

Going Beyond Promoting: Preparing Students to Creatively Solve Future Problems

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Cover Page Footnote

All of the authors served on the faculty of the United States Military Academy at West Point in the Department of Mathematical Sciences during varying windows from the Fall 2016 semester through the Spring 2020 semester. They have each taught the Mathematical Modeling and Introduction to Calculus course for a minimum of three semesters. Additionally, each has served in some kind of leadership role for the course and played a part in shaping it into a course that prepares students to creatively solve future problems. The authors would like to thank the Department of Mathematical Sciences at the United States Military Academy at West Point for their support of these efforts.

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Synopsis

While we cannot know what problems the future will bring, we can be almost certain that solving them will require creativity. In this article we describe how our

course, a first-year undergraduate mathematics course, supports creative problem solving. Creative problem solving cannot be learned through a single experience, so we provide our students with a blend of experiences. We discuss how the course structure enables creative problem solving through class instruction, during class activities, during out of class assessments, and during in class assessments. We believe this course structure increases student comfort with solving open-ended and ill-defined problems similar to what they will encounter in the real world.

Whenever a student enters our classrooms they bring with them the potential to solve one of the world's current or next big problems. The thing is, problems – big or small – do not occur in a vacuum. They happen in the unpredictability of the real world. As instructors we need to keep this in mind because without careful planning our classrooms will only simulate problem solving in a vacuum which is largely useless as soon as our students complete their formal educations. However, no matter how carefully we plan as instructors, our classrooms can only simulate the unpredictable real world as it exists within the bounds of our imaginations.

Unfortunately, we do not know what the future looks like or what problems it will bring. This lack of predictability guarantees that a single simulated real-world situation from the depths of our imaginations can not possibly adequately prepare our students for any problems which they may face once they leave the controls of an educational institution. Similarly, multiple repetitions within simulated real-world situations that are analogues of one another can not possibly be enough either. Therefore, we must carefully design our course to do more.

As instructors, we are tasked with the incredibly challenging and exciting task of preparing our students to be problem solvers, leaders, and decision makers in a future world that is beyond the scope of our imaginations. Given such a task all that anyone can expect of us is to do the best that we possibly can. The question is, what is the best that we can do?

First and foremost it seems imperative that we do not allow ourselves to assume that the solutions to problems of the past or present will suffice for any and all future problems. If we instead assume that tomorrow's problems will require new solutions, then we have in essence assumed that developing problem solvers for the future will not be enough – we instead need creative problem solvers.

1. What is Problem Solving?

The Oxford English Dictionary contains a definition of problem solving dating back to 1854. As a noun, problem solving refers to “the action of finding solutions to difficult or complex issues” and as an adjective, problem-solving is “involved in or rated to this activity; that solves problems” [7]. Over the years, the definition of problem solving has perhaps become less clear. In 1994, Lester noted that this has been an issue for the field of research related to problem solving since at least 1969 [5]. Consequently, we will make use of the Oxford English Dictionary definition throughout this paper.

Of course, as mathematics instructors, we are primarily concerned with mathematical problem solving. As a subset of the broader problem solving, we define mathematical problem solving as problem solving using the tools of mathematics. Speaking specifically of mathematical problem solving Lester notes that knowledge, control, beliefs, and sociocultural contexts all contribute to problem solving performance in interdependent ways [5].

The Association of American Colleges and Universities defines creative thinking as “both the capacity to combine or synthesize existing ideas, images, or expertise in original ways and the experience of thinking, reacting, and working in an imaginative way characterized by a high degree of innovation, divergent thinking, and risk taking” [1]. Considering this definition in conjunction with Lester’s list of factors contributing to problem solving, we see creative problem solving in the context of mathematics as significantly different than problem solving in mathematics. Creative problem solving is solving problems in new and innovative ways, it results in the types of solutions that make the reader stand back and think “Oh, I’ve never thought of it that way.”

Perhaps all problems do not need to be solved in creative ways. However, “complex, novel, ill-defined problems where solutions of quality, originality, and elegance are valued” require creative solutions [9, page 335]. Further, “creative problem solutions must often be formulated in real-world settings” [9, page 335]. Since we as instructors view it as our job to develop the real-world problem solvers of the future, it is clear that we must strive to design our course in such a way as to develop our students as creative problem solvers.

2. Designing A Course That Does More

The first mathematics course that most first-year students take at the United States Military Academy is Mathematical Modeling and Introduction to Calculus, which meets daily for one semester. We have students take a mathematical modeling class first because mathematical modeling is the process through which real-world problems are solved. If our students are going to become the problem solvers, leaders, and decision makers of tomorrow, we want to expose them to the messiness of the real world from the very beginning of their studies so that they will always have an eye towards that as they progress through their studies.

Moreover, we purposefully design our course in such a way with a variety of components in order to not only promote creative problem solving, but to develop our students as creative problem solvers ready to tackle tomorrow's real-world problems. The design of our course begins with extensive professional development for approximately 20 instructors teaching the course each fall. This includes both a six-week summer training program to prepare for the semester as well as weekly meetings throughout the semester. While some of this training is in place to make sure new instructors are ready to step in front of a class when the semester starts, it serves a dual purpose in also allowing the course leadership to help all of the new instructors develop an appreciation for creative problem solving.

