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Virtual Temari: Artistically Inspired Mathematics

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Synopsis

Technology can be a significant aide in understanding and appreciating geometry, beyond theoretical considerations. Both fiber art and technology have been employed as a significant aide and an inspiring vessel in education to explore geometry. The Japanese craft known as *temari*, or "hand-balls", combines important artistic, spiritual, and familial values, and provides one such approach to exploring geometry. Mathematically, the artwork of temari may be classified based on whether they are inspired by polyhedra and discrete patterns or by periodic functional curves. The resulting designs of these categories provide an ancient vantage for displaying spherical patterns. We illustrate a technique that combines the fiber art of temari with interactive computer visualizations. In addition, we provide guided activities to help promote a deeper understanding of how functions and patterns arise on a spherical surface.

Keywords: temari, mathematics, parametric curves, polyhedra, fiber arts, computer graphics.

1. Background

The Japanese *temari* ball can be described as an inner core, wrapped in uniformly-colored thread, forming a sphere which acts as a mesh background for anchoring elegant, hand-embroidered patterns. These patterns are highly geometric and mathematically interesting, as they tend to be repetitive and periodic on the surface of the sphere. Due to their complex intricacy, modern temari are often given as gifts for special occasions, such as weddings, births, or holidays [18]. Becoming recognized as a formal temari craftsperson requires years of training and official certification, further indicating the amount of expertise that must be devoted to the craft (see (`http://www.temarikai.com/ResourcesPages/jtacertification.html`) for more information.)

The temari art form originated from the Asuka Period (538–710) in ancient Japan [18]. It is believed that China introduced temari to Japan as a game involving deerskin balls called *kemari*, or "kick ball", from which the word *temari*, or "hand ball", was later derived [18, 1]. Initially, temari was a craft practiced only by the upper class of Japan, whereby the silk of old kimonos was used to construct a toy that was used in games played by nobles [18, 1, 5, 19]. Steeped alongside Buddhust traditions (Figure 1) and the Japanese philosophy of *mottainai* ("what a waste!"), many Japanese fabric crafts, including temari, are some of the earliest documented cases of fabric repurposing in art [18, 5].

Figure 1: A temari used by the Buddhist monk Ryokan (1758-1831) [10].

During the Edo Period (1603--1868), cotton was introduced to Japan, and *it-*

omari, or *thread balls*, became a craft practiced by the common class [18, 1, 5]. It was also during this period that temari transitioned from a flat, hardy ball used as a toy, into a more decorative ball with deeper cultural significance; the affordability and access to cotton fiber allowed the craft to spread throughout Japan as an artform [18, 1, 5]. Methods for constructing the temari core began to inherit regional significance, dependant on the abundance of natural resources available to the different regions across Japan. Cocoons, clam shells, grass seeds, rice husks, and small stones, have all been documented as being used as temari cores, depending on the region of origin [5]. With the advent of rubber balls as toys, the temari craft became less about toys, and more about art and tradition [18, 5]. Today, itomari are simply called "temari" by Western culture [5]. In modern Eastern culture, temari are given as gifts for life events and holidays. Digital media allowed this artform to flourish, since what was a traditionally regional artform was allowed to spread at a rapid pace. The internet has made it easier to exchange patterns over geographical and linguistic barriers, with patterns often fragmenting away from traditional designs into novel patterns embodying the unique signature of the artist.

Fabric art has been used as a tool for exploring advanced mathematical concepts, such as knitting to explore recursion [23], crochet to explore hyperbolic geometries [11], quilting as a medium for symmetry samplers [9], and even simpler concepts such as quilting to explore fractions [15]. Temari has recently become of interest to mathematicians as an educational tool, due to its connection with spherical geometries; for instance, temari has been used to demonstrate all fourteen of the finite spherical symmetries, incorporating both Frieze groups and Platonic solids in their construction [24, 25]. Temari has also been used in discrete mathematics courses to describe permutations for students [2]. The tactile nature of fabric art allows an individual to directly control how they receive and experience visual input. To this end, individuals who may have an adverse mathematical or adverse artistic mindset benefit from fabric art immensely, as it bridges the gap between these two disciplines.

The toy-like characteristic of a sphere has also been represented digitally in a video game called *Katamari Damacy* [13]. The objective of this game is to roll up pieces of the environment onto a sphere, making the ball as large as possible, a process akin to wrapping a temari core before the embroidery process begins. Lead game designer Keita Takahashi has stated that two major tenets of game design are that games need to be easily understood and enjoyable [12]. *Katamari Damacy* was received favorably by most video game critics and received many awards in 2004 [17]. The game also received further prestige and validation as an artform when it was selected as an installation in New York's Museum of Modern Art in 2013 [21].

One significant bottleneck for fabric art as an educational tool is the amount of time and training it may take to construct a specific pattern or design; an advanced temari artist could conservatively spend over 8 hours working a simple pattern, whereas advanced patterns could take over 40 hours. Another bottleneck is that once constructed, the fabric art may only be on display or in use for one location at a time. Takahashi's tenants of fun and ease-of-use in game design for *Katamari Damacy* are none too different than precepts often embraced in mathematics education. Using technology to simulate temari construction, we hope to mitigate the educational bottlenecks of fabric art, making spherical mathematics fun and accessible for a wide spectrum of people, from artists to mathematicians.

