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A Study of Problem Posing as a Means to Help Mathematics Teachers Foster Creativity

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Abstract

Teaching to develop creativity often requires a shift in instructional tasks. In this paper, we first summarize the body of research related to instructors facilitating and recognizing mathematical creativity. We then provide details as to how one graduate course, designed to help mathematics educators develop a sense of school mathematics from an advanced standpoint, provided opportunities for students to: recognize the difference between problems and exercises, pose problems, reflect on the quality of the tasks they created and review tasks created by others. This series of activities were designed to help the graduate students recognize and appreciate mathematical creativity. We then review the instructional activities in light of the five overarching principles to maximize creativity in K-12 mathematics classrooms suggested by Sriraman [36] and discuss how these might relate to the post-secondary and graduate education of mathematics educators.

Keywords: creativity, problem posing, mathematics teacher education, task generation.

> ... to promote a classroom situation where creative problem solving is the central focus, the practitioner must become skillful in discovering and correctly posing problems [1, page 1].
>
> –Reda Abu-Elwan

1. Introduction

Research suggests that mathematical creativity often results from extended periods of mathematical activity and reflection based on the use of deep and flexible content knowledge [14, 15]. This implies that instruction can influence creativity. However, for teaching to foster creativity in mathematics, there should be purposefully designed instructional tasks. It is doubtful that routine, mechanical exercises would foster creativity. Moreover, mathematical creativity may neither be explicitly promoted, nor fully appreciated, by students when a learning space involves only problem solving, even if the problems are challenging and engaging. For students to get an authentic sense of mathematics and to develop habits that are more likely to lead to an appreciation of mathematical creativity, they need to experience both problem solving and problem posing, as both are "essential aspects of mathematical activity" [22, page 31].

2. Framing the Study

2.1. Problem Posing

Problem posing allows for the creation, adaptation, and application of new mathematical knowledge to fresh situations and contexts [20]. Maybe most importantly, it encourages the self-monitoring and reflecting that are crucial to the development of good problem solvers. Problem posing should be both a means and an object of instruction in mathematical classrooms [18]. Problem posing encourages students to move beyond general assumptions and conclusions [28]. It allows students to test mathematical boundaries and explore mathematical ideas and relationships. In short, experiences that allow students to create and solve their own problems can foster mathematical creativity [24]. Problem posing is an integral skill for mathematics instructors and has been referred to as an "agent of change" in mathematics classrooms [12]. However, it is often only given lip service, even though there is recognition for its role in both teacher education programs and in school curricula [11].

Moreover, there is a lack of research on problem posing, especially related to mathematical instruction and mathematics instructors [33].

2.2. Teaching for Creativity

Creativity is an idea that is used widely but often in a vague or obscure manner [16]. It is typically thought of as something that results in insightful or unusual solutions to problems [2], but numerous definitions for creativity exist [38], since it is a complex construct. As other educational researchers have done, in this paper we will use the idea of *relative* creativity [19, 23]. More specifically, this study will focus on the aspect of relative creativity where "a person may create something that is new to him/her or to his/her peers in a given subculture, but it may not be new to the community of more knowledgeable others" [34, page 27].

One of the key responsibilities of mathematics teachers is to provide meaningful mathematical tasks in the classroom [13]. A starting point for teachers to encourage mathematics creativity may occur by prompting students to "pay attention to their wonderings... [and to] capture their ideas and build on them" [37, page 144]. However, neither the use of ill-posed and open-ended problems nor providing learners independence and prolonged periods of engagement with such problems is common in mathematics classrooms [36].

Silver [35] and Lev-Zamir and Leikin [21] suggest that ill-structured and open-ended tasks may encourage creativity. These types of problems can allow exploration and the use of a variety of methods and even interpretations, which may result in different solutions. While tasks can highlight novelty by involving either unexpected methods or solutions, it is also possible to ask students to generate as many different, but related, problems as possible to a given situation. To help foster students' mathematical creativity, teachers should encourage an exploration of additional methods to solve the problems.

Starko [37] delineates between "creative teaching" and "teaching to develop creativity," the latter meaning to enhance students' creativity [20] Aiken [2] proposed that teachers are key to unlocking creativity in the classroom. Researchers have found that mathematics teachers concur and consider themselves primarily responsible for fostering creativity in their students [5, 17, 20]. Discordantly, studies examining teachers' conceptions of creativity [5, 16, 20, 21] have reported that prospective mathematics teachers have a limited understanding of creativity and how to encourage it in the classroom [5], even though it is something that can, and should, be made part of teacher education programs [40, 41]. Moreover, few teachers feel trained in fostering students' creativity [16]. In this paper, we examine the tasks prospective and practicing mathematics instructors generated and solved in a graduate mathematics education course that first differentiated between problems and exercises and then followed with weekly problem posing requirements to see if the tasks generated were likely to foster creativity.

