mathematical beliefs. In other words, mathematical beliefs are not a predictor of elementary pre-service teachers' mathematical creativity. (See Table 3.)

9.0.2. *What relationship exists between elementary pre-service teachers' mathematical creativity and mathematical anxiety?*

Correlation coefficients were computed between mathematical creativity and mathematics anxiety scales. The results of the correlational analyses pre-

	A	B	C	D
A	1	-	-	-
B	0.256	1	-	-
C	0.319	-0.025	1	-
D	0.071	0.281	-0.084	1

Table 3: Correlations among the Pre- and Post- scores for mathematical beliefs and creativity. A is Pre-Fluid Mathematical Beliefs, B is Pre-Mathematical Creativity, C is Post-Fluid Mathematical Beliefs, and D is Post-Mathematical Creativity. Here $N = 32$, and the 0.319 correlation has a $p < 0.005$.

	E	B	F	D
E	1	-	-	-
B	0.022	1	-	-
F	0.587	-0.223	1	-
D	-0.179	0.281	-2.92	1

Table 4: Correlations among the Pre- and Post- scores for mathematical anxiety and creativity. E is Pre-Mathematical Anxiety, B is Pre-Mathematical Creativity, F is Post-Mathematical Anxiety, and D is Post-Mathematical Creativity. Here $N = 32$, and the 0.587 correlation has a $p < 0.001$.

sented in Table 4 show that one out of the six correlations were statistically significant and were greater than or equal to 0.350. For the pre-test, the correlation between mathematical creativity and mathematical anxiety scales was not significant $r = 0.022$, $p = 0.453$. The correlations of mathematical creativity with mathematics anxiety measures on the post-test tended to be lower and not significant. That is, the correlation between mathematical creativity and mathematical anxiety scales for the post-test was not significant either $r = -0.292$, $p = 0.052$. In general, the results suggest that if mathematical creativity is higher, then mathematics anxiety will be lower (or vice versa).

9.0.3. What relationship exists between elementary pre-service teachers' mathematical creativity and mathematical anxiety?

A paired-sampled t test was conducted on the mathematical anxiety scores to evaluate whether the means of the pre-test was significantly different from the post-test. The pre-test sample mean 26.56 (SD = 4.85) was significantly different from the post-test sample mean 25.66 (SD = 5.78), $t(31) = 25.12$, $p = 0.01$. The 99% confidence interval for mathematical creativity mean ranged from 24.21 to 28.91 on the pre-test and 22.85 to 28.46 on the post-test. The effect size of d was 4.44. Figure 5 shows the distribution of the mathematical anxiety scores. The results suggest that a punctuated, intentional experience with mathematical creativity decreases elementary pre-service teachers' mathematical anxiety.

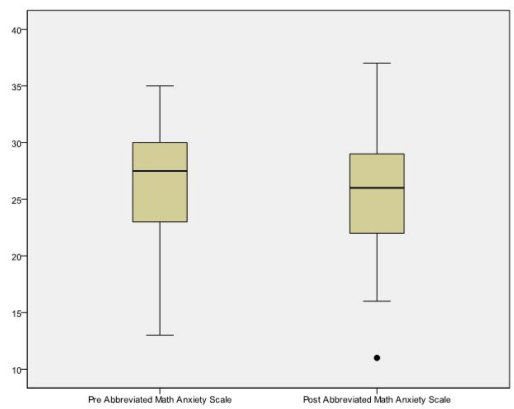

Figure 5: Boxplots of Pre- and Post-Mathematical Anxiety Scores.

10. Discussion

10.1. What effect does a punctuated, intentional experience with mathematical creativity have on elementary pre-service teachers' mathematical creativity?

To answer the question of what effect does a punctuated, intentional experience with mathematical creativity have on elementary pre-service teachers' mathematical creativity, this study showed two intriguing findings. First, using the counterbalanced quasi-experimental design and an ANOVA data analysis, the results suggested that mathematical creativity can be developed or fostered. Second, after examining the CAMT pre- and post-test data, the mathematical creativity scores significantly increased, which further support this notion.