2. Mathematics

We hope to construct some basic connections between fabric art and digital media in a manner that makes spherical functions easy to understand inspirational. We will categorize, parameterize, and describe 5 groups of mathematical constructs as a model for patterns commonly found in temari artwork: rings, bands, hypocycloids, Lissajous curves, and discrete patterns. We will then provide guided exercises that we hope will inspire a deeper understanding of mathematics, regardless of the reader's mathematical background.

Without loss of generality, it should be noted that due to the nature of normalized, spherical equations, that any of the x, y, or $z(t)$, functions in the following parameterized equations may be interchanged across the Cartesian variable space, based on preference. We also will construct all parameterizations so that the virtual temari balls are centered at the origin, and have a radius of 1.

2.1. Rings

Temari preparation is an intricate aspect of the craft, which requires precise measurement, as the preparation of the ball helps to determine what patterns will be observed on the final product. After wrapping a temari ball in thread, the ball gets subdivided using what are referred to as Simple and Combination divisions (Figure 2a,c). Simple divisions only have a north and south pole with an equator. Combination divisions have multiple poles equally spaced across the temari. These divisions are also classified based on the number of vertices at the poles. The subdivision process in temari preparation (see Figure 2, [18], [5], and [20] for further information on how this process works) makes the pattern look as though there are thread rings partitioning the ball either across longitudinal or latitudinal axes. Such rings on a three-dimensional sphere may described using only two trigonometric functions in Cartesian coordinates:

$$\begin{aligned} x(t) &= \cos(t), \\ y(t) &= 0, \\ z(t) &= \sin(t), \\ 0 \leq & t \leq 2\pi. \end{aligned}$$

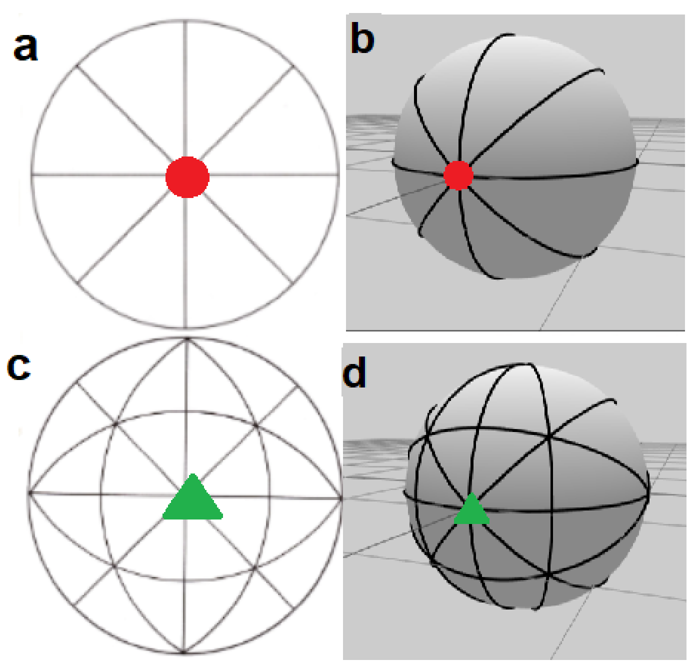

Figure 2: Typical temari divisions and virtual counterparts [20], used with permission, all rights reserved.

Figure 3: Illustration of the maki kagari technique. The central, green band on a temari (left) is created using the *maki kagari* technique [20], used with permission, all rights reserved, and simulated with a compressed helix (right).

2.2. Bands

A technique commonly practiced in temari construction is known as *maki kagari*. The goal of this technique is to create a thick band of thread around the center of the temari, splitting the ball into hemispheres (Figure 3). We used the equations for a compressed helix to simulate this technique:

$$
\begin{aligned}
x(t) &= \cos(t)/N, \\
y(t) &= (R/\pi)t/N, \\
z(t) &= \sin(t)/N, \\
-A\pi &\le t \le A\pi
\end{aligned}
$$

where

$$N = \sqrt{1 + (R/\pi)^2 t^2}$$

is a normalization factor so that the bands of the helix hug the surface of the sphere, R is a parameter determining the thickness of the thread, and A is an even integer describing the total number of thread wraps circling the center of the ball ($A/2$ threads on each side of the temari equator, respectively).

2.3. Star patterns

Hypocycloids, or Spirograph curves, are generated by tracing a single point on the circumference of a rolling, smaller circle within a larger circle (Figure 4, left). The results are designs that appear to be star-like patterns (Table 1). Many temari patterns are the result of stitching straight lines of thread between the poles and some predetermined nodes along the longitudinal bands of the division threads (Figure 5). The result appears similar to a hypocycloid projected onto a sphere. The following set of equations use trigonometric functions to generate these patterns:

$$\begin{aligned} x(t) &= (1 - A/B)\cos(t) + A/B\cos((1 - B/A)t), \\ y(t) &= \pm\sqrt{(4(1 - A/B)(A/B))} \cdot \sin(B/(2A)t), \\ z(t) &= (1 - A/B)\sin(t) + A/B\sin((1 - B/A)t), \\ R &= A/B < 1, \end{aligned}$$

where A and B are referred to as *toy parameters*. We choose to call these parameters toy parameters due to their highly unconstrained nature, and the fact that the user may feel free to adjust these parameters until they get a pattern they find aesthetically pleasing. Hypocycloids have what is known as the multiple-generation property; the reduced form of the ratio $R = A/B$ determines the uniqueness of the pattern observed on the spherical surface. R must also follow the constraint $R < 1$ for real-curve results.

