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Engaging the Paradoxical: Zeno's Paradoxes in Three Works of Interactive Fiction

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Abstract

For over two millennia thinkers have wrestled with Zeno's paradoxes on space, time, motion, and the nature of infinity. In this article we compare and contrast representations of Zeno's paradoxes in three works of interactive fiction, *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere*. Each of these works incorporates one of Zeno's paradoxes as part of a puzzle that the player must solve in order to advance and ultimately complete the story. As such, the reader must engage more deeply with the paradoxes than he or she would in a static work of fiction. In addition, each of the three works presents a different perspective on the intellectual challenges associated with the paradoxes.

1. Introduction

Zeno's paradoxes are a collection of scenarios that raise difficulties with the notion of infinity, particularly with the assumption that space and time are infinitely divisible. These paradoxes have proved to be intellectually fruitful for millennia: Thinkers from Aristotle [2] to Bertrand Russell [6] have wrestled with the difficulties inherent in the scenarios posed by Zeno. Certain mathematical questions, such as what precisely it means to sum an infinite series, or to let a variable quantity become arbitrarily close to zero, are related to Zeno's paradoxes as well.

Zeno's paradoxes are in the form of stories that anyone can easily read. They have also appeared in various forms in popular culture throughout the years, such as in Tom Stoppard's play *Jumpers* [10] and the comic strip *Dilbert* [1].

These representations are all static: Someone might read the paradoxes and think about them, or perhaps watch actors discussing them, but unless she debates them with others or actively chooses to consider them further she remains a somewhat passive observer of the intellectual challenges they raise.

However, there are also portrayals of Zeno's paradoxes in *interactive fiction*. Unlike static fiction, interactive fiction requires the reader to make choices that advance and potentially change the arc of a story. In this article we compare and contrast the representations of Zeno's paradoxes in three works of interaction fiction: Brian Moriarty's *Beyond Zork* [5], Harry Giles and Joey Jones's *The Chinese Room* [3], and my own *A Beauty Cold and Austere* [9]. Each work encapsulates one of Zeno's paradoxes within an obstacle, or puzzle, that the player must solve in order to complete the game. Solving the puzzle associated with a particular paradox requires that the reader think carefully about the ideas raised by that paradox. Thus these three works of interactive fiction force the reader to engage Zeno's paradoxes more deeply than a work of static fiction can. In addition, each solution represents a different perspective on the intellectual challenges associated with the paradox.

In Section 2 we describe the Dichotomy paradox and the Achilles paradox, the two paradoxes that appear in our three works. In Section 3 we discuss the interactive fiction genre and give an overview of the three works under discussion. Section 4 discusses how the three works represent Zeno's paradoxes, and Section 5 goes into more detail about how the works engage the paradoxes interactively. In Section 6 we describe the solutions to the puzzles associated with the paradoxes and discuss how they represent different approaches to the challenges raised by the paradoxes. Finally, Section 7 contains some concluding remarks.

2. The Dichotomy and Achilles Paradoxes

Two of Zeno's paradoxes, the Dichotomy paradox and the Achilles paradox, are featured in the three works we consider.

The *Dichotomy paradox* starts with the observation that an object traveling a certain distance must first reach its halfway point. However, before the object can reach its halfway point it must have traveled halfway to the halfway point, or a quarter of the total distance. But the object must have also traveled one-eighth of the total length before reaching the quarter point, one sixteenth of the distance before reaching the one-eighth mark, and so forth.

From this perspective, there can be no first interval over which the object travels (any candidate interval for “first” could be subdivided). Without an initial distance interval, Zeno argues, motion cannot occur.

A variation of the Dichotomy paradox takes the “traveling halfway” idea in the other direction. After an object reaches the halfway point to its destination, it must then travel halfway from the halfway mark, to a point that is three-quarters of the total distance. Then it must again travel halfway to its destination, reaching the seven-eighths point. After that it must travel half of the remaining distance, to the fifteen-sixteenths mark. Continuing with this thought process, we see that motion over a finite distance requires traversing infinitely many distance intervals. The paradox’s argument is that this is impossible and, therefore, motion cannot occur.

The *Achilles paradox* posits a race between the swift-footed Achilles and a tortoise. Since the tortoise is slower, it is given a head start. When the race begins, Achilles runs until he reaches the tortoise’s starting location. However, by that time the tortoise has moved on to a second location along the race track, and so Achilles is still behind the tortoise. Then Achilles must run to the tortoise’s second location. Again, though, the tortoise has now moved further down the track and so is still ahead of Achilles. In fact, each time Achilles runs to the tortoise’s most recent location, the tortoise has ambled still further along the race course and so always remains ahead of Achilles. Zeno’s conclusion is that a fast runner can never overtake a slower runner in a race.

Since all physical evidence indicates that motion does actually occur and that faster runners can overtake slower runners, the real intellectual challenge with these two paradoxes is finding the logical flaw in their arguments. (There are some exceptions: For example, the conclusion of Zeno himself appears to have been that time does not exist. [6, page 54]) In the remainder of this article we shall see how our three works of interactive fiction take up the intellectual challenge of these paradoxes.

3. Interactive Fiction

Interactive fiction is a rather broad term that has been used to describe works as different as Terry Winograd’s natural language processing program SHRDLU [4, pages 83–85], Choose Your Own Adventure books [4, page 71], interactive film such as the “Bandersnatch” episode of Netflix’s *Black*

Mirror series¹, and computer-based text adventures such as *Adventure* [4, Chapter 3] and *Zork* [4, Chapter 4]. Nick Montfort, in *Twisty Little Passages: An Approach to Interactive Fiction*, uses a narrower definition, focusing on “computer programs that display text, accept textual responses, and then display additional text in reaction to what has been typed.” [4, page *vi*]. Such a definition would include SHRDLU, *Adventure*, and *Zork*, while excluding Choose Your Own Adventure books and “Bandersnatch.”

