Innovative Induction and Mathematical Code Switching

Benjamin Dickman

The Hewitt School

Erik Nauman

The Hewitt School

Follow this and additional works at: <https://scholarship.claremont.edu/jhm>

Part of the [Arts and Humanities Commons](#), and the [Mathematics Commons](#)

Recommended Citation

Benjamin Dickman & Erik Nauman, "Innovative Induction and Mathematical Code Switching," *Journal of Humanistic Mathematics*, Volume 10 Issue 2 (July 2020), pages 258-290. DOI: 10.5642/jhummath.202002.13. Available at: <https://scholarship.claremont.edu/jhm/vol10/iss2/13>

©2020 by the authors. This work is licensed under a Creative Commons License.

JHM is an open access bi-annual journal sponsored by the Claremont Center for the Mathematical Sciences and published by the Claremont Colleges Library | ISSN 2159-8118 | <http://scholarship.claremont.edu/jhm/>

The editorial staff of JHM works hard to make sure the scholarship disseminated in JHM is accurate and upholds professional ethical guidelines. However the views and opinions expressed in each published manuscript belong exclusively to the individual contributor(s). The publisher and the editors do not endorse or accept responsibility for them. See <https://scholarship.claremont.edu/jhm/policies.html> for more information.

Innovative Induction and Mathematical Code Switching

Cover Page Footnote

The first author wishes to acknowledge Rochelle Gutiérrez for fruitful conversation around, among other topics, the distinction between mathematical code switching and translanguaging. Moreover, both authors wish to thank the reviewers for their feedback, especially as pertains to concerns around ways in which mathematical code switching can be invoked to reinforce or exacerbate power imbalances.

Innovative Induction and Mathematical Code Switching

Benjamin Dickman

The Hewitt School, New York, USA
benjamindickman@gmail.com

Erik Nauman

The Hewitt School, New York, USA
enauman@hewittschool.org

Synopsis

In the first part of this paper, we provide an example of a project designed to foster mathematical creativity among students at an independent, all-girls school in the Northeastern United States. The mathematical motivator for the project is a polyomino proof by induction first formulated by Solomon Golomb. We explain how the project has been implemented over the past two years at the school's Innovation Lab in collaborative work between a mathematics instructor and an educational technologist, provide instructions and background information to facilitate the implementation of this project at other learning sites, and show examples of student work along with a discussion of their reactions and takeaways. We close by naming the practice of “mathematical code switching” and situate it within Rochelle Gutiérrez's discussion of *creative insubordination in mathematics teaching*.

Keywords: creativity, innovation, mathematical code switching, STEM education, educational technology

1. INTRODUCTION

This article represents the collaboration across two years by its authors — a mathematics instructor and an educational technologist — as we sought to incorporate a project that would promote creativity at the beginning of a

high school math course. We open here with a few frequently asked questions about creativity, discuss the project we have designed along with student results/reactions and how it can be implemented at other learning sites, and close by introducing the idea of *mathematical code switching* as situated within Gutiérrez's research on creative insubordination [17].

Figure 1: A student-created 8×8 L-tromino board with one L-tromino.

Frequently Asked Questions on Fostering Creativity

The question of how to foster creativity among (mathematics) learners is difficult to answer for a variety of reasons, which range from pedagogical to philosophical to pragmatic to pedantic. Here are a few of the responses we have encountered in discussions around how to engage with mathematics learners in ways that promote creativity.

1. What does *creativity* mean?
2. Where can I find materials that will make my students more creative?

3. I have heard about some possible ways to engage students mathematically — maybe even creatively — but I am worried that they would be frowned upon since they do not connect with the ways that others (e.g., administrators, parents/guardians/caretakers) think about math.
4. How can I measure creativity to find out if my students are really becoming more creative?
5. What can I do if I am not a creative teacher, a creative mathematician, or even a creative person?

In this article, we will give our answer to the first question by articulating a definition for creativity; we will respond to the second question by providing and discussing materials used in a high school course as a three day project co-led by the authors (a mathematics instructor and an educational technologist); and we will discuss strategies for the third question: in particular, by naming the practice of *mathematical code switching*. We will not give direct answers to the fourth or fifth question, but have included them here due to the frequency with which they arise. Bearing that in mind, let us open with a few remarks about these questions — one on measuring creativity and the other on being, or not being, creative — before moving on.

Creativity Beyond Individuals

The study of *creativity* is often traced back to Guilford's APA presidential address [16], in which the goal was to identify and measure the traits that characterize a creative individual. This approach was consonant with Guilford's earlier psychometric work assessing military personnel (e.g., [15]). Subsequent investigations of creativity have proceeded in other ways; for example, Rhodes introduced his 4P framework around whether creativity is placed in a person, process, procedure, or (environmental) press [26]. An additional dimension that has emerged over time has been the distinction between Big-C Creativity and little-c creativity, which refers to the notion that creativity can be thought about as existing across a continuum from eminent to everyday [21]. Other studies have explored such topics as the collaborative emergence of creativity among groups of individuals [30], identifying creativity with problem solving (e.g., [38]) or problem finding (e.g., [11]; for problem posing in mathematics, see, e.g., [33, 39, 8]); and challenges to uniformly positive views by discussing the dark side of creativity [6].