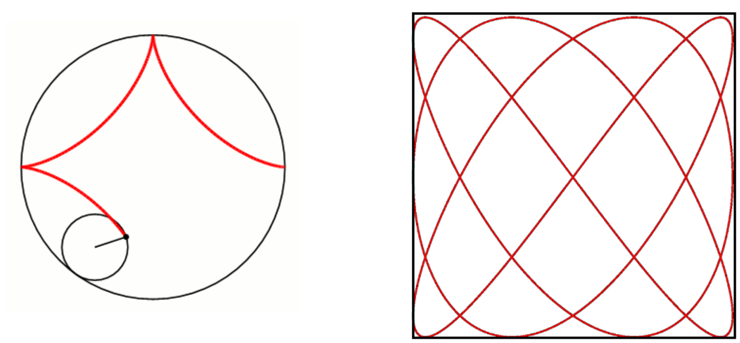

Figure 4: Hypocycloid versus Lissajous curves. Hypocycloids, or Spirograph curves, are generated by tracing a single point on the circumference of a rolling, smaller circle within a larger circle (left) [22], used under Creative Commons license. Similarly, Lissajous curves are generated tracing a small circle bounded within a rectangle (right) [3], used and modified under Creative Commons license.

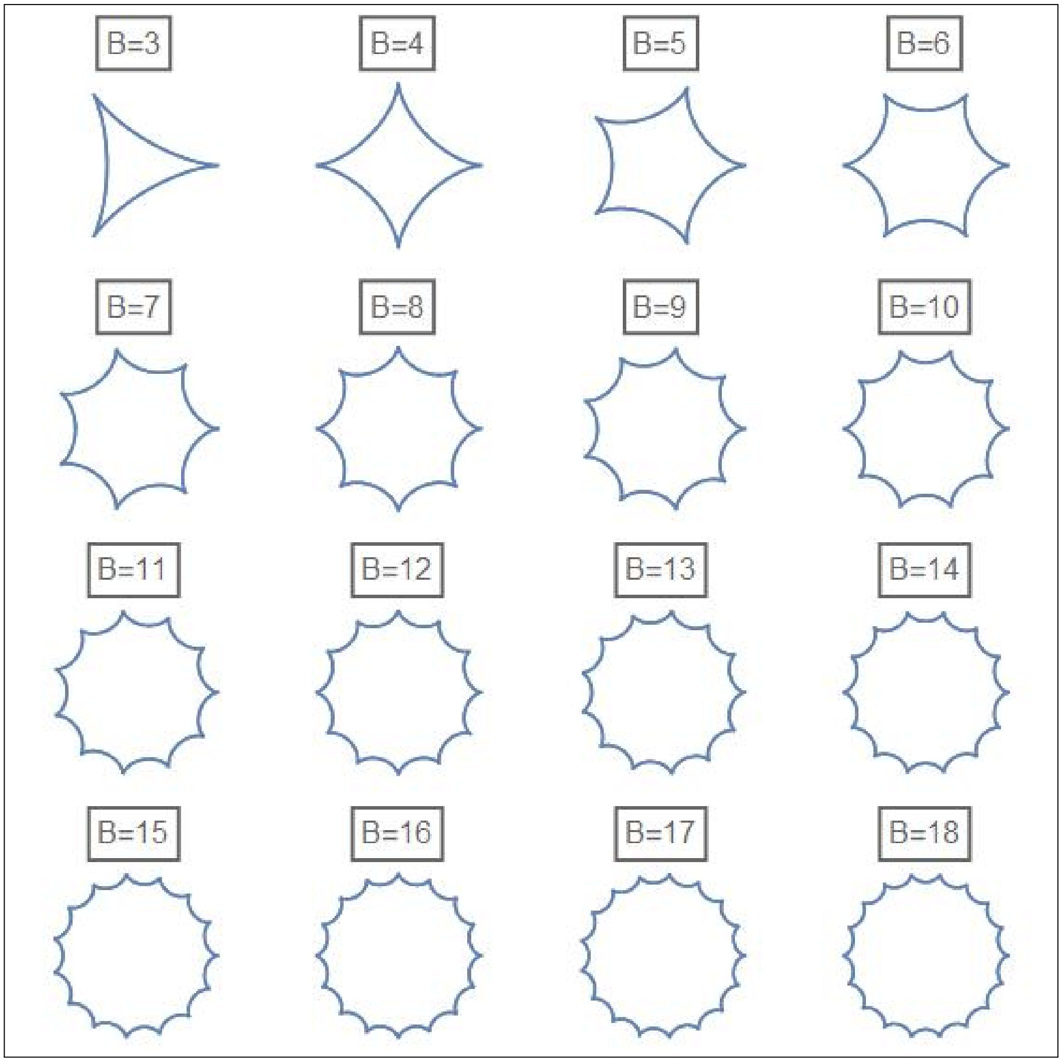

Table 1: Hypocycloid patterns in a circle. Some typical cycloid patterns generated when the parameter A is fixed at 1, and B is allowed to vary over the integers from 3 to 18. In this special case of the cycloid patterns, the parameter B corresponds to the number of points on the cycloid.