3. Methods

3.1. Context

The data for the study came from homework submissions in an online graduate mathematics education course designed to help secondary instructors consider school mathematics from an advanced standpoint. Specifically, the course addressed teachers' horizon content knowledge [4] on topics typically associated with linear functions and slope in secondary mathematics. Note that the prospective and practicing mathematics instructors in the course are referred to as "students" in this paper, and each is assigned a letter (e.g., Student A). At the beginning of the course, students engaged in a discussion forum to address literature regarding the difference between mathematical problems and exercises. Subsequently, the assigned readings that followed were mathematical content-based writings with embedded tasks that the instructor had created.

The weekly readings required students to submit solutions to the embedded tasks in addition to generating two problems (not exercises) with solutions. The differentiation made between problems and exercises in the course was that exercises are tasks for which the solution path, even if tedious and lengthy, is known, and problems are tasks for which the solution path was not immediately obvious. The content of the course started with common ideas and relationships in a two-dimensional environment and transitioned into a three-dimensional environment. Consistent with previous studies [10], studying student's engagement with three-dimensional ideas can lead to innovation in students' problem solving; additionally, making the connections between related two- and three-dimensional ideas is something research suggests needs to be more explicit in mathematical instruction [26].

For this study, each reading will be referred to by its number (e.g., reading 1); additional information regarding the readings, including examples of embedded tasks in the readings, is given in Table 1.

The readings were designed to facilitate prospective and practicing teachers' recognition and appreciation of mathematics as being more than solving exercises that require simply repeating memorized procedures. This often happens with the topic of slope [9]. The technique in the readings' examples was to model problem posing techniques, such as using the what-if-not strategy [7, 6]. This highlighted mathematical ideas and relationships related to the familiar topic of slope. Then, students were asked to generate tasks, so they would recognize how the concept of slope permeates through mathematics [30, 32, 31].

The data for this study comes from the eight assignments where students were to pose (and solve) two problems related to eight different course readings, each of which contained embedded, instructor-created tasks. Table 1 on the next page outlines the weekly topics covered and provides representative examples of the instructor-created tasks that were embedded in each reading.

3.2. Coders and Data Set

Two of the students in the course (the second and the third author) were selected by the instructor (the first author) to work as part of the research team with her after the conclusion of the course. The two student-researchers were selected on the quality of their submissions on all tasks throughout the course to serve as coders. The instructor worked with the two coders on how to utilize the coding schemes described in the next section. Since neither coder had access to other students' posed problems, the course instructor de-identified all submissions, eliminated the two coders' submissions, and used the remaining submissions as the original data set.

The course consisted of 25 graduate mathematics education students. Each held the equivalent of a Bachelor's degree in Mathematics. The two coders previewed the data for the other 23 students in the course by scrutinizing the submissions associated with the first, second, and final readings. The instructor-researcher reviewed the two sets of notes created by the two coders, and the entire research team decided to eliminate data from students who a) had submitted exercises rather than problems; b) were missing any of these three submissions; c) made multiple, substantial mathematical errors in these

Table 1: Reading Topics with Examples of Embedded Tasks in Readings.

Reading Number	Reading Topics	Example of Embedded, Instructor-Created Task in Reading
1	Basic Figures and Distance in Two Dimensions	Consider the segment connected by the points (x_1, y_1) and (x_2, y_2) where x_1, x_2, $y_1 > 0$ and $y_2 < 0$. The midpoint of this segment could fall in which quadrants? Explain the conditions necessary for each of the possible quadrants.
2	Basic Figures and Distance in Three Dimensions	Would a single point and a fixed distance determine a unique segment in 2-space or 3-space like it does in 1-space when given the length of the segment and location of its midpoint? Explain your answer.
3	Geometric Intersections in Two and Three Dimensions	When two lines intersect, their intersection is a point. What geometric figure is the intersection of two planes? List all possible figures considering all possible intersections of three planes.
4	Linear Equations across Dimensions	Graph the following equations on the same xy-plane: $x - y = 1$, $x + y = 1$, $-x - y = 1$, and $-x + y = 1$. Then graph the equation $\|x\| + \|y\| = 1$ on a second xy-plane. Explain how the two graphs are similar and how they are different. Explain why these similarities and differences occur.
5	Intersecting Planes in Three Dimensions	Consider the line defined by the intersection $x + 2z = 4$ and $y = 2$. If the line exists, describe it including all its intercepts and determine the general form for any point on the line in terms of just the variable x.
6	Slope in Three Dimensions	Given a plane with $mx = 2$ and $my = -1$, how would you describe the slope in the direction when x increases by 2 and y increases by 3? If the point $(0, 0, 3)$ were part of the plane, find two other points that would also be part of this plane.
7	Slope Revisited - Directional and Partial Derivatives	You woke up with a wind of 5 mph and a temperature of 40°F. So, it felt like 36°F using the wind chill chart (which was provided). By noon, there were changes in temperature and wind, and the wind chill was then −4°F. If you recorded the initial information using the ordered triple $(5, 40, 36)$, determine the other ordered triples for the three scenarios where the wind chill is −4°F and explain the changes that would have occurred in each scenario to go from feeling like 36°F when you woke up to feeling like −4°F.
8	Parallel Figures	Would you consider concentric circles parallel? What about concentric spheres? Explain your reasoning for each.

submissions; or d) had failed to submit solutions for any of the posed problems for these submissions. To achieve a more focused sample of current and future high school mathematics teachers, the instructor-researcher removed the remaining three students from the reduced data set who worked at the postsecondary level. This resulted in utilizing the data from 13 students for the final data set. The unit of analysis was a single submission, which included two posed problems and their solutions. While there should have been 8 submissions from 13 students for 104 submissions that were analyzed, one student (Student J) only made 7 submissions. Therefore, the final data set contained 103 units that were used for analysis.