So how can this appreciation be achieved? Quite simply, by experiencing 20 different perspectives of problem solving! During the summer program, all instructors will have three opportunities to plan and teach a lesson from the course to all of their peers. Instructors are given three to four lesson objectives but are free to design and teach the lesson however they see fit. The result of this freedom is an array of creative and unique approaches to problem solving that every instructor can participate in. Whereas some instructors approach problems algebraically, others will approach visually or analytically. Some instructors levy technology or animation, whereas others rely on discussion or hands on learning. Whatever the methods used, 20 different instructors will come up with 20 different lesson plans to achieve the same lesson objectives, and all who participate benefit from the diverse creativity displayed.

Further, through the weekly meetings, instructors discuss creative lesson

plans that went well (or not so well) within their classrooms. Often, leadership will discuss creative lesson ideas for upcoming lessons that have been used in the past. With full freedom to design approaches to problem solving that work for each individual instructor, the key idea with both summer training and weekly meetings is to gain an appreciation of creative problem solving through exposure to multiple approaches to problem solving. We want all of the instructors to gain this appreciation because Lester's thorough review of the literature reveals that "for students to benefit from instruction [on problem solving] they need to believe that their instructor thinks problem solving is important" [5, page 666].

Beyond its instructors, the design of our course effectively allows for creative problem solving throughout in order to provide opportunities for creative thought in the classroom that may lead to comfort for future real-world endeavors [3]. In our course, we prepare our students to be creative problem solvers by providing a structure for creative thought, and then letting the students explore in carefully designed, open-ended problems with multiple acceptable answers. These problems range from in-class exercises, to projects, to in-class graded events. We also have students explore a wide breadth of problems as well as explore problems in-depth with repeated solution methods.

3. Structure for Creative Problem Solving

While it may seem counter-intuitive to provide structure to promote creativity, structure and organization have been found to be connected to successful problem solving [4]. Additionally, we have found that while some students thrive in a mathematics classroom environment where creativity is encouraged and rewarded, many more will flounder because being creative can be uncomfortable. By providing a widely applicable structure, provided in Figure 1, in the spirit of Polya's *How to Solve It* [10], we are providing a scaffolding for creative problem solving which functions in two ways. Firstly, it helps students who are comfortable jumping into creative approaches a way to harness – without discouraging – their creativity so that it can be used effectively in solving real-world mathematics problems. Secondly, it requires students who are more hesitant to get creative to take small, structured steps towards getting creative by providing a process to follow which requires a bit of creative thought about the problem along the way.

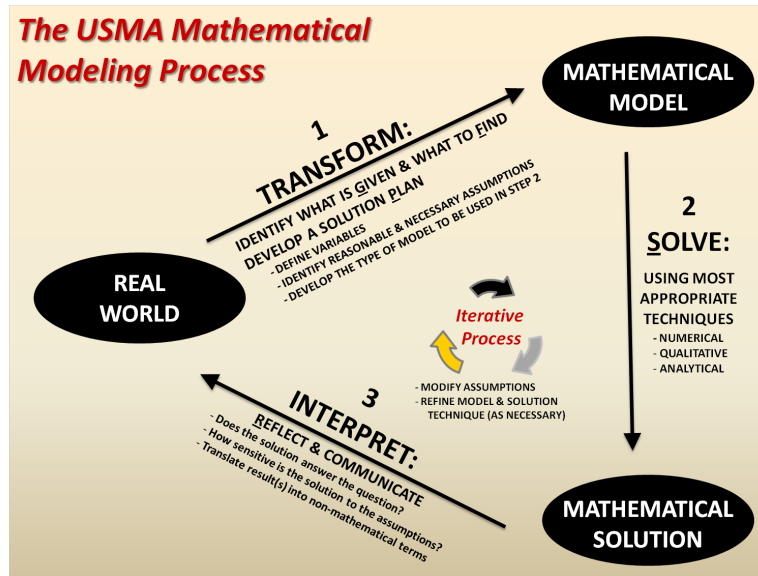


Figure 1: The USMA Mathematical Modeling Process.

The first step of the USMA Mathematical Modeling Process is the Transform step. In this step students take a real-world problem and transform it into a mathematical model. To do so, students identify the given information, what they are attempting to find, and develop a solution plan. Their solution plan should consist of defined variables, reasonable and necessary assumptions, and finally their model. In this step, students can potentially be creative in the type of model that they use, and must be creative in their assumptions.

In the solve step, students use appropriate mathematical techniques and the model they developed to find a numerical answer to the problem they posed in the transform step. These techniques may be numerical, qualitative, and/or analytical in nature. When people think about mathematics, this is often what they think of because this is the step that is most often practiced in formal education. Consequently, this is the step that our students are most comfortable with. It is also the one that requires the least creativity.

The third step of the USMA Mathematical Modeling Process is the Interpret step. In this step students reflect and communicate the results of the Solve step in non-mathematical terms. Doing so translates our solution back into the real-world context. Students must also determine if their mathematical solution makes sense and truly solves the original problem posed.

In doing so, they should consider how closely their mathematical model reflects the real world and then think about the impacts of any assumptions they made. If the model does not capture all elements of the real world, students are encouraged to reiterate through the modeling process to improve their model as many times as necessary. This step requires creativity in their interpretation of the results, recasting the mathematical solution in an improved form for all to understand.