Figure 5: Illustration of star-like patterns on a temari. Star-like patterns (left) [20], used with permission, all rights reserved, are simulated using cycloids (right).

2.4. Lissajous curves

Whereas hypocycloids are generated by bounding within a circle, Lissajous curves, though theoretically similar, are instead, bounded by a rectangle (Figure 4, right). These curves may then be projected onto the surface of a sphere, resulting in another classification of curves commonly seen in temari artwork:

$$\begin{aligned} x(t) &= \sin(At)\cos(Bt) \\ y(t) &= \cos(At) \\ z(t) &= \cos(At)\cos(Bt), \end{aligned}$$

where A and B are, again, toy parameters. Some two-dimensional projections of curves generated by the toy parameters are provided in Table 2.

An interesting note on all hypocycloid and Lissajous curves is that when parameters are irrational, the resulting curves will never terminate at their starting point. In this sense, the domain of the variable t may be allowed to vary as little or as much as desired, causing curves to *fill up* the ball on an infinite domain. Additionally, some instances of Lissajous curves (Table 2, $B = 2$ through 16) are not patterns one would typically observe in temari artwork. We plan to address both of these substantive points in future work.

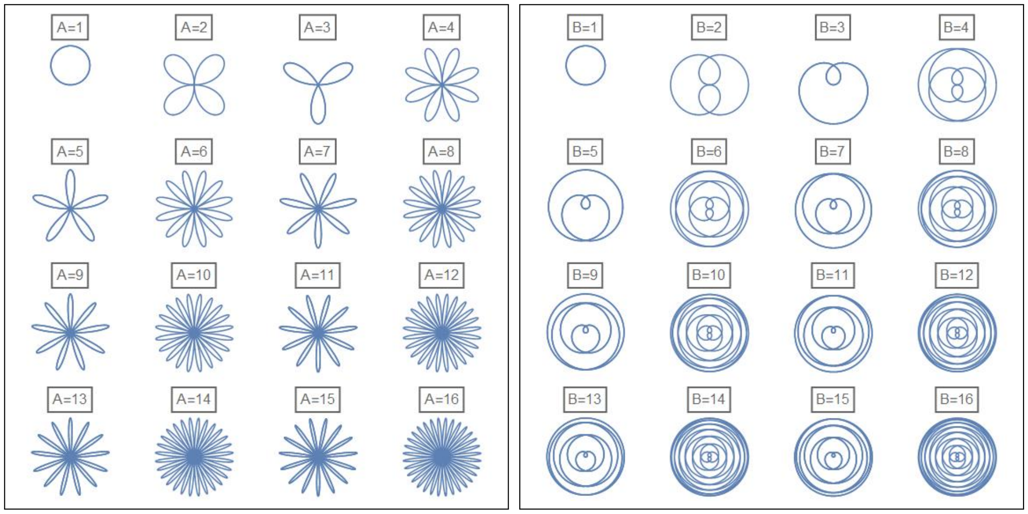

Table 2: Lissajous Patterns. Some two-dimensional projections of Lissajous patterns that can arise when either A (right) or B (left) are fixed at 1, and the other parameter is allowed to vary over the integers from 1 to 16.

Figure 6: Lissajous patterns on a temari. Some patterns observed in temari might be perceived as petals (left) [20], used with permission, all rights reserved, and simulated using Lissajous patterns (right).

2.5. Discrete Patterns

Many complex divisions of temari begin with selecting a set of rotationally symmetric points on the surface of a sphere and connecting the points to partition the surface into congruent spherical polygons. Frequently, the locations of these points are chosen so that the resulting pattern will have a great amount of rotational symmetry around many axes of the sphere, which leads to arrangements such as the vertices of a Platonic, Archimedean, or Catalan solid. Other arrangements of points which possess mainly rotational symmetry around a single axis may correspond to vertex arrangements of polyhedra such as bipyramids, trapezohedra, prisms, or antiprisms. See Figure 7.

Figure 7: Discrete patterns. Discrete patterns in temari are often the result of complex division patterns (left) [20], used with permission, all rights reserved, and are simulated by tiling polyhedra on a sphere (right).

3. Programming

Inspired by the beauty of temari and excited by the possibility of using mathematics to model and explore such patterns, we have created an interactive application to visualize these curves. The application is called *EMARI*: Exploratory Mathematical and Artistic Rendering Interface. In order to be maximally accessible in terms of technology, the application is web-based: EMARI runs in any web browser and requires no downloads or installation. The user interface has been thoughtfully designed in the hopes of being usable and of value to multiple audiences: artists and mathematicians, students and teachers, amateurs and professionals alike. We have strived to include a balanced number of features, great enough to provide a wide range of artistic freedom to customize the appearance of the temari, while small enough to avoid overwhelming the user. With the current set of features, which are described in detail later in this section, the application can be used to experiment, create, educate, and inspire. The app, currently hosted at the website https://stemkoski.github.io/EMARI/, is pictured in Figure 8 on the next page, which illustrates a scenario where both a parametric curve and a discrete pattern have been added.