More generally, Sawyer remarks succinctly that “defining creativity may be one of the most difficult tasks facing the social sciences” [31, page 11]). In our project, we avoid the 4P framework and the Big-C to little-c continuum; in particular, we present a conception of creativity that does not locate creativity within people: in this way, the question of measuring a student’s creativity, and the consideration of what to do if educators do not believe themselves to be creative, are both rendered moot. Although we intend to make a case for why this mathematical activity reflects participation in creativity, we nevertheless encourage readers to reflect on why so many have the knee-jerk reaction to ask of creativity how it can be measured or evaluated through some standardized metric (for more, cf. e.g., [2, page 452]).

Working Definition of Creativity

Our goal in this article will be to situate creativity within a participatory model (cf. [19, 20]). In particular, our own working definition of creativity is as follows.

Creativity: Purposeful work that is novel and valued within a domain/area, which emerges over time through the participation of many actors in many roles.

For remarks on the meaning of purpose and how it evolves in the context of creativity studies, we refer the reader to Gruber and collaborators (e.g., [13, 14]). Salient to our definition is that we shift from the adjective ‘creative’ to the noun ‘creativity’, such that we choose not to apply the former to some object but rather to encourage *participation* in the latter. Thus, we speak of ‘participating in creativity’, and intend to discuss concretely how we have aimed to do so with our students in their mathematical studies.

2. INDUCTION IN THE MATHEMATICS CLASSROOM

The mathematics covered in our course consists of an introduction to proof by the principle of mathematical induction (PMI) as taught in an all-girls school in the Northeastern United States. The course title is Problem Solving & Problem Posing; the class has a pre/corequisite of Calculus; its structure is to cover a variety of mathematical topics outside of Calculus (e.g., proof-writing, graph theory, combinatorics, elementary group theory, number theory) and to facilitate students’ grasp of resources to call upon when facing a non-routine problem (e.g., [25, 32]) as well as how to pose problems of their own (e.g., [4]).

Figure 2: Two student-created 8×8 L-tromino boards.

The principal goal of this course is for students to expand their conception of mathematics beyond solving equations or measuring angles, and to view themselves as generative contributors to mathematics who are capable of novel (i.e., new to them) and valuable (i.e., valued by them) ideas. As we are primarily concerned with students' own self-assessments of novelty and value — rather than assessments from peers, experts, or gatekeepers from a larger field — the result is a form of creativity outside of the Big-C to little-c continuum (although connected to *mini-c* creativity, cf. [3]).

Tackling Problems Pre- and Post-PMI

PMI is introduced in our classroom as a method to prove statements about the natural numbers, and we begin with students by verifying past assertions that were either proved using alternative approaches or investigated with conjectures.

We include below two statements that we explored without PMI and then proved with PMI; brief summary accounts are provided for the former, and succinct proofs are provided for the latter.

Example 1.: The sum of the first n natural numbers is $n(n+1)/2$.

Exploration without PMI.: We used a geometric representation to show that the sum of the first n odd numbers is n^2 ; next, we observed that summing the first n evens can be done by adding 1 to each of the first n odds, thereby adding an additional n for a total of $n^2 + n = n(n+1)$. Finally, we observed that the first n natural numbers can be summed by taking the evens' sum and dividing by 2, which gave us the desired formula of $n(n+1)/2$.

Proof with PMI.: The formula holds for $n = 1$ since $1 = 1(1+1)/2$. Assuming the formula holds for the first k naturals, observe that adding $k+1$ gives $k(k+1)/2 + (k+1) = (k+1)((k+1)+1)/2$. QED.

Example 2.: The sum of the first n perfect squares is $n(n+1)(2n+1)/6$.

Exploration without PMI.: We listed the first few sums of squares with the goal of finding a formula by inspection: 1, 5, 14, 30. With the hope that a formula would involve some product of terms, we proceeded with a slightly wild approach (although tame from the perspective of someone who already knows the formula: cf. [9]) of multiplying each term by 6 to introduce more factors: 6, 30, 84, 180. Next, we factored the first four terms as follows: $1(2)3$, $2(3)5$, $3(4)7$, $4(5)9$. From here, we could guess the pattern of $n(n+1)(2n+1)$, and divided by 6 to undo our initial multiplication.

Proof with PMI.: The formula holds for $n = 1$ since $1 = 1(1+1)(2+1)/6$. Assuming the formula holds for the first k naturals, observe that adding $(k+1)^2$ yields $k(k+1)(2k+1)/6 + (k+1)^2$, which can be rewritten as $(k+1)((k+1)+1)(2(k+1)+1)/6$ as desired. QED.

Students completing their first proofs by induction often respond with uncertainty as to whether they have shown anything at all. In each of the number-theoretical examples above, the pre-PMI exploration allowed students to engage with the material using authentic mathematical habits of mind [7], whereas the PMI proofs were initially perceived as an application of legerdemain. The same can be said of other explorations when compared with PMI proofs; for example, combinatorial arguments to show $\{1, \dots, n\}$ has 2^n subsets or that an n -gon where $n \geq 3$ has $n(n-3)/2$ diagonals may

position students to participate in mathematical creativity, whereas the corresponding proofs by induction can look stale or procedural, and lack the *aha* insight familiar to writings on mathematical creativity (e.g., [24]; see also [37]).