We design our course so that the first week of class is primarily devoted to introducing the USMA Mathematical Modeling Process. We do this because “most students benefit greatly from planned problem-solving instruction” [5, page 666]. However, despite this imposed structure, solving real-world problems creatively with mathematics can be an intimidating process, especially for a first-semester college student. Many students at this level have never been asked to be creative in their mathematics courses and have never thought of solving mathematics problems as anything more than an algorithmic process. Without any further guidance and encouragement to engage with the structure, many students will fail to risk taking the step into creativity required to solve open-ended or ill-defined problems mathematically. Therefore, noting that “teaching students *about* problem-solving strategies and heuristics and phases of problem solving [like Polya] does little to improve students’ ability to solve mathematics problems in general,” we instead teach via the USMA Mathematical Modeling Process [5, page 666].

4. Open-Ended and Ill-Defined Problems

In order to teach via the USMA Mathematical Modeling Process, we must have problems which require such a process. Therefore, we design our course around solving open-ended, and sometimes ill-defined, real-world problems. These problems require students to be creative because they are “ambiguous and include conflicting assumptions and information that may lead to different solutions” [11, page 9]. From the first day of class all the way through the semester to the final exam we are making use of the USMA Mathematical Modeling Process as we work with our students to solve these problems. We design our course this way because we know that “students must solve many problems to improve” in their problem-solving abilities [5, page 666].

By beginning our course with open-ended, ill-defined problems, we invite students to begin letting go of some of the formulaic views of mathematics that have been ingrained throughout their K-12 experiences, which do not always encourage and reward creativity. We want to make it clear from the beginning of our course that not only will creativity be encouraged, it will be rewarded.

So, what do open-ended problems look like? We answer that question through an example.

We have all seen the “guess how many gumballs or jelly beans are in the jar” contests, and some of us might have even participated. If we were to participate and wanted to submit an answer more probable than a guess which contained our lucky numbers, we would try to model this situation. Perhaps one would count the columns and rows of gumballs or jelly beans and attempt to formulate a mathematical volume equation to predict the total number. Perhaps one would lift the jar to estimate its weight and base their assessment on relative weight. Or, perhaps one would assume the person who filled the jar used one standard-sized bag of gumballs or jelly beans and would then base their guess on the advertised number of gumballs or jelly beans in a standard-sized bag. There are multiple ways to model this situation and develop an educated guess. The classic gumball/jelly bean jar contest is a simple, open-ended problem requiring some creativity and critical thinking skills to increase the likelihood of accuracy.

Although at the end of the gumball/jelly bean jar contest there is only one winner, in mathematics class with open-ended problems, we can have multiple winners. For those who see mathematics as either black or white, this might take some adjustment. How can multiple students have the correct answer if their answers are not the same? The students will soon discover that having multiple correct answers is okay. The various answers stem from differing approaches to solving the problem. Supposing the underlying mathematics is correct, we can have multiple correct and acceptable answers.

When we refer to open-ended problems, we mean just what they sound like – problems where the end state is open to interpretation. There are endless possibilities of correct answers. Open-ended problems encourage students to think critically and be creative with their problem solving skills. Open-ended problems highlight the possibility of multiple correct approaches, as well as, the possibility of multiple correct answers.

Good examples of open-ended problems for the mathematics classroom generally involve simple, slightly vague, everyday life questions. These questions tend to remove the anxiety students might have over the mathematics itself and allow students to instead focus on setting up the problem and developing a model. A few of our favorite open-ended, ill-defined, problems which we design our course around in order to develop our students as creative problem solvers follow.

- *How much peanut butter will the Corps of Cadets consume in a year?* In developing a model to solve this problem, some students will consider how many students are in the Corps; how many students sit at each table; how many meals will each student attend (in a day, week, or year); how many students eat peanut butter; what is a typical serving size; what units should be considered (teaspoons, ounces, jars, pounds, etc); what is the size of the jar (how many servings, ounces, pounds, etc); and even the possibility that students take jars of peanut butter from the tables back to their rooms, and more.
- *How many doctors are there in Boston, Massachusetts?* Things students could take into consideration include the population of Boston, the number of hospitals in Boston, the patient-to-doctor ratio, and what counts as a doctor, to name a few.
- *If you could take an elevator to the moon, what floor would the moon be on? How long would it take to get to the moon?* Students must consider their own personal experiences with the speed of an elevator (estimated floor per minute or feet per minute rate), what units to use in computing, what units to use in reporting their answer, does the elevator go to the moon non-stop with no stops, and many, many more.
- *How long will a tube of toothpaste last?* In developing a model to solve this problem students may address how often a person brushes their teeth, how much toothpaste a person uses each time, how many people are sharing this tube of toothpaste, what is the size of this tube of toothpaste, will the person roll the end of the tube to squeeze out every last bit, and more.

We design our course around open-ended, ill-defined, problems because not only do they encourage creativity, they also challenge students to think critically and reason logically while being creative. Creativity alone does not

make a creative problem solver. Creativity must be harnessed and used along side critical thinking and logical reasoning in order to become creative problem solving. If we are to develop creative problem solvers we must not lose sight of these other valuable skills. As stated by the Foundation for Critical Thinking, “critical thinking has three dimensions: the analytic, the evaluative, and the creative. Though we separate these functions for purposes of theoretical clarity, we nevertheless argue that each must be involved if the other two are to be effective” [8, page 22]. Therefore, in our course, as we provide our students feedback to their responses to open-ended, ill-defined, problems, we make sure that we are helping them to learn how to not only be creative, but how to use critical thinking and logical reasoning to truly effectively apply their creativity to mathematical problem solving.