At the top of the app is an interactive 3D viewer that illustrates all of the curves and patterns that have been entered. Depending on whether the user is using a mouse or a touchscreen, they may click/touch and drag to rotate the scene, and use a mouse wheel or pinch gestures to zoom in and out. Buttons below the image enable the user to change the size of the viewport, save the current view to an image file, or add additional curves or patterns to the scene. Underneath these buttons are various controls to customize the overall appearance of the scene, including buttons that show/hide a grid along the horizontal (XZ-)plane, XYZ-axes, and a unit sphere. The user may also change the color or opacity of the unit sphere, change the background color used for the scene itself, and enable or disable the lighting and shading effects that are used to increase the 3D appearance of the scene.

When a new curve or pattern is added, a corresponding data entry area will be added to the bottom of the application. Since this may result in a large amount of content, this data appears in a scrolling region beneath the 3D scene viewer, so that (at least on larger display screens) the user can scroll any data area closer to the 3D viewer area so that they can more easily view the effects of changing parameters in real time, without having to scroll back and forth between these areas in the application. For both parametric curves and discrete patterns, the user may choose among a variety of preset configurations, and they may adjust the rotation of the graphs around the X/Y/Z axes, the color and thickness (tubular radius) of the curves representing the thread, and the layer, which represents the distance from the unit sphere to the graphs in multiples of each curve's thickness. For parametric curves, the user may enter equations for the x, y, and z coordinates in terms of t and, if desired, in terms of the given parameters A, B, and C, and the thickness parameter R. For discrete patterns, the user may enter vertex data and edge data. Each vertex is represented as an array of coordinates $[x, y, z]$,

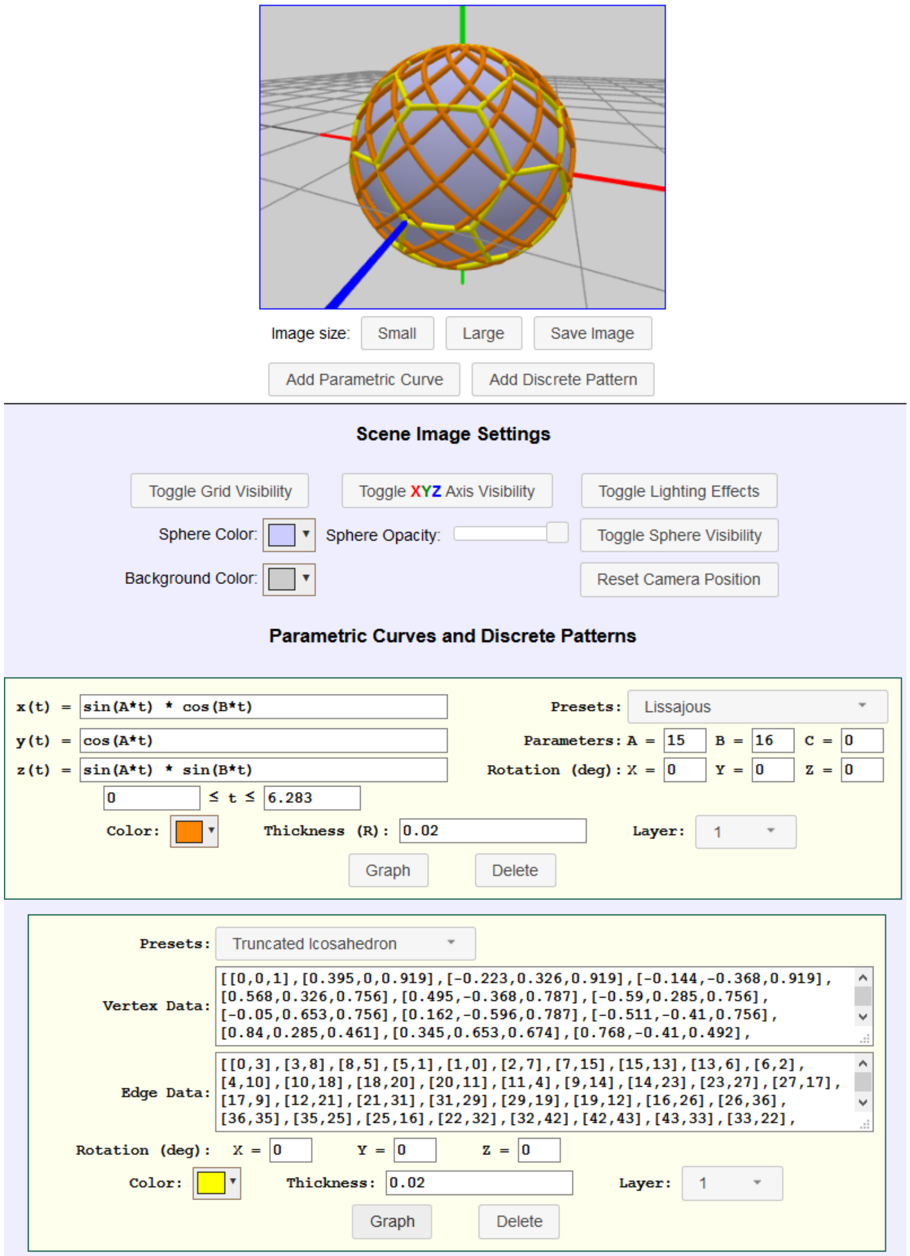

Figure 8: The EMARI Application. For more examples, and in particular for the parameters used to generate the images in this article, see Appendix A. See Appendix B for some sample questions to use EMARI in student investigations.

and the vertex data is an array of vertices, while each edge is represented as an array of two vertex indices, and the edge data is an array of edges. The vertex and edge data appear in text areas that can be scrolled or enlarged if desired. Finally, individual curves and patterns may be removed from the scene by pressing the delete button at the bottom of the corresponding data entry area.