3.3. Data Coding

The two coders jointly analyzed the 103 submissions using two coding schemes described below. The coders reviewed all problems and solutions in order to assign codes to the submissions. Since each submission included two posed problems and solutions, the coders assigned the inclusive set of codes for both problems to the submission. In rare cases of disagreement or uncertainty, the instructor-researcher intervened until consensus was reached.

The first coding scheme was based on the initial work of Sriraman [36] that was slightly revised by Moore-Russo and Demler [27] to include six principles to facilitate creativity. Each submission was assigned all of the following facilitation of creativity codes that the two coders thought were evidenced: *gestalt-related*, *aesthetic-encouraging*, *risk-allowing*, *boundary-pushing*, *uncertainty-tolerating*, and *alternative-angling*. A brief description of each code is outlined in Table 2. Each submission was also coded using Silver's [35] three requirements for tasks that are commonly associated with creativity. Unlike the facilitation of creativity principles, there were no instances in which a submission was assigned two task requirement codes. Outlined in Table 3 are brief descriptions of each of the task requirement codes, which include *fluency*, *flexibility* and *novelty.*

4. Results

In this section we present the results of the analysis of student submissions on two levels. We present trends and patterns observable across all the 103 submissions; we then examine the data at the level of the individual students.

Table 2: Facilitation of Creativity Codes from Sriraman [36] and Moore-Russo and Demler [27].

Facilitation of Creativity Principle	Code Description
Gestalt-related	requires significant time, energy, effort, persistence; type of task students need to "chew on"
Aesthetic-encouraging	provides space for work that would be deemed fun, beautiful, aesthetically pleasing
Risk-allowing	explicitly encourages students to take risks or to use atypical thinking
Boundary-pushing	helps students push boundaries of what should be known or what has been considered in class; must be based on foundational ideas in class and might only be pushing the student's or class's boundaries
Uncertainty-tolerating	purposely involves open-ended, messy, ill-defined ideas or communication that force students to tolerate ambiguity and uncertainty
Alternative-angling	explicitly asks students to think about the problem/idea at hand in a slightly different, nontrivial way than material or other tasks presented in class or previously experienced

Table 3: Three Codes for Task Requirements adapted from Silver [35].

Task Requirement	Code Description
Fluency	explicitly requires generating a number of solutions or ideas; purposed or obvious expectation for multiplicity in ways that task is completed or communicated
Flexibility	requires shift in approach or method used to solve task from what is traditionally done/expected or what has been previously experienced
Novelty	explicitly asks for solutions involving original, unique, or unexpected methods and solutions

4.1. Facilitation of Creativity

4.1.1. Analysis of Entire Data Set

As illustrated in Table 4, the most frequently assigned code for all 103 submissions in the facilitation of creativity framework was *boundary-pushing.* This held true for the student submissions associated with all eight readings, in both the early and later weeks of the course. For each student in the course, the submissions he or she made were more likely to facilitate creativity through *boundary-pushing.* Total counts on the assigned codes show that problems posed by students in the course were assigned the code of *boundary-pushing* more than twice as often as any other facilitation of creativity principle code. For example, the following problem submitted from reading 5 by Student B is representative of the student submissions that were coded as *boundary-pushing.*

> We are given two lines M and N. Any point on line N can be determined by $(3+t, 3+3t, 4-t)$, and any point on line M can be determined by $(2-u, 1-2u, 6+2u)$. Determine whether these lines would be parallel, intersect or skew? Show a graph to prove your solution (*Student B, Reading 5 submission*)

In this submission, Student B utilized an algebraic representation of a line that was developed through the tasks embedded in reading 5. The problem posed by Student B required using this algebraic representation to explore properties of lines previously described only from a geometric perspective.

A distant second, *alternative-angling* was the next most common facilitation of creativity principle code assigned. With the exception of the first assignment, at least one student submission was coded as including *alternative-angling.* For example, the following problem submitted from reading 2 by Student G was assigned the code as *alternative-angling.*

> Think of the classroom as part of the first octant. Describe in words the new location of the classroom if the classroom is reflected about the floor. Would the students and instructor still be facing one another? (*Student G, Reading 2 submission*)

Reading 2 and the embedded tasks required students to explore the octants in three-dimensional space by experimenting with the ways in which objects and orientation worked for reflections. This submission was assigned a code of *alternative-angling* because it involved a transformation that required considering the relative position of objects in a slightly different, nontrivial way.