As instructors, we are also always cognizant of just how daunting open-ended problems can seem. Our feedback to students who are struggling to figure out where to begin is to reassure them that there is no single way to solve the problem so the only way to guarantee they are wrong is to fail to begin, so start big, start small, it does not matter, just start. More hesitant students might start with a very basic model. Other students who are confident in their mathematics skills might start with a complex model. Both types of students will reach a correct answer. Because open-ended problems can have multiple correct answers, they are extremely engaging. We have witnessed in our classrooms semester after semester how much students’ confidence in the classroom increases along with their participation once they realize there is more than one way to solve a problem.

As students gain confidence and participate more, it becomes impossible to allow everyone who wants their input heard to share with the whole class when working as individuals. Therefore, we design our course with teamwork in mind. While many students are hesitant to work in teams, we choose to force them to do so because working as a part of a team is an essential life skill. We have seen that as part of a team, even the most hesitant students eventually begin to contribute their independent thought process to a small group collaboration and develop a well thought-out result. When explaining to their peers how they arrived at their answer, students are forced to justify factors chosen for their model and describe their approach to solving the problem. The students are no longer learning from just the instructor, but are actively teaching each other to creatively solve problems, making the classroom a safe space for students to express their individual thought processes.

5. Graded Events

Unfortunately, the real world rarely feels safe. So while it is imperative to provide support and help students feel comfortable with creative problem solving, we would not be doing our jobs as instructors if we stopped there. A student who has only had to solve problems in a low-stress, safe environment is not prepared to solve real problems in the real world. Therefore, we design our course with many opportunities for our students to grow, apply, practice, and hone their creative problem solving skills in various settings through various activities which invoke varying levels of stress and require varying depths of knowledge. We do this not to torture our students, but in the hopes that they will leave our classrooms ready to apply their creative problem-solving skills in whatever situations they may find themselves.

5.1. *Projects*

We design our course to include two projects which provide students the opportunity to creatively solve open-ended problems with a moderate amount of stress, but with mostly unlimited resources. This allows the students to think creatively about their solution with only moderate time pressure. Peterson references a 2006 study by Oldham that “found that the time pressure experienced by employees evidenced a curvilinear relationship with supervisory appraisals of creativity” [9, page 336]. That is to say, the moderate time pressure experienced in completing a project provides an optimal opportunity for creative problem solving for our students. Consequently, we have the highest expectations of our students’ creative problem-solving performance on their projects.

When designing the projects to include in our course, we work to craft them to be open-ended, ill-defined, real-world problems which require our students to apply the creative problem-solving skills that they have been developing throughout the course.

In the first project, we often ask students to engage with a real-world financial situation. For example, in one of the projects we have assigned, we gave students a incredibly low interest rate loan and asked them what they would do with it, how they would pay it off, and what investments they might make. This challenges them to think about multiple aspects of lending, spending, and savings. There is not a single correct answer to this problem because

everyone can approach it differently. Some will spend more, others will save more. As long as they show that they have successfully applied the course material in some way while solving the problem they will be rewarded for good mathematical modeling. Additionally this project has the added benefit of exposing first-year college students to a life situation many of them have never encountered.

In the second project, we typically require students to improve a process or system. For example, one project described a supply chain ordering system. Students had to use the information provided to build a multi-variable discrete dynamical system to model the situation. Using their model they first had to analyze the system to determine how often they would have shortages or overages. Then we asked them to improve the ordering system and show through analysis that their system reduces the amount of shortages and overages. Every student may come up with a different ordering system. Consequently, each student may have a different answer. All of those answers have the potential to be correct assuming their modeling techniques and mathematics are sound.

Sometimes the process or system we ask students to improve involves having them think creatively about policy decisions and hypothesize how policy might affect a real-world situation. For example, we assigned a project that examined the opioid crisis and how people move between being opioid free, taking prescription opioids, abusing prescription opioids, and taking illegal opioids. Students were asked to create increasingly more sophisticated models to answer questions from policy makers about the scope and scale of the epidemic. The project culminated with the students having to make a policy recommendation, assuming the implications of their policy recommendation, and then reporting to the policy maker how their policy recommendation would improve the situation. Again in this project, there is not a single correct answer. Some students recommended harsher prison sentences, some recommended more funding for rehab, others more funding for addiction blocking medications. As long as these policy recommendations were reasonably modeled then the work was considered correct.

This theme of no single correct answer is central not only to the design of our projects, but to the design of our course. We want students to analyze a real-world system, recommend improvements to that system, and then model the changes after their improvements. By design this leads to multiple correct

answers. This fosters creative solutions that even we as instructors would not expect. For example, when recommending changes to the supply chain system, some students recommended changing the time the order is placed, as well as the quantity ordered. Instead of focusing on students arriving at a correct answer, we instead focus on the correct representation of the student's idea in a mathematical context. When students see that they can apply creative ideas in a wide breadth of contexts and achieve full marks it encourages their creativity.