Users may interact with the application at a variety of levels, depending on their background. We recommend new users begin by experimenting with the different presets in both categories, and adjusting the values of the parameters and rotation about the axes. When adding multiple curves, we recommend changing the graph colors, so that they are more easily distinguishable from each other, and also for the aesthetic joy of creating a colorful composition. Advanced users may want to experiment with their own sets of parametric equations or arrays of vertices and edges; it should be noted that no *normalization calculations* take place behind the scenes to force parametric curves to lie on the unit sphere, and thus this application could theoretically be used to graph any desired curves. Additionally, due to the inherently discrete nature of all computer graphics, the tubes representing the temari threads are in fact triangulated surfaces, and extremely large ranges of t values could result in curves that appear more angular than smooth; very large numbers of sample points are used when rendering each curve in the hopes that such an unsightly curve will rarely be seen by most users.

4. Temari as a Vehicle for Mathematical Creativity

Designs and patterns of geometry have long served a purpose of engaging students who may otherwise be adverse towards mathematics, due to their creative nature [3, 26]. More recently, there has been interest in the pedagogical benefits of interfacing mathematics, computer sciences, and artistic expression, as a vehicle for cultural equity in the classroom [6, 14, 4]. Part of this educational phenomenon stems from the affordability and accessibility of personal computers, three-dimensional printers, and virtual reality devices [16, 7]. EMARI taps into historical and modern creativity techniques implicit in this pedagogical phenomenon to provide a concrete work of art that the user can easily share with peers. EMARI provides a framework for mathematically creative customization that might otherwise be impossible due to the meticulous training and resources required by the temari fiber artform.

The authors had to carefully consider a huge landscape of parametric equations normalized to align with the surface of the unit sphere and then compare them to samples of temari patterns, in order to choose the classes of functions included in the EMARI app, making every attempt to balance simplicity of mathematical representation, alignment with temari patterns, and versatility in producing aesthetically pleasing designs. The authors acknowledge that the functions embedded in the EMARI app are merely a starting point for the multitude of beautiful, virtual analogs, present in traditional temari patterns. In addition, throughout the design process, the authors experimented with factors such as the radius of the tubes rendered around parametric curves in order to simulate thickness of thread, whether or not to include a unit sphere and coordinate axes for reference, and whether to include realistic lighting and shading effects to help provide a sense of depth. Ultimately, upon reflecting that each user will have their own aesthetic desires and purposes, the decision was made to allow the user to enable or disable and customize these aspects whenever possible. The EMARI interface encourages mathematical discovery and creativity by allowing users to tweak everything from mathematical parameters and functions to thread width, thread color, and how these thread layers sit on the temari ball. The classes of curves outlined in this paper are, by no means, exhaustive, and the authors encourage the reader to research their own temari pattern-inspired mathematical designs, to help further expand upon this body of work.

5. Future Work and Closing Remarks

Just as a good mathematical question inspires further questions and new directions for research, the EMARI application has many avenues along which development could be continued. Taking inspiration from temari, one could introduce the ability to repeat a simple pattern (such as a small star) at a given set of vertices, or streamline the process for adding multiple instances of the same curve or pattern with an additional amount of rotation or scaling applied to each new instance. Taking inspiration from mathematics, one could introduce new categories of curves: procedurally generated curves, fractal-like patterns, or Turing patterns. To increase the ability of EMARI explorers to communicate and share their creations, it would be worthwhile to add the ability to save and load the data underlying their creations as a custom file type, or export their creations to a file type amenable to 3D printing.

The emphasis of this paper has been defining and modeling temari art with functions and patterns. One advantage that digital media has over fabric art is that some of the models in EMARI may be difficult or impossible to construct with a needle and thread; irrational parameters used in the hypocycloid and Lissajous curves would lead to patterns which traditional temari patterns cannot capture. Additionally, the curves observed in the right-hand panel of Table 2 would be very difficult to construct using the conventional division patterns observed in Figure 2. We believe that EMARI allows temari artists to explore complicated patterns before they sink hours into defining the pattern and construction on their own accord. Finally, the flexibility that EMARI provides with the placement of edges and vertices on the sphere might help temari artists visualize asymmetric or unconventional tilings on the sphere, which could lead to extraordinarily unique, albeit nontraditional, temari patterns.

In closing, the reader may have noticed that the acronym EMARI is inspired in turn by our original source of inspiration, temari, but we hope that deeper similarities hold true. Just as the temari art form continues to evolve, we hope that these investigations will continue to do so as well. In addition, just as temari were once considered a toy, it is our hope that EMARI will encourage active use, creative play, and experimentation. Finally, recalling that temari are given as gifts, we like to think of EMARI as our gift to the mathematical and artistic communities, and we hope that these efforts further strengthen the beautiful bridges that connect these two communities.