All other facilitation of creativity principles were rarely noted. Only three submissions were described *as gestalt-related*, and there were only two submissions for the *uncertainty-tolerating* principle. It is noteworthy that no student submission in this sample data set was identified as either *risk-tolerating* or as *aesthetic-encouraging*. The following submission from reading 6 by Student L is one of two submissions coded as *uncertainty-tolerating*.

> Given the directional slope of a line in 3-space, where the rise is with respect to the z-axis and the run is with respect to the xy-plane, find m_x, m_y, and three points that would fall on a line with the given directional slope of $13/\sqrt{17}$. (*Student L, Reading 6 submission*)

The problem was assigned a code of *uncertainty-tolerating* because of the ambiguity inherent in the problem. Crafting a solution requires making a series of minimally guided choices that become part of the problem and influence the eventual solution.

Table 4: Frequency of the Facilitation of Creativity Principles in Student-Generated Problems.

Submission-Associated Reading	**Facilitation of Creativity Principles**					
	Boundary-pushing	Alternative-Angling	Gestalt-Related	Risk-Allowing	Uncertainty-Tolerating	Aesthetic-Encouraging
Reading 1	5	0	0	0	0	0
Reading 2	6	5	0	0	0	0
Reading 3	6	2	0	0	1	0
Reading 4	3	1	0	0	0	0
Reading 5	5	2	1	0	0	0
Reading 6	5	2	0	0	1	0
Reading 7	7	1	1	0	0	0
Reading 8	5	4	1	0	0	0

Despite the dominance of *boundary-pushing* as the principle most leveraged to facilitate creativity in the problems students submitted, the data show a slight increase in the variety of the facilitation of creativity principles used by students in their later submissions. Beyond this observation, there are few temporal trends in the data set across readings.

4.1.2. Analysis by Student

The data reveal differences in patterns of the distribution of facilitation of creativity principle codes in students' submissions as displayed in Table 5. In general, most students whose submissions evidenced use of facilitation of creativity principles in the tasks they generated did so across at least half of their submissions.

Table 5: Counts for Types of Facilitation of Creativity Principles in Student Submissions.

Student	Reading 1	Reading 2	Reading 3	Reading 4	Reading 5	Reading 6	Reading 7	Reading 8
A	1	1	1	0	0	0	1	0
B	1	0	1	0	1	0	0	1
C	0	1	2	1	0	0	1	1
D	0	0	0	0	0	1	1	1
E	0	1	0	0	1	1	2	1
F	0	3	0	1	0	2	0	1
G	1	1	1	0	0	1	0	1
H	1	1	1	0	2	1	1	0
I	0	0	0	0	1	0	0	0
J	1	2	0	0	-	0	0	2
K	0	1	1	1	2	1	1	1
L	1	0	1	0	0	1	1	0
M	0	1	0	1	1	0	1	1
Totals	6	12	8	4	8	8	9	10

Student K stands out as having a code assigned for all submissions except for the first submission (associated with reading 1). By contrast, Student I has only a single code that was assigned for the submission associated with reading 5. Student F had fewer submissions coded for creativity than Student K, but more submissions by Student F evidenced multiple elements of creativity facilitation.

For example, the submission by Student F for reading 2 was identified as *boundary-pushing*, *alternative-angling*, and *uncertainty-tolerating*. In this submission Student F provided two vertices of a triangle in three dimensions and asked for the coordinates of a third vertex that would result in a right triangle. The submission was coded as *alternative-angling* because rather than posing questions about a given figure, the problem required generating a figure. While students were familiar with triangles in the xy-plane, they had little experience embedding these shapes in three dimensions. Visualizing possible triangles in three dimensions with minimal information required an exploration that built upon students' foundational knowledge; therefore, this submission also warranted the code *boundary pushing*. This problem differed from other submissions involving coordinate geometry in its lack of specificity, and that is what justified the code of *uncertainty-tolerating*. The problem stem did not indicate whether the given vertices were the endpoints of the hypotenuse or a leg; therefore, the location of the right angle was ambiguous. Locating a possible third vertex required recognizing that ambiguity, selecting a role for the given segment (as being the triangle's hypotenuse or not), and pursing a method of solution based upon that selection.

The data also suggest change in specific students over time. Submissions sets from Student E showed more elements of creativity in the later submissions; five of the six codes were assigned to submissions associated with the last four readings. Also of note is that Student D's first five submissions did not have any codes assigned, but the last three did.

4.1.3. Analysis by Associated Reading

The data suggest that the presence of facilitation of creativity principles in student submissions may be linked to the specific topic presented in the reading that had been assigned. For the submissions associated with readings 2 and 8, a greater number of students' individual submissions evidenced some facilitation of creativity principle. In student submissions linked to reading 2, 12 codes were assigned to 9 different students, and in student submissions linked to reading 8, 10 codes were assigned to 8 students. The student submissions from these readings showed a greater variety of facilitation of creativity principles than did other readings. In most submission sets, only one or two students were coded with anything other than *boundary-pushing*; however, in sets 2 and 8, half of the codes were identified as facilitation of creativity principles other than *boundary-pushing*. At the other extreme, in student submissions linked to reading 4, only 4 student submissions were coded as having facilitation of creativity principles, and each submission was assigned only a single code.