5.2. Creative Problem Solving in Assessments

While the projects that we include in our course design are a chance for students to really challenge themselves individually, real-life problems are rarely solved in isolation. Therefore, we design our course to include several assessments which involve working as both an individual and as a team to solve an open-ended, potentially ill-defined, real-world problem under time constraints with limited resources. We do this because in the real-world there are constraints which must be considered and “there is reason to suspect that people's skill in identifying and working within these constraints might influence their ability to produce the kind of high quality, original, and elegant solutions that are the hallmark of creative thought” [9, page 335]. Consequently, “providing people with training in working with constraints might improve the efficacy of process execution thereby contributing to the production of high quality, original, and elegant solutions” [9, page 336].

We design our course to include these opportunities and this training as assessments because while we can encourage creative problem solving throughout our class time and other assignments, we know that the majority of students are primarily motivated to learn and grow in the ways required by assessments. Consequently, to truly encourage students to make gains in creative problem solving, we must design assessments that reward it. Traditional assessments fall short of this goal. Therefore, we developed a new kind of assessment that we call Discovery Learning Assessments (DLAs) which we piloted with positive results in the Fall 2017 semester [2]. A sample DLA can be found in Appendix A. DLAs directly challenge the fundamental assumption of education, that individual assessments are required to assess student achievement. Working in teams of three or four, students worked both individually and collectively to solve an open-ended, ill-defined, real-world problem.

However, the inclusion of the team component is only part of the re-invention of assessments that DLAs represent. Almost all academic classes are designed in a way that scaffolds student learning towards overall course learning objectives. Lessons are arranged such that each successive one adds a degree of complexity or a new idea. When using traditional assessments, periodically learning pauses so that students can take an assessment, which allows both student and teacher to gauge the level of understanding on the material taught. But why does learning have to pause during assessments? DLAs are specifically designed to be the next topic in the student's step progression towards the overall course objectives.

The problems presented in DLAs are designed to be both a summative and formative assessment. Students are required to individually show that they have learned the course material to date and then they are asked to think beyond what they have already learned in the classroom and use their prior knowledge and experiences to discover an added degree of complexity as a team. Each student has the same questions for the individual portions and all teams share the same team questions. From semester to semester, DLAs will also typically change as the course evolves.

This makes for a rich learning environment because students come to class with an already established worldview, formed by years of prior experiences and learning. Their understanding of an open-ended, ill-defined, real-world problem is constructed based on those past experiences. The team component of DLAs encourages creative problem solving by requiring students with various worldviews to collaborate to solve a problem. This forces students to discuss their thought process, hear alternate viewpoints, and ultimately gives more meaning to the problem. In a team environment, students gain the advantage of emotional and intellectual support that allows them to go beyond their present knowledge, to both understand and solve more complex problems.

DLAs are designed with five successive components: A read-ahead, individual component, team component, individual reflection component, and instructor feedback.

The read-ahead is delivered the day before the assessment and provides students with the real-world context for the problem on the DLA. Real-world articles are included to help frame the situation and make the problem experientially real to the student.

The assessment begins in class with an individual component lasting approximately 20 minutes. In this component, students are asked computational questions which prove they have the mathematical grounding to meaningfully contribute to their team. Individuals are also often required to perform the transform step of the USMA Mathematical Modeling Process during this component so that they are brainstorming about how to solve the problem that the team will solve before meeting with their team to discuss it.

Next, students gather in their teams to complete the team component which lasts about 25 minutes. During this component, teams will discuss what they have brainstormed as individuals, compare their ideas, and ultimately merge their ideas into one unified effort. As previously mentioned, this team component of DLAs is the true catalyst for creative problem solving. As students attempt to explain their thought process to their peers, they often receive criticism and feedback that allows them to better justify their ideas. Hearing new perspectives often sparks even more creative ideas from the other group members, leading to an even more creative and better-defined model. Many times, students will get so involved in discussing and developing the ideas presented in their group, they will struggle with time to get all their ideas written on paper. The organized chaos environment that the team component provides allows for the creative collaboration that DLAs were designed to achieve.

The fourth component and final in-class component is an individual reflection which lasts about 10 minutes. After the chaotic team component, this phase allows the student individual time to reflect on the results obtained and think about the implications of their answer. Often, questions pertaining to sensitivity analysis are asked to promote thoughts about creative problem solving. What if we changed “X” in this problem; how would that change your response? What if you wanted to add “Y” to this problem; would that have affected your assumptions? These thought-provoking questions allow the instructor to gauge each student’s individual understanding of the team component, while allowing the student to further expand their creative problem-solving skills by thinking about how different factors would change their model.

The final component of DLAs is instructor feedback. When giving teams of students challenging, open-ended, ill-defined, real-world problems to solve under a time constraint, it is almost always easy to find enough little flaws to

fail almost every team, every time. While it is certainly okay to make note of errors when they occur in order to help the students grow in the future, the positive aspects also need attention. When grading DLAs, especially the team components, it is important to notice and comment on their many displays of good creative problem solving through their modeling process. This feedback encourages the teams to keep building on the good things that they are doing as they develop their creative problem solving skills.

Initially, the concept of DLAs was not perceived well by students. One of the largest concerns was that team members would rely on other students to do a disproportionate share of the graded work. To combat this, peer assessments like the one presented in Appendix B were introduced, allowing students to grade each others contributions to the group. Whether it was the peer assessment or simply peer pressure, we found that most students prepared with the read-ahead and actively contributed to their group. By the end of the course, there were still some students that felt that teams hindered them, but most students responded positively to DLAs.