The three works of interactive fiction that we consider here all fit Montfort’s definition. Indeed, they are all parser-based text adventures: They are all experienced via a computer displaying text for the reader, in which the reader communicates with the computer by typing in commands to be parsed. For example, here is the beginning to the classic text adventure *Zork I*, together with a couple of reasonable commands a reader might try.

West of House

You are standing in an open field west of a white house, with a boarded front door.

There is a small mailbox here.

> *open mailbox*

Opening the small mailbox reveals a leaflet.

> *read leaflet*

(Taken)

“WELCOME TO ZORK!

ZORK is a game of adventure, danger, and low cunning. In it you will explore some of the most amazing territory ever seen by mortals.

No computer should be without one!”

Zork I claims to be a game, and certainly many works of interactive fiction can be viewed as such. We will adopt that convention as well, occasionally referring to the three works we consider here as games.

¹“Bandersnatch” won Best Use of Multimedia at the 2018 XYZZY Awards, which are like the Academy Awards of interactive fiction.

3.1. *Beyond Zork*

Brian Moriarty's *Beyond Zork* was a commercial release by Infocom in 1987. It is a sequel of sorts not only to Infocom's *Zork* trilogy (as the title indicates) but also to the company's *Enchanter* trilogy. Plot-wise, *Beyond Zork* continues the self-aware fantasy comedy of most of the other works in these two series: Magic is collapsing, and so the player must find a fabled artifact that has the capacity to preserve what remains of magical knowledge for a later age. Gameplay mostly entails the reader solving a series of puzzles in order to acquire the artifact and complete the story. *Beyond Zork* also features a system of leveling and skill development commonly found in role-playing games. While *Beyond Zork*'s system is a bit rudimentary, it was also quite novel for a piece of interactive fiction in 1987. Of the three works we consider here, *Beyond Zork* is the most game-like. You can find more information about *Beyond Zork* at <https://ifdb.tads.org/viewgame?id=9h6o1charof548ii>.

3.2. *The Chinese Room*

The opening screen of Harry Giles and Joey Jones's *The Chinese Room* features a quote from 20th-century philosopher Ludwig Wittgenstein: "Never stay up in the barren heights of cleverness, but come down into the green valleys of silliness." This sets the tone for the work, a madcap romp through a large collection of ideas in philosophy. The authors themselves describe *The Chinese Room* as "a comic adventure set in the fantasy world of philosophy thought experiments." In fact, the title "The Chinese Room" is taken from a thought experiment designed by the American philosopher John Searle on the nature of the human mind [7].

In *The Chinese Room* you play as a philosophy student staggering home after a night of drinking. You fall into bed and have a vivid dream featuring Searle's thought experiment, various ethical dilemmas, Descartes's evil demon, Kant's categorical imperative, a Turing machine, and a variety of other philosophers and important philosophical concepts. To complete the work you must solve the puzzles associated with the thought experiments, thereby demonstrating an understanding of the underlying philosophical ideas.

The Chinese Room was released for free as part of the 2007 Interactive Fiction Competition, where it placed 5th out of 27 works. You can find more information about *The Chinese Room*, and you can play it as well, at <https://ifdb.tads.org/viewgame?id=j6vtd2djn6o97a8b>.

3.3. *A Beauty Cold and Austere*

A Beauty Cold and Austere is quite similar to *The Chinese Room*, although it is more serious in tone and focuses on mathematical ideas rather than philosophical ones. The opening screen features a quote from 20th-century mathematician Bertrand Russell that describes mathematics as possessing “not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture... sublimely pure, and capable of a stern perfection such as only the greatest art can show.” This quote, like that from Wittgenstein for *The Chinese Room*, sets the tone for the rest of *A Beauty Cold and Austere*. In the game you play as a student cramming for the final exam in a course on conceptual mathematics. Exhausted, you take one of your roommate’s memory pills, fall asleep, and enter a dream world featuring a variety of mathematical problems. Like *The Chinese Room*, to complete *A Beauty Cold and Austere* you must solve the work’s mathematical puzzles, thereby demonstrating enough of an understanding of their underlying concepts to pass the next day’s exam. In contrast to *The Chinese Room*, though, the mathematics in *A Beauty Cold and Austere* is developed historically, beginning with ideas associated with numbers and continuing through ancient Greek geometry and the Arabic algebra of the Middle Ages to probability, calculus, linear algebra, infinite sequences and series, and more recent mathematical concepts.

A Beauty Cold and Austere was released for free as part of the 2017 Interactive Fiction Competition, where it placed 7th out of 79 works. You can find more information about *A Beauty Cold and Austere*, and you can play it as well, at <https://ifdb.tads.org/viewgame?id=y9y7jozi0176bb82>.

4. Representing the Paradoxes

Each of the three works represents one or more of Zeno’s paradoxes within its game world. In this section we discuss these representations.

4.1. *Zeno in Beyond Zork*

The beginning of *Beyond Zork* features a fair amount of exploration, but the player does not have to solve any puzzles before encountering the following scene.

South Chasm

You're shivering on the south edge of a broad chasm. Clammy mists chill the air, and the ground trembles with the roar of a cataract.

Your heart sinks as you inspect the crude rope bridge spanning the chasm. A notice hangs near the bridge's entrance.

> *read notice*

The notice says,
ZENO'S BRIDGE
Cross at thy Own Risk

> *go north*

Halfway to the North End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. Clammy mist obscures your view of either end.

A closed umbrella dangles uncertainly from one of the ropes.

> *get umbrella*

Taken.

> *go north*

3/4 of the Way to the North End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. Your ears ring from the roar of the water far below.