4.2. Requirements of Tasks

4.2.1. Analysis of Entire Data Set

When considering the data set using task requirement elements, *flexibility* dominated the coding of non-routine problems posed by the students in the course. As noted in Table 6, *flexibility* was evident in a total of 46 submissions, whereas *novelty* was assigned only 6 times. Only a single student submission was coded as requiring *fluency*. We offer the following example submitted by Student A in response to reading 1 as illustrative of the code *flexibility*.

> Let Point M be the midpoint of Segment AB and let M also be the midpoint of segment CD. Points A, B, C and D are unique points.
>
> A. Draw a possible figure to represent segments AB and CD and midpoint M
>
> B. Is it always true that segment AC is equal and congruent to segment BD?
>
> C. Show one or more examples and write a formal proof if congruence can be shown (*Student A, Reading 1 submission*)

Because this problem is associated with a reading focused on points and distance in the coordinate plane, in the submitted solution to the problem Student A reasonably presumed that the segments and points would be drawn in the coordinate plane using the concepts and tools developed in the reading. However, the problem exemplifies a shift in approach because it directs the solver to confirm that two segments are the same length through geometric proof instead of using the algebraic methods of coordinate geometry emphasized in the reading.

The temporal patterns for the task requirement codes are consistent with those observed when examining the data set for facilitation of creativity principles. While there was no overall pattern of increase in the number of student written problems whose solutions necessitated any of the three task requirements, there was greater variety of requirements coded in later submissions. In each of the last four assignments, one student's submission was coded for *novelty*, and the only coding for *fluency* was assigned in the seventh assignment. The submission that evidenced *fluency* was unique among the sample data in that it explicitly required describing the contextualized meaning of a function across graphical, algebraic, and tabular representations. The tabular representation appeared in reading 7 as data on temperature, wind speed, and wind chill. The submission from Student K required inferring information from the graph of the function in three dimensions in the context of windchill and connecting it to the data table and the embedded tasks within the associated reading.

Table 6: Frequency of the Task Facilitation Codes in Student-Generated Problems.

Submission-Associated Reading	**Task Requirement Elements**		
	Flexibility	Fluency	Novelty
Reading 1	6	0	0
Reading 2	7	0	1
Reading 3	7	0	0
Reading 4	5	0	0
Reading 5	4	0	1
Reading 6	8	0	1
Reading 7	5	1	1
Reading 8	4	0	2

As with the coding for facilitation of creativity principles, some patterns are associated with particular course readings. The submissions associated with readings 2 and 6 stand out as meriting the greatest number of task requirements noted for 8 and 9 individual student submissions, respectively. Additionally, the submissions associated with both these readings included the coding for *novelty*. The submissions associated with readings 4 and 5 were assigned fewer codes than other sections, but the gap between these sections and the sections with more numerous codes was less striking than for the facilitation of creativity principles coding.

4.2.2. Analysis by Student

At the student level of analysis, there was no evidence of any individual student's evolution throughout the course. There were students whose submissions consistently warranted coding for task requirements as can be noted in Table 7. For example, Student M posed problems that received a task requirement code in 7 of the 8 submissions, and Student G's efforts were assigned a code on 6 out of the 8 submissions. At the other extreme, Student D had only a single code assigned for task requirement for submission series.

Table 7: Counts for Types of Facilitation of Creativity Principles in Student Submissions.

Student	Reading 1	Reading 2	Reading 3	Reading 4	Reading 5	Reading 6	Reading 7	Reading 8
A	1	1	1	0	0	0	1	1
B	1	0	0	0	1	1	0	1
C	0	1	1	1	0	0	1	1
D	0	0	0	0	0	0	1	0
E	0	1	1	0	0	1	1	0
F	0	1	1	1	0	1	0	1
G	1	1	0	0	0	1	1	1
H	0	1	0	0	0	1	1	0
I	0	0	1	1	1	0	0	0
J	1	1	0	0	0	1	0	0
K	0	0	1	1	1	1	1	0
L	1	0	0	0	1	1	0	0
M	1	1	1	1	1	1	0	1
Totals	6	8	7	5	5	9	7	6

4.2.3. Analysis by Associated Reading

Although the range of code totals of each submission set is smaller in the task requirement scheme than with the facilitation of creativity principle scheme, the data suggest the possibility of a relationship between the presence of task requirement elements and the topic in the associated reading. With 8 and 9 task requirement codes respectively, the submissions associated with readings 2 and 6 stand out as having the greatest number of total codes related to task requirement. Because each student submission was assigned at most one task requirement code, this means that readings 2 and 6 evidenced the greatest number of students whose submissions earned task requirement codes.

As shown in Table 6, each of these sets of submissions from all students for readings 2 and 6 include a single code for *novelty* with the *flexibility* code occurring most frequently. The submission set associated with reading 7 is noteworthy because it is the only submission set with codes for all three task requirements. The submissions associated with readings 4 and 5 both have the lowest code totals of 5 for task requirement elements. Submissions for reading 4 present no variety in codes; the only task requirement identified in these submissions was *flexibility*.