The import of the following data must be stressed. First, note at time zero that Group A's initial mean score was lower that Group B's mean score, as seen in Figure 5. Then Group A received the treatment, while Group B did not. Both were assessed at time one. Significant gains were accrued to Group A's mean score by nearly double, whereas the mean score of Group B made no significant change. Next Group B received the treatment and Group A did not. At time two, the mean score for Group B increased significantly, while Group A's mean score slightly decreased. Nevertheless, the first treatment group started with a lower mean score and finished with a larger mean score, even with a modest decline at the end of time two.

This data suggested that a punctuated, intentional experience with mathematical creativity increases elementary education pre-service teacher's mathematical creativity. With the tapering mean score of Group A at time two, it may lead one to inquire if prolonged, intentional experiences with mathematical creativity are required to maintain these gains. To phrase it differently, does mathematical creativity atrophy and diminish over time or in certain impoverished environments when it is not exercised?

Importantly, the data suggests that pre-service teachers' mathematical creativity can be significantly enhanced in a relatively short period of time. If this is indeed the case, how might this translate into school settings in the mathematics classroom? Could it be that punctuated, intentional experiences with mathematical creativity have the potential to redesign the landscape of the mathematics classroom?

Furthermore, perhaps to extend the principle of this finding to school mathematics, could it not point to the need for continual, prolonged experiences with mathematical creativity? For instance, if a third-grade student made gains in mathematical creativity, but the next year was not exposed to an environment that fostered mathematical creativity, would not the student's mathematical creativity atrophy and diminish by the end of the fourth-grade year? This hypothetical scenario solicits a solution which requires mathematics teachers at all levels or grades to afford students the opportunities to intentional experiences with mathematical creativity.

Then using the CAMT pre- and post-test data, a paired-samples t test was conducted to evaluate mathematical creativity as to whether the means of the pre-test was significantly different from the post-test. The results indicated that the pre-test sample mean for mathematical creativity (M = 35.13, SD = 10.56) was significantly different from the post-test sample (M = 40.24, SD = 11.42), $t(31) = 19.99$, $p < .01$. The effect size of d was 3.53. The 99% confidence interval for mathematical creativity mean ranged from 30.01 to 40.24 on the pre-test and 34.81 to 45.88 on the post-test.

As detailed earlier, Sriraman [21] espoused that mathematical creativity and giftedness can be harmonized at the K-12 level using five principles: The Uncertainty Principle, The Scholarly Principle, The Free Market Principle, The Gestalt Principle, and The Aesthetic Principle. The current study suggested that a punctuated, intentional experience with mathematical creativity (problem posing, divergent thinking, alternative algorithms, and invented strategies), corresponds to many of Sriraman's five principles. By fostering an environment that tolerates ambiguity through open-ended or ill-posed problems the Uncertainty Principle is supported. The Scholarly Principle states that creativity challenges existing trains of thoughts and extends current knowledge. Creativity thrives where risk-taking is encouraged and atypical or divergent thinking is promoted. The Gestalt Principle contends that with the freedom of time and movement creativity can flourish. This is the Free Market Principle. To behold solutions, methods, problems, or ways of thinking as objects of beauty is the Aesthetic Principle. In part, this study has maintained these five principles, and the data suggests that mathematical creativity has been developed.

10.2. What effect does a punctuated, intentional experience with mathematical creativity have on elementary pre-service teachers' mathematical beliefs?