> *go north*

7/8 of the Way to the North End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. The deafening roar of water is giving you a headache.

> *go north*

15/16 of the Way to the North End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. Clammy mist obscures your view of either end.

As we can see, Zeno's bridge in *Beyond Zork* is a representation of the variant of Zeno's Dichotomy paradox we described in Section 2. Each time the player goes north, she can only travel halfway from her current point to her destination.

4.2. Zeno in The Chinese Room

The Chinese Room begins with the reader in a location labeled “The Chinese Room,” which is an implementation of Searle’s thought experiment from [7]. After solving the puzzles associated with the Chinese Room and escaping, the player soon encounters the following scene.

Beach

You find yourself on a beach – a strip of silvery sand bordering an ocean that seems truly boundless. You try and focus on the horizon, but find your attention wavering whenever you get close to pinning it down, as if there were a blind spot in your consciousness. The sea shimmers and shifts under your gaze. You shake your head, blinking, and make your way along the beach, feet sliding in the fine-grained sand, towards a few small figures further along.

As you approach them, the figures – two men dressed in white togas – turn to look at you. “By Zeus, Achilles!” says one to the other, “It looks like an adventurer has arrived.”

Before you can get a word in, the man – an elderly, squat little Greek – greets you effusively:

“Welcome, adventurer! Well come indeed, oh yes. Very, very fortuitous indeed . . . ” He tails off, and the other man glances at you, embarrassed.

“Hallo,” he says, shuffling on his feet. “I’m Achilles.”

The older man is Zeno, as the reader discovers after a few lines of dialogue. *The Chinese Room* explains the problem they are currently facing.

“We’re holding a race!” cries Zeno. “A great and magnificent race, of profound importance. But we’ve only got one racer.”

“Me,” says Achilles.

“And you can’t hold a proper paradox with only one racer! It would be like holding a symposium with only one philosopher. Very boring. Not enough contradictions.”

As Achilles explains, the reason they have only one racer is that their tortoise is “on tour with her Aunt Hillary in France... Said she was sick of being exploited by our patriarchal obsession with paradoxes and chauvanistic exploitation of her abilities, and wanted to be somewhere where she could truly be herself.”

Zeno and Achilles are more than happy for you to offer yourself as a racer in the tortoise's place. In keeping with ancient Greek athletic tradition, Achilles runs in the buff. *The Chinese Room* describes the race thusly:

When Zeno shouts the go, there's a moment of blurred confusion as you both launch off. Your mind can't manage any complicated tasks, like managing sensory apparatus, when it's pumping your arms and legs this hard. When the mess of sensations starts to coalesce, you realise that Achilles has streaked (in every sense of the word) off in front of you, and is already nearly at the finish line. Oh, well. At least you tried.

Achilles loops around the flagpole and jogs back to you. Soon he's bouncing up and down at your side, jiggling in a disconcerting way. "Come on!" he says. "You're halfway there!"

At this stage, you don't think your brain can possibly cope with issuing any instructions other than GIVE UP or KEEP RUNNING.

> *keep running*

You plummet onwards down the track, Achilles keeping pace beside you.

"Only 1/4 a millos to go! Don't want to give up, do you?"

> *keep running*

You plummet onwards down the track, Achilles keeping pace beside you.

"Only 1/8 a millos to go! Don't want to give up, do you?"

> *keep running*

You plummet onwards down the track, Achilles keeping pace beside you.

"Only 1/16 a millos to go! You won't give up now, will you?"

> *keep running*

You plummet onwards down the track, Achilles keeping pace beside you.

"Only 1/32 a millos to go! Keep it up!"

Eventually the player must either GIVE UP or collapse from exhaustion.

Despite the scenario involving a race with Achilles, *The Chinese Room* does not here feature a faster runner trying and failing to overtake a slower runner. Instead, the faster runner Achilles handily wins the race, and the slower runner—the reader—travels halfway from his current location to the ending location with each action. Thus the paradox presented here is not the Achilles paradox but rather a variation of the Dichotomy paradox—the same variation, in fact, that appears in *Beyond Zork*.

4.3. *Zeno in A Beauty Cold and Austere*

At the start of *A Beauty Cold and Austere* the reader is in her dorm room. After taking the memory pill and falling asleep, the reader needs only solve one minor puzzle in the dream world before the following scene becomes available.

Stadium

This is a stadium in ancient Athens. The stadium itself consists of raised ground on either side of a race track about 200 meters long. The rest of the Lyceum is back east.

Achilles sits by the side of the race track with his head in his hands.

> *greet Achilles*

Achilles looks up. “I’m the fastest man in Greece. Then this guy Zeno challenged me to a 200-meter race with a tortoise, under two conditions: (1) I had to be willing to give the tortoise a head start of 100 meters, and (2) every time I got to where I last saw the tortoise I had to look up and mark its position. Of course I can beat a tortoise, even with that much head start, right? But no. Watch.”

Achilles snaps his fingers, and from somewhere in the grass of the raised ground comes a tortoise. The tortoise ambles to the 100-meter mark, and Achilles heads to the start. They take off. When Achilles reaches the halfway mark, he looks up, and the tortoise is ahead of him. Achilles notes its position, races to that position, and looks up. The tortoise is still ahead of him. He does this again. The tortoise is still ahead. And again. The tortoise is, once more, ahead. Every time Achilles looks up he’s closer to the tortoise, but he never quite catches up to it! Finally he stops running and looks over at you forlornly. The tortoise crosses the finish line, flashes a chelonian smile at Achilles, and disappears to wherever it was before.

Achilles says, “You see? I can’t catch up to the tortoise if every time I run to where it was before, it’s still ahead of me!”

As this passage indicates, *A Beauty Cold and Austere* features a direct implementation of the Achilles paradox: Achilles and a tortoise run a race in which the tortoise has a head start. Each time Achilles reaches the location at which he last observed the tortoise, it has moved further down the race-track. As this happens every time Achilles runs a segment of the race, the tortoise always remains ahead of him.

