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Teaching From the Unknown

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Synopsis

The goal of teaching is to transform our students' understanding, much as the goal of acting is to transform the audience's reality. In this note we use the setting of mathematics to explore connections between teaching and acting and how such connections can help our students learn not only mathematics but also about the nature of mathematics.

"*To teach is to create a space ...*"
–Parker Palmer

A central challenge for a good stage performance is to make it all happen for the first time, again and again. Performers must recreate the text and keep it fresh night after night. An essential technique to accomplish this is to "live in the unknown." Hamlet can't play four and a half hours moping about as if he's going to die. The classroom is also a type of performative space where teachers make known ideas come alive, again and again.

Many of us naturally teach from the unknown as we reveal our mathematics line by line, step by step. We carefully set the stage and work through the script, building up to the 'aha' moment when the strands of the proof come together for the triumphant conclusion — our denouement. We present from the unknown to engage students in the ideas and let them experience the feeling of creating something as if for the first time. Teaching from the unknown also narrows the gap between teacher and student — given that we both have the same tools on the table, what can we make of the ideas at hand? Just as performers want the audience to live in the moment and wonder how the story's themes will develop, we too want our students to live in the moment and experience the mathematical ideas unfolding and combining in new ways to create yet more threads in their mathematical tapestries.

But the more I've taught the same material over the years, the more the mathematical ideas and proofs have become familiar and almost self-evident. What once was mind-expanding (whoa, eigenvalues!) was becoming, dare I say, ordinary (hmm, just eigenvalues). I feared this familiarity was starting to negatively affect my teaching and that some of the freshness and excitement of the ideas was missing. However, if I were to "live in the unknown" and teach from that space, I realized I could create an environment where the freshness and excitement would return. Moreover, the longer I taught, the more my increased understanding of the material would allow me to better play it from the unknown, just as performers use their depth of understanding of the narrative to heighten audience engagement, emotional connection, and response.

This new perspective became particularly clear when I was teaching an honors calculus class for the third year in a row. The course is designed for students who have already completed a year of high school calculus and covers the usual first-semester topics but with a late-transcendental approach. From their prior experience students were familiar with the content but mostly as a jumbled bag of techniques that they were anxious to apply. This time our goal would be to "build calculus from scratch" and only allow ideas to surface that we had previously developed.

For example, a common proof that $\sin x$ is differentiable leads to the limit of $\sin h/h$ as h tends to 0. "How are we going to compute this limit?" I could ask, to which several students would offer up an enthusiastic "L'Hopital's rule!" Their enthusiasm would wane as I would react looking puzzled and perplexed. We would have only just introduced the definition of derivative, so their "rule" would be completely unknown, justifying my bewilderment. "What is this hospital rule you speak of? How does it work?" I would ask. Invariably, someone would press through my spurious confusion and suggest we differentiate the numerator, at which point they would catch on to the logical inconsistency and begin to wonder how we might genuinely establish the limit.

Here is another example: since we were following the late-transcendental approach of defining the natural logarithm as an antiderivative of $1/x$ (and using this to define e), I could play the unenlightened professor when students referred to the natural logarithm before we had discussed integration. "What is this 'natural log' you speak of? Inverse of e^x? What is this number e you refer to?" I was living in the unknown, and this was forcing students to re-examine their understanding of calculus.

Students quickly caught on to my game and it changed the way they thought about mathematics. By living in the unknown they began to see that we were not just throwing techniques at problems; we were building a coordinated and consistent body of ideas with far-reaching consequences. Their focus shifted from "how" to "why," which added meaningful insight to their understanding of the nature of mathematics. To advance through their high school curriculum these students had concentrated on classifying and solving various calculus problems. While this built their confidence and set up a natural affinity for mathematics, most had not allowed themselves to really question why a given definition is needed, why a given proof works, or why a given theorem is so powerful. Consequently, they were missing a crucial aspect of the mathematical beauty of calculus. Moreover, they were missing out on a meaningful learning experience of what it's like to be a mathematician and do mathematics. After all, the unknown is where mathematicians live as we seek to uncover new insights and truths. By teaching from the unknown we help engage students in the joy of discovery that permeates the mathematical life.

This calculus course may seem like an unusual example for playing off of students' familiarity with the mechanics of calculus, but most math courses start with some common prior knowledge from which to build, and this creates the potential to teach from the unknown. Sometimes I even purposely misdirect students to downplay a connection in order to heighten the impact when it suddenly reappears to link the formerly disparate ideas. For example, when introducing linear systems of differential equations, straightforward examples can motivate the ansatz $\mathbf{y}(t) = e^{\lambda t}\mathbf{v}$ for a solution of the linear system $\mathbf{y}' = A\mathbf{y}$, but students are not expecting to see eigenvalues and eigenvectors appear – that's linear algebra, not differential equations! The subsequent calculation substituting the ansatz into the system that leads to the unexpected appearance of $A\mathbf{v} = \lambda\mathbf{v}$ and the magical connection with linear algebra is like a paradigm-shifting end of Act I that expands preconceived boundaries and entices the learner further into the adventure.

Many complementary themes link the art of classroom teaching with acting. For example, like it or not, when class starts, it's showtime! Regardless of pedagogy, teachers still need to create a space and sustain a process that connects their students with the subject matter. Theatre professor Ann Woodworth from Northwestern University notes many other similarities:

- Both revolve around the art of communication, including both verbal and nonverbal aspects;
- Both focus on known texts and making the inactive, active;
- Both involve important transitions ("Acts");
- Both are goal oriented: engage the "audience" and develop their interest in the subject matter;
- Both involve co-existing in a shared physical setting with the teacher/ performer positioning themselves within or around an "audience."

Like performers, teachers must also live in the moment and dynamically adjust their strategies in response to perceived student cues. Some of the most rewarding teaching experiences can arise from this delicate interplay of our individuality with each class's unpredictable and ephemeral reality.

Before pursuing a career in magic, renowned magician Teller (of Penn & Teller) taught Latin for several years. When asked to reflect on how performance played a role in his teaching he observed "[t]he first job of a teacher is to make the student fall in love with the subject. That doesn't have to be done by waving your arms and prancing about the classroom; there's all sorts of ways to go at it, but no matter what, you are a symbol of the subject in the students' minds" [1]. Central to Teller's educational philosophy is Whitehead's "rhythm of education" which sees intellectual progress proceeding in three stages: romance, precision, and generalization [1, 2]. These stages parallel the progression sought by performers who seek to transform audience members from the infancy of their arrival state to the complexity of their exit state with renewed perspective and deeper humanity for the themes explored. Is that not also an aspirational goal for our classes?

Living in the unknown is a crucial technique for stage performers, and I believe it can be an equally effective technique for teaching mathematics. Given the common goal of transformation, I wonder what else we can learn from our theater colleagues to positively impact our students' learning.

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