References

[1] Bernadette A. Berken, "Ethnomathematics: unlocking the wonders of mathematical ideas," in 82nd Annual Meeting of the National Council of Teachers of Mathematics, Philadelphia PA, 2004. Available at https://www.maa.org/sites/default/files/pdf/cms_upload/Berkenhandout252426.pdf, last accessed on July 28, 2020.

[2] Debra K. Borkovitz, "A Temari Permutation Sampler," in *Bridges 2017 Conference Proceedings*, pages 363–366, 2017. Available at https://archive.bridgesmathart.org/2017/bridges2017-363.pdf, last accessed on July 28, 2020.

[3] William M. Carroll, "Using a Polar Coordinate System in the Classroom." *The Arithmetic Teacher* **37**, no. 8 (1990): 42.

[4] James Davis, Michael Lachney, Zoe Zatz, William Babbitt, and Ron Eglash. "A cultural computing curriculum." In *Proceedings of the 50th ACM Technical Symposium on Computer Science Education*, pp. 1171-1175. 2019.

[5] Anna Diamond, *The Temari Book: Techniques and Patterns for Making Japanese Thread Balls*, Lark Books, Asheville NC, 1999.

[6] Ron Eglash, Audrey Bennett, John Drazan, Michael Lachney, and William Babbitt. "A mathematical tool kit for generative justice." *ETD-Educação Temática Digital* **19**, no. 3 (2017).

[7] Laura Freina, and Michela Ott. "A literature review on immersive virtual reality in education: state of the art and perspectives." In *The International Scientific Conference eLearning and Software for Education*, vol. **1**, no. 133, pp. 10-1007. 2015.

[8] Susan Goldstine, "A recursion in knitting," in *Bridges 2016 Conference Proceedings*, pages 395–398, 2016. Available at https://archive.bridgesmathart.org/2016/bridges2016-395.pdf, last accessed on July 28, 2020.

[9] Susan Goldstine, "A survey of symmetry samplers," in *Bridges 2017 Conference Proceedings*, pages 103–110, 2017. Available at https://archive.bridgesmathart.org/2017/bridges2017-103.pdf, last accessed on July 28, 2020.

[10] Google Arts and Culture contributors, A temari used to be owned by Buddhist monk Ryokan, Sanuki kagari temari, *Google*, available at https://artsandculture.google.com/asset/a-temari-used-to-be-owned-by-buddhist-monk-ryokan-sanuki-kagari-temari/8wGiW5giR2dMtg, last accessed on July 28, 2020.

[11] David W. Henderson and Daina Taimina, "Crocheting the hyperbolic plane," *Mathematical Intelligencer*, Volume **23** Number 2 (2001), pages 17–28; available at http://pi.math.cornell.edu/~dwh/papers/crochet/crochet.html, last accessed on July 28, 2020.

[12] Alfred Hermida, "Katamari creator dreams of playgrounds," *BBC News* at BBC.com, November 10, 2005; available at http://news.bbc.co.uk/2/hi/technology/4392964.stm, last accessed on July 28, 2020.

[13] *Katamari Damacy* [Game], Namco, 2004, PS2.

[14] Michael Lachney, William Babbitt, Audrey Bennett, and Ron Eglash. "A Voice to Talk About it: Cosmetologists as STEM Experts in Educational Technology Design and Implementation." *European Journal of Open, Distance and E-Learning* **22**, no. 2 (2020): 41-55.

[15] Kristin Kaster Lamberty and Janet L. Kolodner, "Exploring digital quilt design using manipulatives as a math learning tool," in *Proceedings of ICLS*, vol. **2**, International Society of the Learning Sciences, 2002, pages 1–7.

[16] Robert L. Martin, Nicholas S. Bowden, and Chris Merrill. "3D printing in technology and engineering education." *Technology and engineering teacher* **73**, no. 8 (2014): 30.

[17] Metacritic contributors, "Katamari Damacy for PlayStation 2," *metacritic* reviews; available at https://www.metacritic.com/game/playstation-2/katamari-damacy, last accessed on July 28, 2020.

[18] Barbara B. Suess. *Temari Techniques: A Visual Guide to Making Japanese Embroidered Thread Balls*. Breckling Press, 2012.

[19] Ginny Thompson, "Making the dodai mari," web resource, 2014. Available at http://www.temarikai.com/HowToPages/marimaking.html, last accessed on July 28, 2020.

[20] Ginny Thompson, *Temari Kai*, web resource, 2014. Available at http://www.temarikai.com/, last accessed on July 28, 2020.

[21] Jeff Grubb, "Katamari Damacy is arty enough for the New York Museum of Modern Art," *venturebeat.com*, July 30, 2012. Available at https://venturebeat.com/2012/07/30/katamari-damacy-is-arty-enough-for-the-new-york-museum-of-modern-art/, last accessed on July 28, 2020.

[22] Wikipedia contributors, *Hypocycloids*, Wikipedia, The Free Encyclopedia, available at https://en.wikipedia.org/wiki/Hypocycloid, last accessed on July 28, 2020.

[23] Susan Goldstine, "A recursion in knitting," in *Bridges 2016 Conference Proceedings*, pages 395–398, 2016. Available at https://archive.bridgesmathart.org/2016/bridges2016-395.pdf, last accessed on July 28, 2020.