5. Engaging the Paradoxes

In the previous section we saw how *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere* represent Zeno’s Dichotomy and Achilles paradoxes. As these are works of interactive fiction, though, they do more than just represent those paradoxes. They all present the player with a puzzle that features the paradox, and the reader must solve the puzzle in order to advance and ultimately win the game. In other words, all three require the reader to *engage* with Zeno’s paradoxes. In this section we discuss in more detail exactly how this engagement works in *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere*.

5.1. *Beyond Zork* engages with Zeno

We have already seen some of how *Beyond Zork* allows the reader to engage with Zeno’s Dichotomy paradox. With each successive attempt to GO NORTH, starting from South Chasm, the reader travels first halfway to the north end, then $3/4$ of the way, then $7/8$ of the way, then $15/16$ of the way, and so forth. Thus the reader is actually, within the game world, experiencing the effect of the Dichotomy paradox.

Beyond Zork is even more clever about its representation of the Dichotomy paradox, though. Once the reader is on Zeno’s bridge, there is another option for movement besides going north: The reader could also attempt to go south, back to the start of the bridge. Here is how *Beyond Zork* handles this action, starting at $15/16$ of the way to the north end.

15/16 of the Way to the North End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. Clammy mist obscures your view of either end.

> *go south*

17/32 of the Way to the South End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. Your ears ring from the roar of the water far below.

> *go south*

49/64 of the Way to the South End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. Both edges of the chasm are obscured in the clammy mist.

Here *Beyond Zork* is, in keeping with its variant of the Dichotomy paradox, implementing the idea that each movement on Zeno's bridge takes one halfway to one's goal. If one starts $15/16$ of the way to the north end and heads south toward the other end of the bridge, the halfway point is $15/32$ of the way to the north end, or, equivalently, $17/32$ of the way to the south end. Similarly, if one starts $17/32$ of the way from the south end and heads south, one has $15/32$ of the total length of the bridge to traverse to reach the end. Thus the halfway point is $15/64$ of the total distance to cross the bridge, as measured from the south end, which means the reader is now $49/64$ of the way to the south end. While not technically part of the Dichotomy paradox as envisioned by Zeno, this is a natural extension of the ideas underlying that paradox. This deepens the player's experience of the effect of the Dichotomy paradox.

Beyond Zork does more to engage a player with the Dichotomy paradox than implement this consistent way of handling an attempt to go south while on the bridge, though. The game also presents him with a puzzle associated with the Dichotomy paradox, a puzzle he must ultimately solve to complete the game: How does one reach the other side of the bridge? The commercial version of *Beyond Zork* even came with a map giving an overview of the game world, and according to the map there is clearly a ruined castle or city on the other side of the bridge. Thus while someone could read about Zeno's paradoxes in a non-interactive work and find them intellectually interesting in a passive sort of way, the player of *Beyond Zork* is motivated to discover some way to resolve Zeno's Dichotomy paradox and cross the bridge—or, at the very least, somehow bypass the bridge—in order to reach the intriguing location on the other side.

5.2. *The Chinese Room engages with Zeno*

In a manner similar to traveling on the bridge in *Beyond Zork*, *The Chinese Room* allows the reader to engage with Zeno's Dichotomy paradox by choosing KEEP RUNNING at each stage of the race. Achilles continually updates the reader as to his progress: "You're halfway there!", followed by "Only 1/4 a millos to go!", "Only 1/8 a millos to go!", "Only 1/16 a millos to go!", "Only 1/32 a millos to go!", and so forth. By choosing KEEP RUNNING as the action, the reader experiences the effect of the paradox within the story of *The Chinese Room*.

While *The Chinese Room* does not allow the reader to reverse direction as *Beyond Zork* does, the work does contain a feature by which the reader may delve more deeply into Zeno's paradoxes. The THINK ABOUT command, which *The Chinese Room* mentions in its help menu, gives the reader information about the various philosophical conundrums that appear in the work. For example, after losing the race with Achilles, the reader could try the following.

> *think about Zeno*

Zeno of Elea (c. 490 BC - c. 430 BC) was a very early Greek philosopher who devised a set of paradoxes (apparen [sic] philosophical contradictions, or, more generally, insoluble philosophical problems) designed to challenge ideas of the divisibility of space and time. Many philosophers would hesistate to say that they have been completely solved, but approaching the paradoxes has led to some of the foundations of modern mathematics.

Your failed race with Achilles seems to be demonstrating Zeno's dichotomy paradox, which is the deceptively simple observation that, in order to reach B from A, one must first reach the halfway point between B and A. And in order to get from there to B, one must reach halfway towards B again (now 3/4 of the way between B and A). Continuing this to the 7/8 point, the 15/16 point, we see a problem – how can we ever reach B, given that we must always reach half the distance still to travel first?

Achilles, and his friend the tortoise, did in fact feature in one of Zeno's paradoxes, where, rather to Achilles's embarrassment, he was beaten by the slow-footed creature in a race. Being a tad inhibited

in the speed department, the tortoise was given a headstart in the race to give it a fighting chance. Obviously, in order to overtake the tortoise, Achilles first had to reach this headstart point. But by that time the tortoise had, albeit slowly, moved a little further down the track. Achilles had to catch up again. By which point the tortoise had moved on. And so on and so forth – Achilles, seemingly, can never overtake the tortoise. The lesson would seem to be: there's something wrong with the idea of approaching space as infinitely divisible in this way – or, as in calculus, perhaps taking things to infinity is the only way of progressing. Or perhaps it's just that, when racing ancient Greek sprinters, always get a head start.