In general, beliefs are hard to change [42], [43], [44]. Nevertheless, beliefs can be permeated under certain conditions. As Murphy and Mason stated [42, page 311], "although changes [in beliefs] can occur by chance, serendipity, [or] without awareness, only high levels of cognitive, metacognitive, and motivational engagement lead to deeper and longer lasting change." In the present study, beliefs about mathematics did change. At the commencement of the study, the students mean average score was 150.19 on a scale ranging from 40 to 240. At the conclusion of the study, the mean average score of the students was 185.63. This result suggests that a punctuated, intentional experience with mathematical creativity affects elementary pre-service teachers' mathematical beliefs. Analogous to this finding, Hart [45] considered problem situation or dilemmas to change beliefs while examining alternative and invented algorithms. Silver [46, page 22] claimed that "one needs to understand the activities or practice of persons who are makers of mathematics." Could it be that understanding the cognitive process to differing methods to mathematical problems is perhaps a genesis or catalyst to changing the beliefs that mathematics is not fixed but fluid in nature?

10.3. What relationship exists between elementary pre-service teachers' mathematical creativity and their mathematical beliefs?

Although the results of Schommer-Aikins, Duell, and Hutter [47] suggested that beliefs factor into problem-solving performance, in this current study beliefs were not directly linked to mathematical creativity, which is sometimes viewed as a subcategory of problem solving. In a qualitative study, Sriraman [20] found that beliefs regarding the nature of mathematics played a role into how mathematical creativity was intricately involved. Even though the pre-service teachers in the present study increased their fluid or informal view of mathematics, it did not correlate with mathematical creativity. While the two variables that were measured correlated at the beginning and end of the study, neither of the correlations were significant.

10.4. What relationship exists between elementary pre-service teachers' mathematical creativity and mathematics anxiety?

Haylock [48] confirmed his hypothesis that the highly mathematically creative students would have lower anxiety compared to their contemporaries. On the contrary, the present study could neither confirm nor deny such claims. The data did not prove to be significant. Differing instruments (for measuring mathematical creativity and anxiety were used) and sample populations, however, were considered in both studies. In spite of that, the current study suggested that a punctuated, intentional experience with mathematical creativity lowered mathematical anxiety. It should be noted that for the post-test, mathematical creativity and anxiety possessed a slight correlation ($r = -0.292$ and $p = 0.052$). This finding suggests that higher mathematical creativity scores correlate with lower mathematical anxiety scores, although it is not significant.

11. Conclusion

The expectation was sustained by the data that a punctuated, intentional experience with mathematical creativity would foster mathematical creativity. It was predicted and supported by this initial study that a punctuated, intentional experience with mathematical creativity would affect pre-service teachers in viewing mathematics as more fluid than fixed. In addition, the data suggested that a punctuated, intentional experience with mathematical creativity lowers mathematical anxiety. However, it was not substantiated that either mathematical beliefs or mathematical anxiety predicted mathematical creativity as conjectured by the researcher.

The findings of this study point to three particular implications. First, punctuated, intentional experience with mathematical creativity appears to change the perspective of pre-service teachers' beliefs about mathematics, which aligns with NCTM's [2] vision for mathematics. Second, if a punctuated, intentional experience with mathematical creativity fosters mathematical creativity, then conceivably this is a means to maintain and develop mathematical capital and prowess. Third, although mathematical anxiety appears to be pandemic for nearly all levels of mathematics classes, a punctuated, intentional experience with mathematical creativity may, in part, lower mathematical anxiety for some at these levels.

In conclusion, intentional experiences with mathematical creativity provide hope. It potentiates change in the mind that mathematics is, in fact, a creative endeavor. Therefore, mathematics can and should be approached with alternative algorithms, differing representations, invented strategies, problem posing, and multiple methods based upon the learner's prior knowledge and experiences. Transforming mathematical beliefs into a problem-solving paradigm that coincides with NCTM's vision may also provide an avenue to alleviate mathematical anxiety. Could it be that mathematical capital and prowess is harvested through these intentional experiences with mathematical creativity? How might the mathematics educators of today change the course of mathematical learning for the next generation? This study has suggested that there can be hope to foster mathematical creativity, beliefs can be bent toward NCTM's vision for the mathematics classroom, and mathematics anxiety can be alleviated.

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