6. Iterative Creative Problem Solving on Assessments

In addition to the single class-long DLAs described in the previous section, we have designed our course to include multi-day DLAs as its major exams. These multi-day DLAs are simply a scaled-up version of the DLAs that were previously described. We use the 55-minute team components of these DLAs to ask a more challenging question than we do in the approximately 25-minute team components of the shorter DLAs. We take advantage of the opportunity to have students work on more challenging problems together by designing them to be iterative over the course of the entire semester. We design our course this way because it has been shown that “problem solving ability develops over time” and we have noticed that when students see a real-world problem more than once it gives them the opportunity to harness their natural creative thoughts about the situation and to truly develop as creative problem solvers [5, page 666].

Every real-life problem has multiple possible tools that can be used to solve it. We design our course to give our students the experience of approaching the same scenario from multiple directions, requiring multiple tools with added complexity as each tool is introduced over time. On the team components of

our multi-day DLAs, students are asked to solve the same real-world problem on each multi-day DLA making use of their most newly acquired mathematical tools each time. As the semester progresses, both our expectations and their preparation for solving the problem grow.

Before implementing the iterative problem-solving component with our multi-day DLAs, student solutions to real-world problems on assessments had some undesirable traits. Often students would read too much into the problem, considering rare or unlikely events. Students would then use these events to bypass the heart of the problem asked. For example, if a student were asked to explain a declining fish population, they might make assumptions about hurricanes or unlikely predators as explanations for the decrease, while ignoring a birth rate that does not fully replace the existing fish population. While a student's explanation concerning hurricanes is not necessarily incorrect, it certainly side steps the mathematical richness of the problem if they do not go on to explore how the unpredictability of weather may variably impact parameters in a model over time. In short, students were being creative in their assumptions, but they were not harnessing that creativity to become creative problem solving.

To enable our students to develop as creative problem solvers, by working to harness their creativity, we began designing our course to implement the iterative component to our multi-day DLA team components. To do so, we use real-world situations that can be approached from different directions. One way that we do so is to ask new questions on subsequent assessments based in the same real-world scenario. By using the same scenario to ask questions from new material, we encourage students to both think critically about their past creative answers and to revise those answers. This allows students to not only focus on the techniques they are learning in class, but also to explore the real-world scenario more in-depth. This allows students to truly harness their creativity into creative problem solving.

One real-world scenario that we have taken this iterative approach with is the snapping turtle population at a nearby reserve. We gave the students some read-ahead material to familiarize them with relevant information regarding snapping turtles and factors that contribute to their population changes. In their first assessment, just as we had observed in the past, many students tried to highlight unlikely events as explanations for changes in the snapping turtle population. Students attributed changes in the population to some new

unknown predator or bad storms that had impacted the overall turtle population. They ignored information provided in the read ahead that discussed turtle egg hatch rates as a significant factor. However, in future iterations on subsequent assessments, we saw fewer students use unlikely events to explain their reasoning. Instead, they began to think more critically about underlying factors that would cause change, harness their creativity, use provided resources, and develop as creative problem solvers.

Additionally, in each assessment we increase the precision of information about the scenario provided to the students, allowing us to ask significantly more complex questions. For example, in the first assessment we provided a single egg hatch rate for snapping turtles and in the second assessment we provided different reproduction rates for different aged turtles. This allowed us to motivate students to build on their prior knowledge and consider multiple age-dependent populations, which added more complexity and more closely resembled the real world.

The growth that our students showed as creative problem solvers through this iterative problem-solving process was truly incredible. They had modeled the snapping turtle population in a series of ever increasingly complex and challenging assessments. By the final assessment, we provided no new information for the scenario. Instead we had students recreate a few of the previously used models, and then asked open-ended questions about the strengths and weaknesses of the models. The results we saw were incredibly encouraging. The critical thinking and creative problem solving skills that teams of students displayed in presenting and defending what they believed mattered most in their models, and subsequently evaluating each of the models based on those beliefs, were often beyond our expectations.

7. Conclusion

In our course, Mathematical Modeling and Introduction to Calculus, as in many mathematics courses, we have a higher-order learning goal of developing our students into creative problem solvers. Since we have not yet figured out how to see the future, we do everything we can to prepare our students to creatively solve unknown problems in unseen conditions. As instructors, we must create experiences that foster creativity throughout the entire course. One single event does not promote excellence in creative problem solving nor

prepare our students to be problem solvers, leaders and decision makers in a future world that is beyond the scope of our imaginations. It is the breadth and depth of the experiences offered that allows our students to grow in their creative process. We intentionally create the opportunities throughout our course that will not only promote creativity but truly prepare them to utilize creative problem solving in their every day lives. We challenge you to do the same.

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A. Sample Discovery Learning Assessment (DLA)

The following is a complete example of a Discovery Learning Assessment (DLA) worth 50 points and given during a 55-minute class with time for a 1 minute transition between each of the three in-class components.