Thus *The Chinese Room* allows the reader to delve more deeply into Zeno's paradoxes by giving additional background on them.

There is also the question of motivation for engaging with Zeno's paradoxes in *The Chinese Room*. While this work (unlike *Beyond Zork*) contains no promise of a new, interesting location for resolving the paradox, Zeno and Achilles are clearly asking for the reader's help. In order to deliver that assistance the reader must choose to participate in the race and thus engage with the paradox. In addition, Zeno offers a prize for the player if she can beat Achilles: "a large glass cup with a laurel leaf carving." He also (perhaps somewhat inconsistently for the story but necessary for game playability) allows the player to keep racing Achilles until the player wins. The puzzle in *The Chinese Room* associated with Zeno's paradoxes, then, is for the player to figure out some way to beat Achilles in the race.

5.3. *A Beauty Cold and Austere engages with Zeno*

A Beauty Cold and Austere does not have a mechanism like *Beyond Zork's* bridge or *The Chinese Room's* race with Achilles that lets the reader actively experience one of Zeno's paradoxes. Instead, the reader is a passive observer of Achilles's failed attempt to defeat a tortoise. However, much like *The Chinese Room*, *A Beauty Cold and Austere* does feature a mechanism that allows the reader to find out more information about the mathematical concepts encountered in the work. The reader enters the dream world carrying the textbook for the math class she is taking, and this book serves as a reference work by which the reader may look up mathematical ideas.

Thus after observing Achilles's failed attempt to defeat the tortoise, the reader might try the following.

> *look up Achilles in math book*

The story of Achilles and the tortoise is a paradox related to the nature of infinity. It was devised by Zeno of Elea.

In the paradox, Achilles is racing a tortoise who is given a head start. Zeno claims that Achilles can never catch up to the tortoise because "In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead."

The paradox of Achilles and the tortoise, as well as Zeno's paradoxes in general, are important in mathematics because they helped force mathematicians to consider carefully what they mean by continuous quantities that are infinitely divisible, as well as infinity itself.

Some mathematicians believe that the limit concept, which not only provides a rigorous method for handling infinite quantities but also put calculus on a firm theoretical foundation, resolves the paradox of Achilles and the tortoise. However, limits were not developed until over 2000 years after the death of Zeno.

As with the command to THINK ABOUT ZENO in *The Chinese Room, A Beauty Cold and Austere* lets the reader engage more deeply with Zeno's paradoxes by giving background information on them.

A Beauty Cold and Austere also allows for an unsuccessful but interesting attempt to help Achilles. After solving a few puzzles the reader can acquire a pair of exhaustion sandals. Attempting to understand what these are might lead to a play session like the following.

> *examine exhaustion sandals*

This pair of exhaustion sandals is the embodiment of Eudoxus's principle of exhaustion.

> *look up principle of exhaustion in math book*

The method of exhaustion was a technique used by various ancient Greek mathematicians to find areas and volumes of geometric objects like circles, spheres, cones, and cylinders. It can be thought of as a precursor to more modern techniques that use limits.

> *look up limits in math book*

The limit is one of the most important concepts in mathematics. Limits allow us to define exactly what we mean by concepts like “arbitrarily large” and “infinitesimally small.”

Limits had been used implicitly in calculus since the time of Newton and Leibniz, but it was not until over 150 years later that the definition of a limit was made precise by Weierstrass, thereby placing calculus on a more solid theoretical foundation. Limits are also used to make sense of sequences and infinite series. Some argue that limits resolve Zeno’s paradoxes.

Taken together, the math book entries hint that the exhaustion sandals might be useful to Achilles.

> *give sandals to Achilles*

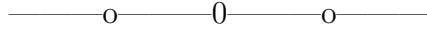
Achilles looks at the exhaustion sandals rather skeptically (“I always run barefoot,” he says), but he agrees to try them. After lacing them up, he snaps his fingers, and the tortoise appears. They find their places as before, and take off. Achilles runs to the halfway mark, looks up, and sees the tortoise ahead of him. He notes the tortoise’s new position, races to it, and sees the tortoise still ahead. Achilles doesn’t give up, though, and every time he looks up he finds himself closer to the tortoise. As they approach the finish, Achilles is so close to the tortoise you can’t see much distance between them. However, the tortoise crosses the finish line just barely ahead of Achilles.

The tortoise disappears again, and Achilles jogs back to you. He returns the exhaustion sandals. “Thanks for letting me try the sandals. I felt myself almost overtaking the tortoise, but the sandals weren’t quite enough.”

Thus *A Beauty Cold and Austere* allows the player to engage more deeply with Zeno’s paradoxes by allowing him a reasonable but unsuccessful attempt at helping Achilles—an attempt that hints at a potential resolution to the Achilles paradox that involves limits.

Similar to *The Chinese Room*, the motivation for engaging with Zeno’s Achilles paradox in *A Beauty Cold and Austere* involves a race with Achilles. However, instead of searching for a way to beat Achilles in the race, the puzzle presented to the player entails helping Achilles defeat the tortoise.

No motivation beyond assisting Achilles is given, though: The game seems to rely on the player's good will—or, more likely, the convention in computer-based games that helping another character will bring the player some sort of reward.



Before we move on to the next section and see how *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere* allow the reader to resolve (or not resolve) Zeno's paradoxes, we should mention that the parser-based nature of these works is ideal for capturing one important aspect of the paradoxes. The paradoxes deal with discrete actions: In the Dichotomy paradox, for example, one act of movement takes you halfway to your goal. With parser-based interactive fiction like the three works considered here, the player must enter each of his actions separately on a command line. Thus story world actions are inherently discrete. This means that the choice to GO NORTH (in *Beyond Zork*) or KEEP RUNNING (in *The Chinese Room*) can easily be implemented as a discrete action that results in traveling halfway to one's goal. By way of contrast, it is much more difficult to imagine how an author would successfully implement the Dichotomy paradox in an interactive open-world game such as *Fortnite* or *The Witcher* that features (at least the simulation of) continuous movement. By the same token, *A Beauty Cold and Austere*'s failure to take advantage of the parser in this way could be considered a missed opportunity to engage further with Zeno's paradoxes.