Our goal, therefore, was to locate a statement whose exploration could be done with recursive reasoning, in a manner that would allow students to feel that induction is a natural strategy to justify the assertion. We found such an example in the writing of Golomb [12]; it is a problem that was popularized by Gardner [10], visualized by Nelsen [23], and extended by Starr [35] and Costello [5]. The original phrasing by Golomb (see also [22]) is as follows:

It is impossible to cover an 8×8 board entirely with trominoes, polyominoes of 3 squares, because 64 is not divisible by 3. Instead, it shall be asked: Can the 8×8 board be covered with 21 trominoes and 1 monomino (a single square)? [12, page 20]

In this particular excerpt, Golomb is talking about trominoes that look like extended dominoes with three unit squares in a row rather than two. He demonstrates that this is only possible when the monomino is placed in one of four different locations, and then begins his discussion of L-shaped trominoes:

When another type of tromino is considered, the result is surprisingly different: No matter where on the checkerboard a monomino is placed, the remaining squares always can be covered with 21 right trominoes, i.e., L-shaped trominoes. [12, page 21].

Our class endeavored to prove the latter statement by using Pólya's generalization heuristic [25] and, in particular, proving that for all natural numbers n , a board with dimensions $2^n \times 2^n$ can be tiled with one monomino and the rest being L-trominoes. The standard checkerboard result then follows in the case of $n = 3$, since it has dimensions $2^3 \times 2^3$, i.e., 8×8 . To prove this by induction, we considered $n = 1$ and showed any 2×2 board can be tiled with a monomino and an L-tromino. For the 4×4 board, we subdivided it into four 2×2 boards, and supposed the monomino was placed in any one of the quadrants; we could then place a single L-tromino so as to cover one square in each of the other three quadrants, whence the problem reduces to solving four copies of the 2×2 case. We reasoned similarly for the 8×8 board: subdividing into four 4×4 boards, placing the monomino in one of

the quadrants, and placing an L-tromino so as to cover one square in each of the other three quadrants. The inductive step is summarized elegantly in Nelsen's proof without words [23] described in Figure 3:

Figure 3: Inductive step for tiling a $2^n \times 2^n$ board with a monomino and L-trominoes.

Although we could effectively go through examples of this result by drawing $2^n \times 2^n$ boards by hand, we wished to create a physical copy (cf. [35]; see also Figures 1 and 2) that would allow students to operate with the boards in a manner intended both to (*re*)humanize [18] the inductive exploration at hand and to satisfy the need for *play* in mathematics [36]. Fortunately, our institution has access to an Innovation Lab in which this was possible.

3. INDUCTION IN THE INNOVATION LABORATORY

In this section, the second author provides further insights and background on how the project developed across its first two years of implementation. We also provide pointers to Appendix A, which contains the handout used by students to begin the project.

Hands On Process Over Three Classes

The students visited the Innovation Lab, which houses a laser cutting machine, to make their own L-tromino boards and pieces over three class periods. They created their graphic designs using Adobe Illustrator software. Adobe Illustrator vector graphics created instructions for our 30 Watt Universal laser cutting machine by means of color and line thickness.

Figure 4: A student-created 8×8 L-tromino board with a single monomino in the top-left corner.

Any graphic designs in black were etched on the surface of the material placed in the machine. Any line shape in red with a hairline thickness (0.001 pt) was set to cut all the way through the material. Any line shape in blue with a hairline thickness was set to score the shape in the surface of the material, following the precise path of the line graphic. Eight-inch square plywood boards were prepared ahead of time with an inset pocket for the L-tromino pieces to be cut from acrylic plastic in colors chosen by the students.

Students followed a handout (Appendix A) that guided them with text and illustrations through three stages of making their designs. Beginning with their own copy of a template Illustrator file that had been prepared ahead of time to have pre-drawn grid guidelines for the 8×8 rows and columns, and with an artboard size set to the cutting dimensions of the laser cutting machine (in our case 24 inches by 12 inches), students first created red lines to mark out their L-tromino shapes and one monomino shape.

They then created a grid of blue score lines and any black graphics or text that they wanted to be etched into the inset pocket of their board. Finally they created any black graphics or text that they wanted to be etched on the border of their board. These graphics were created on three different layers in the Illustrator document, as each layer had to be sent separately to the laser cutter; the first to cut acrylic pieces, the second to score and etch the inset pocket, and the third to etch the border of the board, which was at a different material height and thus could not be done at the same time as the inset pocket graphics. As students began completing the first stage of their designs the laser cutter could begin cutting their acrylic pieces, and students could individually choose what color acrylic they wanted to use, sometimes choosing a different color for the monomino piece to create a contrast between this piece and the L-tromino pieces.