4.3. Analysis Across Coding Schemes

While we acknowledge that the two coding schemes overlap, analysis of the data at the student level suggests that the two coding schemes captured distinctly different aspects of problem posing. Comparing the coding patterns in Table 8 for individual students offers evidence of the differences between the two coding schemes.

Table 8: Counts Comparisons for Two Coding Frameworks by Student.

Code	Student												
	A	B	C	D	E	F	G	H	I	J	K	L	M
Facilitation of Creativity													
Boundary-Pushing	4	3	4	2	5	3	3	5	0	3	5	3	4
Alternative-Angling	0	1	1	1	0	3	2	0	1	1	2	0	2
Gestalt-Related	0	0	0	0	1	0	0	0	0	1	1	0	0
Uncertainty-Tolerating	0	0	0	1	0	1	0	0	0	0	0	1	0
Risk-Allowing	0	0	0	0	0	0	0	0	0	0	0	0	0
Aesthetic-Encouraging	0	0	0	0	0	0	0	0	0	0	0	0	0
Task Requirement													
Fluency	0	0	0	0	0	0	0	0	0	0	1	0	0
Flexibility	3	3	5	1	3	5	5	3	2	3	3	3	6
Novelty	1	1	0	0	1	0	1	0	1	0	0	0	1

Using the facilitation of creativity framework, Students I and K differed significantly. As noted earlier, Student I was assigned only one code, whereas Student K was assigned eight codes, and this student's series of submissions were assigned at least one code for all but one reading. However, in the second coding scheme using the task requirement framework, Students I and K appear more alike and more like typical students with the total number of codes differing only by one. In the facilitation of creativity framework, Student K stands out as having the largest number of codes as well as the highest incidence of codes yet, in the task requirement framework, this student's total of 4 codes positions the student as typical for the class.

Similarly, while the task requirement framework positions Students D and M at the two extremes, the difference is much less striking in the task facilitation framework. Their task facilitation code totals differ by only two.

4.4. Errors in Student Submissions

Some student submissions contained errors in either the problem statement or the presented solution. Some errors involved explicitly false statements or conclusions, while others took the form of incomplete problem statements. For example, one submission neglected to specify the second coordinates for points in two dimensions. Some problem statements were ambiguous due to poor articulation or inexact usage of mathematical language. The most egregious errors were identified during the initial stages of analysis. Students whose submissions contained substantial errors were eliminated from the final sample of 13 included in this study.

Errors were more frequent in the submissions associated with reading 8. Because reading 8 involved extended the concept of parallel to objects other than lines, many students' submissions included problems that required an examination of distance. The most prevalent error in these was the failure to consistently measure distance along a line normal to the curve. This error manifested in different forms. For example Student C provided the graphs of the functions $f(x) = \sqrt{x}$ and $f(x) = \sqrt{x} + 3$, then asked if these graphs are parallel. In the solution Student C concluded the graphs are parallel because they are always separated by distance of 3 units, erroneously equating the vertical shift with the distance between the curves. A few students asserted that various concentric polygons were parallel because the distance between the sides was constant, but they failed to consider how to measure the distance between the vertices of the concentric polygons.

The nature of the error in a submission determined how the error affected coding that submission. If the error occurred in the provided solution, the error often did not seem to alter the coding of the problem. The problem involving the comparison of shifted graphs of the square root function was assigned the codes of *boundary-pushing* in the facilitation of creativity scheme and *flexibility* in the task requirement scheme. Arguably, the type of error in the provided solution lent credence to claim that the problem did push the boundaries of what should be known and required a shift in approach or method. For submissions in which the solution revealed additional assumptions, the submission was coded as if those assumptions had been explicitly stated in the problem stem. When inexact usage of mathematical language obscured the statement of the problem, the coders used the provided solution as suggestive of the correct problem statement. Often the solution included a graph that clarified the problem statement. In the event that the problem stem itself contained an error or false statement, the problem was not assigned a code in either framework. Each student submission included two problems; because students who made substantial errors were eliminated from the sample set, it was rare that both problems contained errors that made coding problematic.

5. Discussion and Implications

As stated in the Methods section (Section 3), this course was intentionally designed to address the students' horizon content knowledge. Ball, Thames, and Phelps define horizon content knowledge as "an awareness of how mathematical topics are related over the span of mathematical topics included in the curriculum" [4, page 403]. The authors contend that mathematics teachers rely upon horizon content knowledge when "extending procedures and concepts... while preserving properties and meaning" [4, page 402]. In this course, students were asked to extend common procedures and concepts in two dimensions to three dimensions. The study explores the tasks prospective and practicing mathematics instructors generated and solved to see if the tasks generated were likely to foster creativity.

5.1. Coding Patterns Linked to Readings

The coding of the data reveals patterns associated with particular readings. For the facilitation of creativity framework, student submissions that were coded as having a greater number of facilitation of creativity principles linked to readings 2 and 8. For the second coding scheme that used the task requirements framework, the submissions associated with readings 2 and 6 emerged as having the highest number of codes. All three readings were based on material in two dimensions that is ubiquitous in high school algebra and geometry curricula. The research team hypothesizes that the students' familiarity and comfort with this material in two dimensions gave them the confidence to take risks when extending the concepts and procedures to three dimensions.