6. Resolving or Escaping the Paradoxes

A key aspect of the representations of Zeno's paradoxes in *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere* is that they are integrated into puzzles; that is, obstacles that the reader must engage with and ultimately overcome in order to complete the story. The solutions to these puzzles thus represent potential ways to think about resolving or at least escaping the intellectual difficulties presented by the paradoxes. In this section we discuss the solutions to these three puzzles.

6.1. *Beyond Zork* resolves the paradoxes by avoiding them

In *Beyond Zork* the challenge associated with the paradox is to reach the other side of Zeno's bridge. We have seen that, in keeping with the ideas underlying the Dichotomy paradox, continuing to travel north will never

allow the reader to reach the north side of the chasm. The reader's situation is even worse, though: Once on the bridge, returning to South Chasm by going south is not possible, either! Thus, after stepping onto the bridge, the reader is actually trapped by the paradox that the bridge represents. The first puzzle that the reader must solve, therefore, is simply escaping the bridge (and thus escaping the Dichotomy paradox).

There are various magical objects available in *Beyond Zork* that will allow the reader, through flight or teleportation, to escape the bridge. However, none of them are available near the beginning of the story, when the player is first likely to encounter the bridge. In fact, the only way to escape the bridge in the early part of the game is to use the one object the reader finds at the bridge: a very mundane umbrella.

49/64 of the Way to the South End

The entire bridge jerks and sways as you struggle to keep your footing on the slippery ropes. Both edges of the chasm are obscured in the clammy mist.

> *open umbrella*

You snap open the umbrella.

> *jump*

You leap off the slippery ropes and drift down towards the raging water.

Rip! A gust of spray tears the broken umbrella from your grasp a little too soon, and you hit the freezing water...

Babbling Brook

A footpath emerges from the shadow of a forbidding forest, curving southwest along the edge of a brook.

There's a broken umbrella here.

You slowly come to your senses.

[Your endurance just went down.]

Jumping off the bridge while holding the umbrella open allows the reader to escape the paradox embodied by the bridge—but at the cost of some of the reader's endurance (one of the statistics tracked by *Beyond Zork*). In addition, the umbrella is now broken and cannot be used to escape the bridge again. Moreover, the reader still has not solved the primary puzzle

associated with the paradox: The reader has not reached the north side of the bridge. (The Babbling Brook location is on the south side of the chasm and likely has already been visited by the reader.)

In order to reach the north side of the chasm in *Beyond Zork*, it turns out that the reader must bypass Zeno's bridge completely. Elsewhere in the story world the reader comes across a wounded pterodactyl. After healing the pterodactyl and thus earning its trust, the pterodactyl permits the reader to ride it a limited number of times. The reader must fly over the chasm on the back of the pterodactyl to reach the north side and the ruins there.

Thus we see that *Beyond Zork* does not allow the reader a way to resolve Zeno's Dichotomy paradox. The only solution to the associated puzzle is to bypass the paradox completely, riding on the back of a prehistoric flying creature! Moreover, entering into the paradox by attempting to cross the bridge actually traps the reader in the paradox, with only a limited number of options to escape. Furthermore, all of the escape methods are either magical or cause damage to the player. Overall, then, *Beyond Zork's* treatment of Zeno's Dichotomy paradox seems to suggest that the paradox is a trap—a trap that the player should avoid engaging with at all. As to the intellectual difficulties raised by the paradox, *Beyond Zork's* solution is simply to refute the paradox's conclusion. As Diogenes the Cynic noted [8, 1012:24–27], the act of moving (e.g. walking, flying on the back of a pterodactyl) inherently proves that motion is possible. Since the paradox's conclusion is that motion is not possible, we know that the paradox must have a logical flaw, even if we cannot pinpoint exactly where the flaw lies. Thus we can bypass the paradox and not bother ourselves with it further. This is a very practical approach to the Dichotomy paradox—one that most people who are aware of the paradox effectively take, as well as one entirely in keeping with the ethos of a game that is trying, at base, to entertain its player.

6.2. *The Chinese Room resolves the paradoxes by pitting one against the other*

In *The Chinese Room* the challenge is to win the race against Achilles. The solution to this puzzle is right there in the Achilles paradox (and in the text response to THINK ABOUT ZENO in the game): Like the tortoise, the player needs a head start. Zeno isn't inclined to give the player one, though. After having lost the race the player might try the following.

> *talk to Zeno*

“What? Oh! Oh, it’s you. Fancy another race then, do you?”

[1] Nah, thanks, I’d only get thrashed again.

[2] I haven’t a chance unless you give me a head start!

[3] Yeah, I think I could give it another shot.

> 2

“Oh no, I’m afraid that would foul all the calculations. We’d only need one of those to compensate for some flaw or other in the race track – and just look at it! Aren’t those lanes perfect? Can you manage without?”

Zeno is inadvertently giving the player a hint as to how to proceed.

> *examine track*

Little more than a few lines in the sand. There’s one just by Achilles and Zeno and the other terribly, terribly far (it seems) in the distance. You can only tell where it is by the flag fluttering away there. In between the two, long lines follow the steep curve of the beach to mark out two lanes, outside and inside. As the beach takes a steep curve for the length of the race track, the inside lane is probably a little bit shorter than the outer.

Since the two lanes curve along the beach, one of them is shorter than the other. This sounds like something worth pointing out to Zeno.

> *talk to Zeno*

“What? Oh! Oh, it’s you. Fancy another race then, do you?”