While the laser cutter cut the acrylic pieces quickly, it took significantly more time to etch the graphics onto the boards — from 15 to 20 minutes each — so most of the students' board designs were etched outside of class time. Fortunately, they were present while at least one of the boards was being etched so that they could witness the seeming magic of laser etching and appreciate the process by which their designs were “printed” onto wood. The experience of witnessing the laser cutter in action helped demystify and make more accessible what the machine does and helped them to see its function as a part of the creative process rather than as an invisible service that they were using.

Procedural Modifications Across Two Years

A central concern in designing the learning environment for an integrated technology project is maintaining a balance between time spent learning and practicing discrete technology skills with time spent exploring and playing with creative ideas relevant to the non-technology learning domain, in this case mathematics. Upon reflecting on the students' first year experiences with the project it was apparent to the instructors that students were confused with the initial document set up tasks, especially with creating the sixteen 22 mm-spaced horizontal and vertical grid guides used to enable the precise placement of their L-tromino cutting lines. Students were confused to the degree that some were experiencing a lot of stress by the time they were able to begin designing their own version of the L-tromino board.

Figure 5: Students watching the laser cutting machine cut L-tromino pieces.

The instructors noticed students producing relatively simple board designs despite the examples that they were given showing checkerboard designs and border enhancements. When encouraged to explore more possibilities some students declined the opportunity.

The second time around some initial steps were removed from the students' work time by providing students with a template Illustrator file with the initial document settings and 16 grid guide tasks already completed. This allowed students to skip the first page and first instruction on the second page of the handout and jump right into designing their own L-tromino and monomino shape cut lines. Allowing students to begin the project designing their own pieces without having to start with tedious tasks that require precision resulted in students having more attention and cognitive resources to explore their own graphic ideas for designing their boards. In one case a student took on the ambitious project of doubling the mathematical dimensions of the grid for her tromino board to make a 16×16 L-tromino board and pieces. Comparing the completed projects from the first year to the second year, we observed a notable increase in the amount of personalization students managed to include in their games in the second year (Figure 6).

Figure 6: Typical board designs from 1st year (left) and 2nd year (right).

Modifications in Process

The first time the instructors embarked on this project the educational technologist was just familiarizing himself with many of the digital fabrication tools in the Innovation Lab. After the first iteration of the project, he was able to make one important modification in the process of preparing materials for the project for the second year. In the first year, he prepared the boards by sawing 8-inch square pieces of 1/4-inch plywood, then using a Carvey CNC milling machine to mill the pockets in which the tromino pieces would be placed. This took a long time: about 45 minutes for each of the 8 boards. And because the pockets were milled with a 1/8-inch milling bit the pocket corners were rounded to the nearest 1/8-inch, which required chiseling out for the trominoes to fit. The second year, a colleague made the helpful suggestion of using the laser cutter to cut the puzzle frames in 1/8-inch plywood and glue them to 8-inch laser cut square backing pieces, creating the same pocket frame in a much shorter time, with the added benefit of perfectly square corners.

4. PARTICIPATING IN CREATIVITY

In making the case that the project at hand facilitates participation in creativity, let us first recall the definition for creativity that we provided in an earlier section:

Figure 7: Two student-created 8×8 L-tromino boards from the second iteration of the project.

Creativity: Purposeful work that is novel and valued within a domain/area, which emerges over time through the participation of many actors in many roles.

We discuss the project here as being located within the domain/area of mathematics, even as it clearly touches upon other areas (e.g., crafts, design, technology) and acknowledge that while we consider creativity as domain-specific (i.e., consider participation in *mathematical* creativity rather than in creativity *writ large*) there is an ongoing debate around domain specificity (see, e.g., [1]). Moreover, we reiterate that our notions of *valued* and *novel* are primarily with respect to students' own self-assessments, and are not a direct function of evaluations by members of a larger set of eminent mathematicians or mathematics educators.

Student Participation

Let us take this opportunity to bring in a few student voices around their participation in this project. The excerpts below were taken from writing completed just after the boards were created in year two of this project, and with all students asked to connect the project with Su's mention of *play* in *Mathematics for Human Flourishing* [36]:

STUDENT 1: Making my tromino [board] was an enjoyable mathematical process because there were no great stakes involved in the task, and the project allowed me to use my imagination and creativity. We were given a lot of freedom within our instructions to design our tromino boards however we wanted. I found that actually having to create the tromino and monomino pieces for the board helped me get a better understanding of how to arrange them because we had to create the pieces from a blank square and maneuver them so that they all fit onto the board. I really enjoy doing puzzles and showing other people how to solve puzzles and I cannot wait to show my tromino board to my family and have them try and figure it out.

STUDENT 2: I really enjoyed making my tromino board and for me it can definitely fall under the PLAY section in [Su's] article. The article explains that play needs to be both fun and voluntary. The tromino boards that we made to me were definitely fun and creative. They were also voluntary because we all had the option to take this class and I know that I would have liked to make this board whether it was for a class or not. The article also says that to be categorized as play there is also some structure. There was structure in what we did because we had the directions that [Mr. Nauman, the educational technologist] gave us and [Mr. Dickman, the mathematics instructor] explained to us [in terms of] what we were doing and why it was possible. This project ... built community both inside our classroom and out as we shared these boards with friends and family ... I really loved this project and I look forward to what is to come with more PLAY in our math class.