A.1. Read Ahead

You are a Nurse Practitioner working at Keller Army Community Hospital (KACH). One day a student who recently returned from a vacation to Argentina arrives in the Emergency Room presenting with severe symptoms of Malaria. The standard treatment for Malaria is Quinine which helps cure the disease but can have serious side effects including kidney damage and heart damage if taken in too high of a concentration. You need to prescribe the correct amount of Quinine so that the amount in the blood stream remains steady without exceeding the ‘maximum safe amount’ as defined by the FDA.

In order to give the best advice to this patient, it is recommended that you review some of your nursing materials provided below regarding how the body metabolizes medicine.

- [TED Talk – How Does Your Body Process Medicine](#)
- [Article – How Does The Body Metabolize Medicine](#)

Expectations for preparation: Techniques include looking at lesson and block objectives, reviewing course material in this block, making connections to the course material covered thus far, doing additional problems that reinforce the connection made to the critical concepts in the block, and lastly making a sheet of notes as needed.

A.2. Individual - 20 Minutes

1. **[10 points]** Answer the following True/False and Multiple Choice questions. For True/False, if the answer is FALSE; correct the statement. If the answer is TRUE, simply state TRUE. For Multiple Choice circle the correct answer.

- (a) True or False. The sequence generated by the below DDS is arithmetic.

$$p_n = 5p_{n-1} + 3$$
$$p_0 = 10$$

- (b) True or False. The following first order linear DDS is classified as homogeneous.

$$p_n = -2p_{n-1} + 6$$
$$p_0 = 2$$

- (c) Multiple Choice. Given the following recursion equation, what is the value of the d parameter?

$$p_n = 7p_{n-1}$$
$$p_0 = 4$$

- i. 7
- ii. -1
- iii. 4
- iv. 0

2. **[5 Points]** Calculate the next 2 terms generated by this DDS. Show all calculation steps.

$$p_n = -1.5p_{n-1} + 4$$
$$p_0 = 6$$

From The Read Ahead

You are a Nurse Practitioner working at Keller Army Community Hospital (KACH). One day a student who recently returned from a vacation to Argentina arrives in the Emergency Room presenting with severe symptoms of Malaria. The standard treatment for Malaria is Quinine which helps cure the disease but can have serious side effects including kidney damage and heart damage if taken in too high of a concentration. You need to prescribe the correct amount of Quinine so that the amount in the blood stream remains steady without exceeding the ‘maximum safe amount’ as defined by the FDA.

3. [10 points] Begin the **TRANSFORM** step of the USMA Mathematical Modeling Process for the following problem.

SITUATION UPDATE

The patient spent 48 hours under observation at the hospital. During their time under observation, they received 30 ml doses of Quinine intravenously for each 24-hour period. After the 48 hours of observation, the student’s condition improved and they were discharged with a prescription of Quinine (liquid capsules) to ensure the parasite was destroyed. You are working with the student’s Primary Care Manager (PCM) on a continuing care plan. Research states the average human liver processes 26% of Quinine in a 24-hour period, turning it into more inert chemicals that are then excreted out of the body. If the amount of Quinine in the body ever exceeds 76 ml then the patient is at risk for serious kidney damage. The Doctor has recommended the patient takes one 20 ml Quinine capsule per day for 2 weeks.

Do you endorse this recommendation? Is this an appropriate amount of narcotic for the patient to take?

- (a) [5 points] Define variables needed to model this problem.
- (b) [5 points] State one assumption needed to model this situation and state why it is both reasonable and necessary.

*A.3. Team – 23 Minutes***————Copy of Situation from Individual Portion is Below ————**

The patient spent 48 hours under observation at the hospital. During their time under observation, they received 30 ml doses of Quinine intravenously for each 24-hour period. After the 48 hours of observation, the student's condition improved and they were discharged with a prescription of Quinine (liquid capsules) to ensure the parasite was destroyed. You are working with the student's Primary Care Manager (PCM) on a continuing care plan. Research states the average human liver processes 26% of Quinine in a 24-hour period, turning it into more inert chemicals that are then excreted out of the body. If the amount of Quinine in the body ever exceeds 76 ml then the patient is at risk for serious kidney damage. The Doctor has recommended the patient takes one 20 ml Quinine capsule per day for 2 weeks.

Do you endorse this recommendation? Is this an appropriate amount of narcotic for the patient to take?

4. **[15 points]** Provide a recommendation to the attending Doctor. Justify your recommendation by showing **ALL** of the steps of the USMA Mathematical Modeling Process.