[1] Nah, thanks, I’d only get thrashed again.

[2] Yeah, I think I could give it another shot.

[3] It’s exhausting! Just how long is that track?

> 3

“Oh, only a millos or so.”

[1] Hmm . . . nah, I’ll pass this time.

[2] Phew! Well, I’ll still give it another go.

[3] And had you realised that one lane is just a little bit shorter than the other?

> 3

Zeno's eyes widen, and he glances round at the steeply curving track. "Curses!" he says. "Quite true! That throws all my calculations out, and renders, I'm terribly sorry, all races completely invalid."

[1] You could just give me – I mean, the person on the outside lane – a bit of a head start.

[2] Well, I'm not racing again, I'm afraid.

> 1

"Why, you're absolutely right! Magnificent! Well, what are you waiting for? Line up, you two!"

With a gleam in your eye, you quickly take your position on the outside lane, just 50 podes ahead of Achilles. When Zeno shouts the go, you pump your legs away without bothering to look behind you.

When you've reached about halfway down the track, you hear a curse to Zeus close behind you. Achilles has made a good deal of headway, but something seems to be holding him back. Every time he reaches where you've just been, you've managed to make a little further hard progress down towards the flag. For each bound he takes, you manage another bound – a smaller, humbler bound – a little further forward. And while you're flagging more than him, and he manages to catch up a little further on each bound, he never quite makes it to you.

Now it's just a matter of endurance. You set your head towards the flag and keep forging on. It gets closer and closer, with Achilles coming up just behind you, a gnat's crotchet away – and you reach the flag! Throwing yourself over the finish line, you grab onto the pole and sink to the ground, exhausted. Achilles throws himself down beside you, and you both gaze up at the fluttering red flag, with that peculiar hole in the centre, waiting for something like breath to return to your lungs.

When you finally make it back to Zeno, he is applauding wildly. "Magnificent! Magnificent!" Even the deject Achilles is overwhelmed as Zeno thrusts the glass trophy into your arms and grabs him by the hand. "Now see here, Achilles," he says, drawing complicated equations in the sand as Achilles looks on, rapt.

[Your score has just gone up by three points.]

The Chinese Room, then, ends up presenting the player with both the Dichotomy and Achilles paradoxes. The player does not resolve either paradox, though. Instead, by convincing Zeno to give her a head start so that she can win the race, the player exploits one paradox (Achilles) in order to escape the problems associated with the other (Dichotomy). Unlike *Beyond Zork*, then, which suggests that the way to handle Zeno's paradoxes is to refuse to even engage with them, *The Chinese Room* instead has the player turn one of them against the other. This enables the player to defeat Achilles, although (again) it does not provide a resolution to either paradox. However, this solution does require the player to engage with and understand both paradoxes. *The Chinese Room* seems to suggest, then, that what is most important about Zeno's paradoxes is not resolving them but rather thinking about them and understanding the issues that they raise. This is entirely in keeping with a work whose goal is to present and explore philosophical thought experiments. Indeed, *The Chinese Room* does not generally take positions on the philosophical problems it presents, instead preferring (as with the race with Achilles) to have its puzzles require an understanding of the philosophical questions raised in order to be solved.

6.3. *A Beauty Cold and Austere resolves the paradoxes via limits*

Like *The Chinese Room*, *A Beauty Cold and Austere* features a race with Achilles to represent Zeno's paradoxes. However, rather than the player trying to beat Achilles in a race, the goal in *A Beauty Cold and Austere* is to help Achilles defeat the tortoise. We have already seen that by giving the exhaustion sandals to Achilles he is almost able to overtake the tortoise. However, as Achilles says, they "weren't quite enough."

A Beauty Cold and Austere's solution to Achilles's difficulty is hinted at in the math textbook. The entry on Achilles says, "Some mathematicians believe that the limit concept... resolves the paradox of Achilles and the tortoise." So limits are needed somehow. Because *A Beauty Cold and Austere* develops the mathematics in the game historically, though, this means that while Achilles is encountered early, the object that embodies the limit concept is not available until much, much later—near the end of the game, in fact. After working his way through (among other topics) ancient Greek geometry, Arabic algebra of the Middle Ages, seventeenth-century probability,

and the calculus of the Enlightenment, the player eventually meets a certain pair of nineteenth-century mathematicians.

Smoky Pub

The smell of tobacco hangs in the air in this small, dark pub. It is nearly deserted; even the bartender appears to have stepped out. An exit leads south.

Cauchy is sitting at the bar, sipping from a glass of wine.

Weierstrass is next to Cauchy, drowning his sorrows in a pint of beer.

Weierstrass is looking for a problem to kick-start his research program. After the player inspires him to work on limits (involving a separate puzzle that we will not detail here), Weierstrass offers his gratitude.

Weierstrass looks at you excitedly... "I owe you more than I can say, but I don't know how best to reward you. Perhaps I can improve something of yours with the power of the limit concept?"

> *give sandals to Weierstrass*

Weierstrass takes the sandals and looks at them carefully. "These embody the principle of exhaustion, don't they?" he says. Then he closes his eyes and concentrates for a moment. The sandals begin to glow, and then they start to morph. Soon Weierstrass is holding a pair of shoes in his hands. "There," he says, opening his eyes and handing the shoes to you. "I've improved your exhaustion sandals to a pair of limit shoes.

"Thanks again," he says. He stands up, an eager look on his face. "I need to explore these concepts further. I've got an idea for a continuous function that is not differentiable anywhere..." Weierstrass downs the rest of his beer, places his mug on the bar, and walks out of the pub. The bartender appears out of nowhere and whisks the mug away before disappearing again.

Now the player can return to Achilles and help him finally defeat that tortoise.