STUDENT 3: I really enjoyed creating my L-Tromino board because it gave me a chance to creatively create my own personalized board game. It also gave me a chance to think more closely about patterns and shapes and how games with shapes and patterns are created. Like Su describes, math is “play,” and when you are “playing” with math you have both structure, by the concepts we learn in math, and its rules, but also freedom because you can experiment with those rules and patterns as much as you like.

It also [leads] you to use your mind and imagination to apply things that you know and learn and “play” with it your own way. I felt like I had the chance to do that with my tromino board, for I was able to take the patterns and concepts we had learned and discussed in class, and apply [them] to create my own way of making and seeing the pattern.

The three sample responses above each use the word “creativity” or “creative” despite the word not arising in their original prompt; the same is true of “enjoyed” or “enjoyable”. Other themes include a desire to engage with their families around this mathematical object/idea (e.g., Student 2 writes that the project “built community both inside our classroom and out as we shared these boards with friends and family”); freedom (Students 1 and 3 explicitly use this word, and Student 2 writes that the project was “voluntary... and I know that I would have liked to make this board whether it was for a class or not”) despite instructions and structure; and ownership (each student refers to it as “my” board, and a number of students chose to personalize them in ways that included meaningful motifs from their own lives, and the optional addition of their initials, names, and nicknames). We believe that these selected responses, which have not been anomalous among all student responses, indicate a clear valuing of their own work.

Figure 8: A student-created 8×8 L-tromino board.

As to novelty, the students are learning a mathematical idea that is new to them, and for which the board can provide a way of more deeply understanding the underlying structure (e.g., Student 1: “I found that actually having to create the tromino and monomino pieces for the board helped me get a better understanding of how to arrange them because we had to create the pieces from a blank square and maneuver them so that they all fit onto the board”). Students were new to proofs by induction; new to this construction; and, in some cases, new to using some of the available tools (digital design programs as well as the laser cutter). This novelty was furthered as students personalized their boards in different ways: In the first year of the project, students created border designs that may have included their name and other personal flourishes (e.g., incorporating foreign language study as in Figure 4); in the second year, students diverged further by modifying the board itself, and one student (see Figure 9) decided to engage in two additionally novel ways: first, by creating her own logo in Desmos to laser cut into the board’s background; second, by applying her structural understanding of the generalized proof to create a 16×16 board and the corresponding pieces.

Figure 9: A student created 16×16 L-tromino board with a background logo that she designed.

Returning once more to our definition of creativity, it is reasonable to ask whether this work is purposeful. Our view is that students who enjoy what they are doing and learning, describe their motivators as intrinsic/voluntary, and continue to deepen their knowledge of mathematics — both specific topics such as proof by induction, as well as “look[ing] forward to what is to come with more PLAY in our math class” — are operating in ways that are purposeful.

Back in the classroom, students were able to engage with this particular proof by induction for the third time. The first time had been in explanations that used our whiteboard; the second time had been in their constructing the puzzle in the Innovation Lab, as the guidelines for the laser cutter required students to tile the board; and, finally, they were able to interact with a physical copy.

Although most student-creators would bring their own boards home, we kept a few tromino boards in the classroom; these game boards became a mainstay among mathematical objects that could be played with by any students who take courses in that particular classroom — generally ranging from grades 7 through 12. On numerous occasions since the first boards were made, these student creations have allowed younger students, with either scaffolding by an instructor or a schoolmate, to learn the inductive/recursive approach to tiling a board with L-trominoes. This has built anticipation among students around their own opportunity to create mathematical objects in the Innovation Lab in the future, and allowed them to see not only the novelty in a mathematical puzzle, but also the value ascribed to this object by both students, who took care to personalize the boards with their own symbols and names, as well as instructors, as these boards found a permanent home among a collection of commercial mathematical puzzles gathered together on a table in the room (see Figures 10 and 11). Finally, the existence of these boards and the subsequent familiarity with these materials enabled students to take a hands-on approach to new creative problems that were posed with polyominoes. For example, The Riddler column at *FiveThirtyEight* posed a problem for which we could use our boards but would need to manufacture pentominoes [27]. With the L-tromino foundation, addressing this latter condition proved to be straightforward, and we were able to laser cut pentominoes for physical game play (see Figure 12) that was featured in The Riddler write-up [28] that followed a week later.

Figure 10: A student-created L-tromino board balanced in the top-left on our play table.

5. MATHEMATICAL CODE SWITCHING

We close by responding to the third item of feedback that we sometimes face when interacting with educators and other stakeholders around fostering creativity in the (mathematics) classroom. As stated in the introduction, this item can be paraphrased as follows:

I have heard about some possible ways to engage students mathematically — maybe even creatively — but I am worried that they would be frowned upon since they do not connect with the ways that others (e.g., administrators, parents/guardians/caretakers) think about math.

Figure 11: Two L-tromino boards located on the instructor desk adjacent to the play table.