[24] Carolyn A. Yackel, "Marking a Physical Sphere with a Projected Platonic Solid", *Bridges 2009 Conference Proceedings*, pages 123–130, 2009. Available at https://archive.bridgesmathart.org/2009/bridges2009-123.pdf, last accessed on July 28, 2020.

[25] Carolyn Yackel, Bridges Art Galleries contribution, 2012. Gallery contribution available at http://gallery.bridgesmathart.org/exhibitions/2012-bridges-conference/cyackel, last accessed on July 28, 2020.

[26] Yilmaz Zengin and Enver Tatar. "The teaching of polar coordinates with dynamic mathematics software." *International Journal of Mathematical Education in Science and Technology* **46**, no. 1 (2015): 127–139.

A. Supplemental Material: Parameters for Generating the Figures

Here, we list the settings used in the EMARI software to generate figures in this paper.

- Figure 2b
  - Thread 1: Preset: Ring, default
  - Thread 2: Ring, Rotation $X = 45$
  - Thread 3: Ring, Rotation $X = 90$
  - Thread 4: Ring, Rotation $X = 135$
- Figure 2d
  - Thread 1: Preset: Ring, default
  - Thread 2: Ring, Rotation $Z = 45$
  - Thread 3: Ring, Rotation $Z = 90$
  - Thread 4: Ring, Rotation $Z = 135$
  - Thread 5: Ring, Rotation $X = 90,\ Y = 60$
  - Thread 6: Ring, Rotation $X = 90,\ Y = 120$

  - Thread 7: Ring, Rotation $X = 90$, $Y = 60$, $Z = 90$
  - Thread 8: Ring, Rotation $X = 90$, $Y = 120$, $Z = 90$
- Figure 3
  - Thread 1: Preset: Bands, $A = 10$
- Figure 4
  - Thread 1: Preset: Stars, Parameter $A = 5$, $B = 13$
  - Thread 2: Preset: Stars, Parameter $A = 7$, $B = 13$
  - Thread 3: Preset: Bands, Parameter $A = 20$
- Figure 5
  - Thread 1: Preset: Lissajous, Parameter $A = 10$, $B = 3$
  - Thread 2: Preset: Bands, $A = 10$
- Figure 6
  - Thread 1: Preset: Dodecahedron, default

B. Supplemental Material: Guiding Questions for Investigation with EMARI

1. To produce the Simple division pattern previously described in Figure 2 in the EMARI application, you can begin by adding a new parametric curve and changing the preset to Ring. Is there any difference between changing the X rotation to 30 degrees versus changing the Z rotation to 30 degrees? If instead you had changed the Y rotation to 30 degrees, why does the curve not appear any different? To complete the Simple division, how many additional parametric Ring curves do you need to add, and by what amounts do you need to rotate them?
2. The formula for the distance from a point (x, y, z) to the origin is $\sqrt{x^2 + y^2 + z^2}$. Since EMARI models a temari ball as the surface of a unit sphere, a point lies on the surface of the temari ball if and only if it has distance 1 from the origin. So for example, the point P = (3, 4, 12) does not lie on the temari ball, but if you scale (divide) the coordinates of this point by its length (a process called *normalization*), the result does lie on the temari ball. Here, the length of P is 13, and therefore the point (3/13, 4/13, 12/13) is on the temari ball. As we will see in the following exercise, a similar process helps us construct curves that

lie on the surface of a sphere.

a. In the EMARI application, create a new parametric curve, and enter the functions:
   x = 2*t -1, **y = 1**, **z = 1** and range **0 <= t <= 1**
   Click the *Graph* button and you will see this is the parameterization of a line that lies above the sphere. Write the formula for the distance from the origin to each point on the curve (which will be a function in terms of t). In the EMARI application, divide each of the x, y, and z functions by this formula (you can enter **sqrt()** to calculate square roots). When you graph this new parameterization, how does the appearance of the curve change?

b. The process to model bands on the surface of a temari begins with a parametric equation of a helix. In the EMARI application, enter the equations:
   x = cos(t), **y = A*t**, **z = sin(t)** and range **-10*pi <= t <= 10*pi**
   Set the parameter **A** equal to **0.02** for now, and graph the function. Experiment with different values of **A** until the helix is vertically compressed such that there are no visible "vertical gaps" as the curve revolves around the y-axis. Next, divide each of the x, y, and z functions by a distance function, as before, so that the graph of the function lies on the surface of the temari ball (simplification will be possible using a trigonometric identity!).

3. Angles have exciting and unexpected behaviors on the surface of the sphere. In this exercise, we will draw two line segments in the EMARI application with an angle between them. To do this, create a new discrete pattern with Vertex Data: **[0,0,1]**,**[1,0,0]**,**[0,1,0]**, and Edge Data: [[**0,1**],[**0,2**]]. Observe that the angle between the two line segments is 90 degrees. Now, create a new discrete pattern with Vertex Data: [[**1,0,0**],[**0,1,0**]], and Edge Data: [[**0,1**]]. Change the color of this new line segment to red.

   c. What is the measurement of the angles formed by the red line segment and each of the two orange line segments?

   d. What common shape appears to be on the surface of the sphere, and what is the sum of the angles of this shape?