> *give shoes to Achilles*

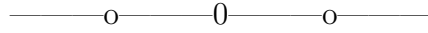
Achilles looks at the limit shoes rather skeptically, but you assure him that these shoes are different - more powerful than the exhaustion sandals - and so he agrees to try them. After lacing them up, he snaps his fingers, and the tortoise appears. They find their places as before, and take off. Achilles runs to the halfway mark, looks up, and sees the tortoise ahead of him. He notes the tortoise's new position, races to it, and sees the tortoise still ahead. You start to think that the limit shoes aren't going to do Achilles any good. About this time, though, their inherent virtue starts to kick in. Yes, there are infinitely many times Achilles must note the tortoise's position, leading to infinitely many distances he must run, but each successive distance is shorter than the one before. The power of the limit shoes successfully sums the lengths of these infinitely many shorter and shorter distances, and Achilles passes the tortoise for the win!

Achilles jogs back to the start, a huge grin on his face, and gives you a crushing hug. Slightly abashed, he steps back, thanks you profusely, and returns the limit shoes. He runs away to celebrate.

As you're recovering from Achilles's excitement, Zeno walks over from where he has been discreetly watching the race. He picks up the tortoise. As he gazes quizzically at it, he's joined by Aristotle. They head off, arguing over whether the limit process actually solves the paradox of Achilles and the tortoise.

We have here, then, a mathematical solution to a puzzle that represents one of Zeno's paradoxes. Instead of solving the associated puzzle via bypassing the paradox entirely, as in *Beyond Zork*, or by turning one paradox against another, as with *The Chinese Room*, in *A Beauty Cold and Austere* the player gives Achilles the power to defeat the tortoise via an object representing the limit concept. The idea here is that, as Russell argues [6], we can address Zeno's objection to an infinite sequence of distances by noting that some infinite sequences can be said to have, in a mathematically precise sense, a finite limit. (For example, the sequence $1/2, 3/4, 7/8, 15/16, \dots$ has a limit of 1.) Thus *A Beauty Cold and Austere* suggests that limits—a mathematical idea not developed until long after Zeno's death—resolve his paradoxes. However, the game does hedge this claim slightly. After Achilles finally wins his race with the tortoise, the game says that Zeno and Aristotle walk off as

well, “arguing over whether the limit process actually solves the paradox of Achilles and the tortoise.” Overall, though, having a mathematical concept resolve the puzzle associated with Zeno’s paradox is entirely in keeping with a game whose goal is to present the history of mathematical thought.



There is one more aspect of the Zeno’s paradox puzzles in these three works that we should mention. While the puzzles are accessible quite early in all three games, the solutions in *Beyond Zork* and *A Beauty Cold and Austere* are not available until mid-game or even later. This means that in these two works the player will likely spend quite a bit of time early in the game attempting to solve a Zeno’s paradox puzzle and failing. (In addition, even though the solution to *The Chinese Room*’s puzzle is available early, a player may not find that solution early. That happened to this player, in fact.) Overall, then, a player may very well become frustrated with the Zeno’s paradox puzzles in these three works. However, even that frustration further represents Zeno’s paradoxes. The paradoxes are intellectually challenging, after all, and thinkers have been wrestling with them for millennia. If a player finds a work’s puzzle associated with Zeno’s paradoxes frustrating, then the work has succeeded in communicating the challenging aspect of those paradoxes.

7. Conclusion

It is perhaps not surprising that *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere* each take an approach to Zeno’s paradoxes that reflects the works’ respective goals. *Beyond Zork*’s take, refuting the paradox’s conclusion while avoiding the in-game physical embodiment of the paradox, is the kind of practical approach one might expect from a fantasy-themed commercial game. On the other hand, *The Chinese Room* and *A Beauty Cold and Austere* were both released for free and thus have somewhat different mixes of goals. *The Chinese Room*, designed to allow the player to explore philosophical thought experiments, is not so much interested in resolving Zeno’s paradoxes as it is the player engaging with and understanding them—understanding them well enough to turn one of them against the other. Finally, the mathematically-focused *A Beauty Cold and Austere* presents the mathematical concept of the limit as a solution to Zeno’s paradoxes.

More than simply representing Zeno's paradoxes and offering approaches or resolutions to them, though, *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere* allow the reader to engage deeply with Zeno's paradoxes in a way that static fiction cannot. While on Zeno's bridge in *Beyond Zork* or racing Achilles in *The Chinese Room*, the reader is actually experiencing the effect of the paradox within the story world. (The inherent discrete aspect of parser-based games matches the discrete nature of the paradoxes and so enhances this effect.) In addition, in all three works the reader must take an active role in solving a puzzle that embodies one of Zeno's paradoxes. This in turn requires the reader to think through the difficulties raised by the paradoxes, rather than just passively reading about them and moving on. We see then that *Beyond Zork*, *The Chinese Room*, and *A Beauty Cold and Austere* (especially when taken together) do a better job of forcing a reader to engage Zeno's paradoxes than a static work of fiction can.

Personal Note: I first played *Beyond Zork* shortly after it was released in 1987. I was 14, and its Zeno's bridge scene was my introduction to Zeno's paradoxes. Within a few years, though, the company that had created *Beyond Zork* went out of business. I assumed that parser-based interactive fiction had died as an art form.

In 2017, though, I discovered the modern interactive fiction community and realized I was wrong about the death of parser-based interactive fiction. On the contrary, people had been producing—for years—incredibly creative works of parser IF, conducting experiments in storytelling, characterization, and gaming in general. I was quickly intrigued by interactive fiction's potential for education, and I wrote *A Beauty Cold and Austere* as an experiment in using IF as to explore mathematical ideas. Since then I have continued to be involved in the interactive fiction community. I encourage interested readers to visit the Interactive Fiction Database and investigate for themselves more of what IF has to offer.

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