Figure 12: Pentominoes designed for hands-on play with The Riddler puzzle.

This is a real concern for those who may hear back that they should be attending more precisely to a preset list of topics, as those are the ones that are best recognized as being mathematical (e.g., solving equations, measuring angles). One way that this pushback arises is in the “Where’s the math?” question, for which Rubel and McCloskey write:

In general, the ‘Where is the math?’ critique is usually used to signal either that mathematics is not foregrounded enough among social phenomena or that the mathematics is not rigorous enough. [29, page 9]

The authors go on to describe further problems with this framing, including a view suggested by the definite article ‘the’ that there is a singular mathematics at play. We note that, in the particular context described here, there is also a more specific criticism that could be levied, whereby stakeholders could ask why, for a course that has Differential Calculus as its pre/corequisite, students are “playing with blocks” rather than pursuing, for example, Integral Calculus, or a course on Big Data and Statistics, or some other more recognizable presentation of mathematics.

To respond to these potential critiques, we advocate using a strategy for creative insubordination [17] that we refer to as *mathematical code switching* (MCS). MCS refers to the use of formal, potentially buzzword-bloated, mathematical terminology to signal connections between covered material with areas of mathematics that have been traditionally considered more rigorous. To give a concrete example, one response to the “playing with blocks” critique could be that our project is enabling students to engage in the recursive thinking that typifies areas of Discrete Mathematics, and allows them to prove by construction a theorem relying on the Principle of Mathematical Induction — which, itself, is one of the Peano Axioms, over which it is equivalent to the Well-Ordering Principle. Critical to this description is, first, that it is true; and, second, that it brings into the discussion a set of mathematical terms intended to shift away from the language of “blocks”, in an effort to convince the interlocutor that there are recognizably formal or rigorous topics at hand.

The idea of MCS is not to establish connections unknown to professional mathematicians, for whom the notions of playing with mathematics may already seem manifestly reasonable [36]. Rather, our intention is to wield mathematical language as one of the *master’s tools* in the sense of Lorde

as referenced in Gutiérrez [17]: We are not, by virtue of charging our descriptions with arcane terminology, dismantling a system that privileges formal mathematical language. Instead, we are adhering to prewritten scripts around what does, or does not, belong in a rigorous mathematics course, and using a select nomenclature to combat the reductive language (e.g., “the math” or “playing with blocks”) found in criticisms that, if they are to take hold, would interfere with our focus on fostering creative participation.

There are, of course, significant limitations to MCS. Beyond its ultimate inability, per Lorde, to dismantle the master’s house, other potential concerns arise. We non-exhaustively mention just two potential issues here.

First, we cannot advocate in good faith for the widespread expectation that others be prepared to engage with MCS in ways that reduce power imbalances. One corollary of using this technique to enable educators to bring in non-standard material is that those who are unfamiliar with certain terminology — how it is not “just folding paper” but rather exploring non-Abelian symmetric groups; or how it is not “just playing with cards” but rather exploring connections between the Euler totient function, the discrete logarithm, binary fraction periods, and Faro shuffles — may find themselves less able to diverge from the canon of mathematical topics. Our wish is to empower educators, students, and other stakeholders — including, but not limited to, administrators — so that pushback can be responded to using MCS as necessary.

A second corollary is that MCS can be wielded in ways that maintain or exacerbate power imbalances; consider a mathematics instructor who responds to a student’s concern by choosing to employ MCS in ways that denigrate that student’s position and dismiss their concerns (e.g., appealing to mathematically unfamiliar ways in which various averages can be computed for a course grade). Alternatively, administrators may attempt to use a form of MCS in order to justify a negative evaluation of an instructor’s contribution to the institution by appealing to Value Added Models or other quantitative measures that rely on inaccessible mathematical or statistical language.

Second, we would be remiss if we did not mention our own positionality as white, cishetero-presenting male teachers. There can be no doubt that these identity factors, among others, allow our divergence from typical curricula to be perceived more often as creative and innovative rather than inappropriately disruptive or pedagogically insubordinate. Just as we advocate for MCS

based on the privileging of formal mathematical language, so, too, must we recognize that our own identities are privileged in ways that make linguistic shifts a viable tool in combating criticism of our work as educators.

CONCLUSION

In this paper, we have endeavored to explain a single project that the two authors — a mathematics instructor and an educational technologist — have designed and modified to foster creative participation among our students. To do so, we have set forth our own definition and framing of participatory creativity; we have described the project and showed examples of student work (and include materials to implement the project in our appendix); and we have described a technique that we label as mathematical code switching (MCS) to respond to outside criticisms of our teaching, although it is a tool that is necessarily imperfect, and an approach around which we hope to see further discussion and analysis. Nevertheless, we are hopeful that other educators can draw from our nascent and ongoing experiences with this particular project, and the evolving language that we use to describe it, in imagining how to facilitate their own learners' participation in mathematical creativity.

Figure 13: A student-created 8×8 L-tromino board.