Reading 2 meticulously extended considering basic objects and measuring distance from reading 1's treatment of similar topics in two dimensions to three dimensions. The more deliberate connections made between reading 1 and reading 2 in addition to the additional time spent considering the same types of mathematical ideas seem to have impacted students' abilities to require more in their generated tasks and to incorporate principles that would be more likely to foster creativity in their classrooms. Reading 6 also extended from readings 4 and 5; although the overlap was not as explicitly presented as was done in readings 1 and 2.

The research team believes that the style of the exposition in reading 8 influenced students' problem posing in this section. The text began with three pages of examples demonstrating various meanings of the term parallel in different contexts. There was no additional exposition, but one of the embedded tasks involved rhumb lines, a topic that few students had previously encountered. Despite their lack of familiarity with rhumb lines, many of the students' submissions associated with reading 8 involved rhumb lines. The researchers hypothesize that by presenting an exploration of the term parallel as preparation for solving embedded tasks without providing any examples, the instructor-researcher implicitly encouraged students to engage in exploration in both their completion of the embedded tasks and their own problem posing. Earlier in the paper we note the higher frequency of errors in student submissions associated with reading 8. The difference in reading material style without guiding examples related to the notion of parallel coupled with the introduction of terminology that was new to all the students in the course (i.e., rhumb lines) are both likely reasons for the increase in mathematical mistakes noted in the submissions. However, the research team observed that in the set of submissions for reading 8, students still demonstrated a willingness to take risks in their own problem posing.

5.2. Coding Patterns Linked to Course Structure

As part of the structure of the course, the instructor-researcher required students to post their best submissions to a Hall of Fame forum. The following week, students also completed and posted solutions to review problems compiled by the instructor-researcher based on the problem submissions from previous semesters. The Hall of Fame and the review assignments were posted to a digital discussion board that was available to the entire class, and students were required to examine and comment upon their classmates' submissions. Both coding frameworks suggest that the Hall of Fame assignment and the review assignment may have influenced student submissions in two ways.

The Hall of Fame assignment was concurrent with the submission associated with reading 4. The data show a decrease in codes across both frameworks for submissions associated with reading 4, and it may be that because of the additional Hall of Fame assignment, students invested less time and energy in writing problems for reading 4. This supposition is supported by the fact that the second Hall of Fame was assigned after the submission associated with reading 8, and submission set for reading 8 does not evidence a similar decrease in code assignment.

The data also show an increase in the variety of codes in both coding schemes in submissions sets following the first Hall of Fame and first Review Assignment. It is noteworthy that within the facilitation of creativity framework, the assignment of the code *uncertainty-tolerating* first occurs after these two public assignments. Similarly, within the task requirement framework, the single code for *fluency* first occurs after the Hall of Fame and Review assignments. This suggests that students' may have been influenced by thoughtfully examining and critiquing other students' submissions.

5.3. Implications of Connections Between the Coding Schemes

In both coding schemes, a single code was most prevalent. For the facilitation of creativity framework, *boundary-pushing* occurred twice as often as any other code. In the task requirements framework, *flexibility* dominated the codes assigned. While the two codes are distinct, they may both reflect a common way in which several students approached problem posing. Many students' submissions took the form of a modifying or building off tasks embedded in the reading. Several students made explicit reference to an embedded task in the reading from which they developed their own problems for submission. While many of these submissions did not deviate sufficiently from the embedded tasks to warrant assignment of a code in either scheme, some did. Because these submissions were rooted in the tasks presented in the reading, they most often represented *boundary-pushing* by extending beyond the material presented in the reading, or *flexibility* by modifying the approach used in an embedded task. This observation along with the observation made about the style of reading 8 suggests that students are strongly influenced by the written materials (i.e., the readings) provided in the course.

We hypothesize that the low occurrence of *uncertainty-tolerating* in the facilitation of creativity framework coding is related to the low occurrence of the *fluency* element in the task requirement framework. The coders observe that even when students created a problem stem that would allow for multiple solutions, they often posed a question designed to elicit a particular answer. For example, in a submission associated with reading 2, Student J asked if a right triangle is uniquely determined in three dimensions by specifying the endpoints of the hypotenuse anticipating a yes or no answer. Had the problem stem been phrased slightly differently, it might have generated multiple answers and could have necessitated a description of the solution space. In the solution for this problem, Student J produced two different right triangles with the same hypotenuse of length $\sqrt{45}$, both of which were drawn on a plane, to justify an answer of no. The student argued there could be more than one triangle with hypotenuse $\sqrt{45}$ because the position of the legs could be interchanged. Student J justified this answer using the Pythagorean theorem to erroneously conclude that the legs must have lengths 3 and 6. In the submission, Student J did not acknowledge or expect others to find that there are infinitely many triangles with this hypotenuse because the legs need not have integer lengths. With slight variations in the problem stem and the student's expectations for its solution, this submission would have received the *fluency* and *uncertainty-tolerating* codes.