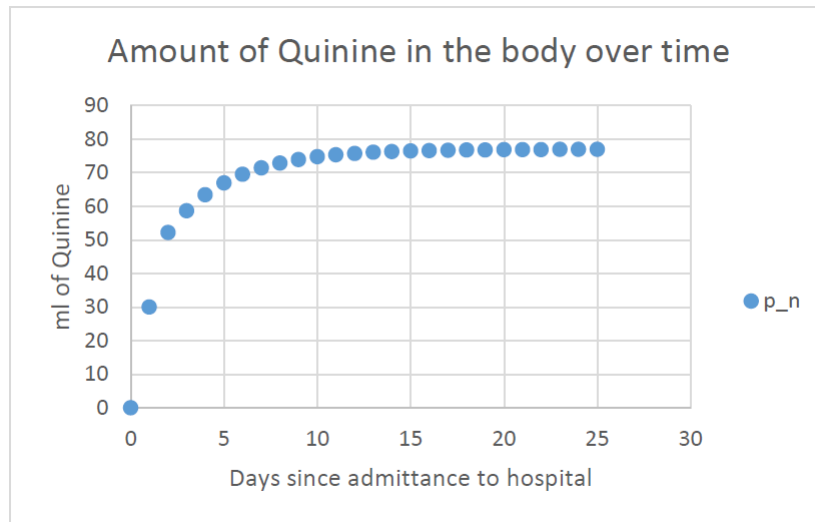
A.4. Individual Reflection – 10 Minutes

—————**Copy of Situation from Individual Portion is Below**—————

The patient spent 48 hours under observation at the hospital. During their time under observation, they received 30 ml doses of Quinine intravenously for each 24-hour period. After the 48 hours of observation, the student’s condition improved and they were discharged with a prescription of Quinine (liquid capsules) to ensure the parasite was destroyed. You are working with the student’s Primary Care Manager (PCM) on a continuing care plan. Research states the average human liver processes 26% of Quinine in a 24-hour period, turning it into more inert chemicals that are then excreted out of the body. If the amount of Quinine in the body ever exceeds 76 ml then the patient is at risk for serious kidney damage. The Doctor has recommended the patient takes one 20 ml Quinine capsule per day for 2 weeks.

Do you endorse this recommendation? Is this an appropriate amount of narcotic for the patient to take?

—————**UPDATE FOR REFLECTION PORTION**—————



Last week you read an article in the New England Journal of Medicine discussing a recent study that suggests there is no evidence that patients will

experience kidney damage if they have only 76 ml of Quinine in the blood. The study recommended to the FDA that the maximum safe amount be changed to 80 ml in the bloodstream. **Use this update and the graph of the recursion equation for this Discrete Dynamical System above to answer the following questions using mathematical analysis.**

5. **[5 points]** If the journal article is accepted as fact, would you change your analysis or recommendation to the Doctor?
6. **[5 points]** If the PCM decided that a 4-week course of treatment was more appropriate than a 2 weeks, would you be concerned about kidney damage if the FDA adopted an 80 ml safe limit? Can you draw any conclusions about the behavior of this system based on your analysis of this question?

B. Sample Peer Assessment

The following is the peer assessment that we created by adapting ideas from [6] in a way that fit our course. Students are rewarded for being a good teammate and for having the self-awareness to realize what kind of teammate they are. What kind of teammate they are is determined based on the average of what their teammates say about them. Self-awareness is measured by looking at the difference of their self-rating in comparison and the ratings from their teammates. A small margin of error in either direction receives full points with a larger buffer given for students being humble in their self-evaluation.

1. What is the name of your first teammate?
2. Rate your first teammate.
 - Excellent: Consistently carried more than their fair share of the workload.
 - Very Good: Consistently did what they were supposed to do, very prepared and cooperative.
 - Satisfactory: Usually did what they were supposed to do, acceptably prepared and cooperative.
 - Ordinary: Often did what they were supposed to do, minimally prepared and cooperative.
 - Marginal: Sometimes failed to show up or complete assignments, rarely prepared.
 - Deficient: Often failed to show up or complete assignments, rarely prepared.
 - Unsatisfactory: Consistently failed to show up or complete assignments, unprepared.
 - Superficial: Practically no participation.
 - No Show: No participation at all.
3. Comment on the rating of your first teammate.
4. What is the name of your second teammate?

5. Rate your second teammate.

- Excellent: Consistently carried more than their fair share of the workload.
- Very Good: Consistently did what they were supposed to do, very prepared and cooperative.
- Satisfactory: Usually did what they were supposed to do, acceptably prepared and cooperative.
- Ordinary: Often did what they were supposed to do, minimally prepared and cooperative.
- Marginal: Sometimes failed to show up or complete assignments, rarely prepared.
- Deficient: Often failed to show up or complete assignments, rarely prepared.
- Unsatisfactory: Consistently failed to show up or complete assignments, unprepared.
- Superficial: Practically no participation.
- No Show: No participation at all.

6. Comment on the rating of your second teammate.

7. What is the your name?

8. Rate yourself.

- Excellent: Consistently carried more than my fair share of the workload.
- Very Good: Consistently did what I was supposed to do, very prepared, and cooperative.
- Satisfactory: Usually did what I was supposed to do, acceptably prepared, and cooperative.
- Ordinary: Often did what I was supposed to do, minimally prepared, and cooperative.
- Marginal: Sometimes failed to show up or complete assignments, rarely prepared.

- Deficient: Often failed to show up or complete assignments, rarely prepared.
- Unsatisfactory: Consistently failed to show up or complete assignments, unprepared.
- Superficial: Practically no participation.
- No Show: No participation at all.

9. Comment on the rating of yourself.

10. Rate your team

- Excellent: We consistently communicated well, effectively resolved our conflicts, and our individual strengths complemented one another's making us stronger together.
- Satisfactory: We usually communicated well and resolved our conflicts.
- Marginal: We sometimes failed to communicate effectively and although we addressed our conflicts, we sometimes failed to resolve them.
- Unsatisfactory: We consistently failed to communicate effectively, failed to address some of our conflicts, and rarely resolved a conflict.
- Completely Dysfunctional: We failed to communicate effectively, failed to address most of our conflicts, and never resolved a conflict. We would have been stronger on our own.

11. Comment on the rating of your team.