References

- [1] Baer, John. "The case for domain specificity of creativity," *Creativity Research Journal*, Volume **11** Number 2 (1998), pages 173-177.
- [2] Beghetto, Ronald A. "Creativity in the classroom," in *The Cambridge Handbook of Creativity*, J.C. Kaufman and R.J. Sternberg, eds., Cambridge University Press, New York NY, 2010, pages 447-463.
- [3] Beghetto, Ronald A., and James C. Kaufman. "Toward a broader conception of creativity: A case for 'mini-c' creativity," *Psychology of Aesthetics, Creativity, and the Arts* Volume **1** Number 2 (2007), pages 73-79.
- [4] Brown, Stephen I., and Marion I. Walter. *The Art of Problem Posing*, Psychology Press, London, 2005.
- [5] Costello, Patrick J. "Tiling Squares with Big Holes with L-trominoes." *The College Mathematics Journal* **48**, no. 4 (2017): pages 259-263. <https://www.tandfonline.com/doi/pdf/10.4169/college.math.j.48.4.259>.
- [6] Cropley, David H., et al., eds. *The Dark Side of Creativity*. Cambridge University Press, New York, NY, 2010.
- [7] Cuoco, Al, E. Paul Goldenberg, and June Mark. "Habits of mind: An organizing principle for mathematics curricula," *The Journal of Mathematical Behavior*, Volume **15** Number 4 (1996), pages 375-402.
- [8] Dickman, Benjamin. *Conceptions of creativity in elementary school mathematical problem posing* (2014), Doctoral dissertation, Columbia University.
- [9] Dickman, Benjamin. "Looking Back to Support Problem Solving," *Mathematics Teacher*, Volume **110** Number 1 (2016), pages 54-58.
- [10] Gardner, Martin. "L-Tromino Tiling of Mutilated Chessboards." *The College Mathematics Journal*, Volume **40** Number 3 (2009), pages 162-168.
- [11] Getzels, Jacob W., and Mihaly Csikszentmihalyi. "The creative vision: A longitudinal study of problem finding in art." *Journal of Aesthetics and Art Criticism*, Volume **36** Issue 1 (1977), pages 96-98.

- [12] Golomb, Solomon W. *Polyominoes: puzzles, patterns, problems, and packings*. Princeton University Press, Princeton NJ, 1996.
- [13] Gruber, Howard E. “The evolving systems approach to creative work.” *Creativity Research Journal*, Volume **1** Issue 1 (1988), pages 27-51.
- [14] Gruber, Howard E., and Sara N. Davis. “Inching our way up Mount Olympus: The evolving-systems approach to creative thinking.” In *The Nature of Creativity: Contemporary Psychological Perspectives* edited by R.J. Sternberg (Cambridge University Press, 1988), pages 243-270.
- [15] Guilford, Joy Paul, and John I. Lacey. *Printed classification tests: Report Number 5*, U.S. Government Printing Office, 1947.
- [16] Guilford, Joy Paul. “Creativity.” *American Psychologist*, Volume **5** Number 9 (1950), pages 444-454.
- [17] Gutiérrez, Rochelle. “Strategies for creative insubordination in mathematics teaching.” *Teaching for Excellence and Equity in Mathematics*, Volume **7** Issue 1 (Special Issue Mathematics Education: Through the Lens of Social Justice, 2016), pages 52–60.
- [18] Gutiérrez, Rochelle. *Rehumanizing Mathematics: Rehumanizing Mathematics for Black, Indigenous, and Latinx Students*. Annual Perspectives in Mathematics Education. NCTM, Reston VA, 2018.
- [19] Hanson, Michael Hanchett. *Worldmaking: Psychology and the Ideology of Creativity*. Springer, New York NY, 2015.
- [20] Hanson, Michael Hanchett. “The ideology of creativity and challenges of participation.” *Europe’s Journal of Psychology*, Volume **11** Number 3 (2015), pages 369-378.
- [21] Kaufman, James C., and Ronald A. Beghetto. “Beyond big and little: The four c model of creativity.” *Review of General Psychology*, Volume **13** Number 1 (2009), pages 1-12.
- [22] Klamkin, Murray S. (1994). “Mathematical Creativity in Problem Solving and Problem Proposing. II.” In *In Eves’ Circles*, edited by Joby Milo Anthony (MAA Notes Volume **34**), pages 73-95.