This practice of building in simplifying assumptions removed the possibility of multiple answers when the potential for uncertainty was present in several student submissions. Some common constraints included working with coplanar points in three dimensions, positioning sides of figures along one of the axes, using points that were the same distance from one of the axes, and only requesting a single example to meet some required condition. These constraints often reduced a complex problem stem that had potential for more facilitation of creativity principles and task requirements to a problem either with a finite number of answers or a rather mundane solution.

5.4. Teacher's Horizon Content Knowledge

While the instructor-researcher was not surprised (due to her years teaching the course), the student-researchers were taken aback by both the mistakes in some of the submissions and the fact that not all students completed every submission. As early as the first stages of the analysis process, it became apparent that many students in the course did not differentiate between exercises and problems. In addition, there were mathematical errors noted in the submissions.

Errors in the students' submissions provides evidence that teachers' horizon content knowledge needs to be addressed. Teacher preparation programs should help future teachers understand the longitudinal coherence of mathematics in order for them to help students form a foundational knowledge that facilitates learning future mathematical ideas. To do this, teachers need a flexible, nuanced understanding of mathematics that reaches beyond what they themselves teach [3].

The prospective and practicing teachers in the course all held the equivalent of a degree in mathematics; therefore, each had successfully completed a number of upper-division mathematics classes. Yet, the discrete, isolated nature often found in post-calculus courses can make it difficult for future secondary mathematics teachers to connect the advanced topics in these courses to what they themselves will teach. For example, teachers should know how a flexible understanding of a topic such as slope leads to understanding derivatives in a calculus class [39] and directional derivatives in a multivariable class [25]. They should also know the role that slope plays in statistics when introducing basic statistical ideas like the line of best fit [29].

In the words of Bruner, "[l]earning should not only take us somewhere; it should allow us later to go further more easily" [8, page 17]. Teachers with horizon content knowledge should grasp how mathematical knowledge is related and connected to more advanced mathematical concepts. While horizon content knowledge was the goal of the course from which the data was drawn, it is evident from the errors made by prospective and practicing teachers in their submissions that this goal is not an automatic "given" that each student in the course achieved to its fullest.

However, the research team recognized that the set of submissions associated with reading 8, even though they had more errors, also marked a set where the student-generated tasks showed some of the greatest potential to facilitate creativity. It seemed possible that the *boundary-pushing* and *alternative-angling* principles that were noted in the submissions actually helped the prospective and future teachers push their own boundaries of what they actually knew by thinking about what is means to be parallel in a different, nontrivial ways that they had never experienced. None of the submissions for reading 8 were coded as *risk-taking*; however, it seems feasible that, at the meta-level, the environment of the course did seem to encourage students to take risks as they engaged in atypical thinking.

6. Limitations, Conclusions, and Future Research

The graduate course in this study began with an emphasis and discussion on the difference between mathematics exercises and problems. This theme reoccurred in the course since students were to consider that distinction when generating and solving problems for their weekly submissions. Because posing and solving problems, rather than completing exercises, is more likely to encourage students to think beyond rehearsed procedures to consider mathematical ideas and connections, it seems likely that the weekly problem posing might encourage creativity as well as accomplish the course goal of extending horizon content knowledge. Results from this study suggest that when the prospective and practicing secondary mathematics teachers are working with familiar material that is deliberately extended in ways they typically have not experienced; they are capable of generating problems that are likely to foster creativity in their own students.

Results also suggest that the course readings and structure impacted the problems that students posed. If either developing teachers' horizon content knowledge or helping teachers better understand how to foster creativity is the goal of a course, then it is important for teacher educators to consider fundamental topics in the secondary mathematics curriculum that teachers know and use, especially topics that extend into other areas of the mathematics curriculum that might come up in later topics. This should help teacher educators consider how to design learning so that the instructional materials and course structure provide the time and an environment that encourages teachers to experiment, take risks, and collectively learn from that experience. Without such experiences, it is unlikely that teachers will feel prepared to provide an environment were students are able to do the same things. Since this study did not note any instances of either *risk-allowing* or *aesthetic-encouraging* in the submissions, more work should be done to consider how teacher education programs provide opportunities for future and practicing teachers to both experiences, and in turn foster, these two principles for facilitating creativity.

One limitation of the study was the absence of explicit information about the students' intentions or goals for their individual submissions. Often the research team was able to infer intent from the provided solutions, but greater clarity on this issue may have allowed for additional coding and, thus, stronger conclusions from the data. Future research might benefit from incorporating task-based student interviews into the course in which students explain the purpose behind their submissions to the researcher-instructor.

Trying to figure out how to foster connections for students as a course moves from topic to topic or as students experience more major transitions across courses, are problems to solve in and of themselves. One area for future research lies in how to study the ways in which the concept of horizon content knowledge is addressed in teacher preparation programs. More specifically studying how horizon content knowledge ties directly to helping future teachers learn ways to foster creativity in their own students should be of interest to those in mathematics teacher education.

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