- [23] Nelsen, Roger. *Proofs Without Words II: More Exercises in Visual Thinking*. Mathematical Association of America, Washington DC, 2000.
- [24] Poincaré, Henri. *Science and Hypothesis*. Walter Scott Publishing Co., London, 1907.
- [25] Pólya, George. *How to Solve it*. Princeton University Press, Princeton NJ, 1945.
- [26] Rhodes, Mel. "An analysis of creativity." *The Phi Delta Kappan*, Volume 42 Number 7 (1961), pages 305-310.
- [27] Roeder, O. "How many numbers contain the numbers of their numbers?" *FiveThirtyEight*, posted on March 8, 2019. Available at <https://fivethirtyeight.com/features/how-many-numbers-contain-the-numbers-of-their-numbers/>. Accessed 25 April 2020.
- [28] Roeder, O. "Get Perplexed With Some Games Of Hex." *FiveThirtyEight*, posted on March 15, 2019. Available at <https://fivethirtyeight.com/features/get-perplexed-with-some-games-of-hex/>. Accessed 25 April 2020.
- [29] Rubel, Laurie, and Andrea V. McCloskey. "The 'Soft Bigotry of Low Expectations' And Its Role in Maintaining White Supremacy through Mathematics Education," *Bank Street Occasional Paper Series*, Number 41 (2019), pages 113-128. Available at <https://educate.bankstreet.edu/occasional-paper-series/vol2019/iss41/10>
- [30] Sawyer, R. Keith, and Stacy DeZutter. "Distributed creativity: How collective creations emerge from collaboration." *Psychology of aesthetics, creativity, and the arts*, Volume 3 Number 2 (2009), pages 81-92.
- [31] Sawyer, R. Keith. *Structure and Improvisation in Creative Teaching*. Cambridge University Press, New York NY, 2011.
- [32] Schoenfeld, Alan H. *How We Think: A Theory of Goal-Oriented Decision Making and its Educational Applications*, Routledge, New York NY, 2010.
- [33] Silver, Edward A. "On mathematical problem posing." *For the Learning of Mathematics*, Volume 14 Issue 1 (1994), pages 19-28.

- [34] Starr, Norton. “Golomb, Gardner, Benjamin and Jones: Midwives to a Puzzle.” In *Homage to a Pied Puzzler*, Pegg, Schoen, & Rodgers, eds., CRC Press, 2009, pages 269-276.
- [35] Starr, Norton. “The Tromino Puzzle” (2012); available at <https://nstarr.people.amherst.edu/trom/intro.html>, accessed on January 29, 2020.
- [36] Su, Francis Edward. “Mathematics for human flourishing,” *The American Mathematical Monthly*, Volume **124** Number 6 (2017), pages 483-493.
- [37] Wallas, Graham. *The Art of Thought*. Jonathan Cape, London, 1926.
- [38] Weisberg, Robert W. *Creativity: Understanding innovation in problem solving, science, invention, and the arts*, John Wiley & Sons, Hoboken NJ, 2006.
- [39] Van Harpen, Xianwei Y., and Bharath Sriraman. “Creativity and mathematical problem posing: an analysis of high school students’ mathematical problem posing in China and the USA.” *Educational Studies in Mathematics* Volume **82** Number 2 (2013), pages > 201-221.

A. Make a Tromino Game

- Open Illustrator.
- Create a new document.
 - The important settings are 600 mm for Width,
 - 300 mm for Height,
 - and RGB Color for Color Mode.
- View→Rules→Show Rulers to turn on rulers.

- Make the border of our trominoes square
 - Select the rectangle tool, hold Shift, and drag a square out from the corner.
 - Each side length should be 176 mm.
 - To be precise, click Transform and type in the exact dimensions.

- To tell the laser to cut this shape:
 - Make the fill equal to none,
 - and make the stroke color red.
 - For now, leave the stroke at 1 pt so that it can be seen.
 - (Later, it will be changed to 0.001 pt, which it must be for the laser to take it as a cut.)
- Now we need grid lines spaced at 22 mm.
 - Drag them out from the rulers.
 - Select each one and click Transform to type in an exact position.
- Now use the Line Segment tools to draw the borders of you monomino and trominoes.
 - The shapes can share a border, as the laser only needs to cut the shared borders once.
 - Select each one and click Transform to type in an exact position.

- To see how you are doing, hide the guides from time to time using View→Guides→Hide Guides or **Command** + C.

- When you finish making your arrangement of trominoes (and one monomino) drag your mouse over all of the shapes to select them.
 - Change the stroke to 0.001 pt. (You will not be able to see your lines but they are there.)

- Find your Layers palette. If you do not see it, click Windows→Layers.
 - Click the little Post It in the lower right of the Layer palette to make a new layer.
 - Select Layer 2 but keep Layer 1 showing so you can see your guides.

- Now it is time to make your board design
 - Using your guides, draw a line across every row and column.
 - Make the stroke blue.
 - Keep it at 1 pt size.
 - Blue will make the laser **score**, and not cut, the line.

- If you want to can make a checkerboard with black squares.
 - Black will make the laser etch.

- Now, on Layer 2, add vertical and horizontal guides at 100 mm and at 200 mm.

- This is a tricky part:
 - Hide your guides.
 - Drag to select all of your lines.
 - Then show your guides.
 - Now move your selected lines so they are centered over the crosshairs of the 100 mm guides.
- Beautiful!
- Now add a 3rd Layer!
 - This will be for any decoration you wish to make around the margin of your board.
 - Anything in the margin should fit between your lines and the border.
- You could put text.
 - If you do text, when you are done click Type→Create Outlines to turn the text into shapes.

- If you use an image from the web it should be black and white.
 - Download it.
 - Click File→Place and select the file.
 - Rotate and resize your image border until you like it.
 - Click Image Trace to make it into a vector graphic.

- Last step!
 - Click back on your Layer 2.
 - Hide Layer 3.
 - With your lines selected, change their stroke size to 0.001 pt.

Now save your file. Upload it to your team drive folder. It is ready for the laser!