Journal of Humanistic Mathematics

Volume 11 | Issue 2 July 2021

Mathematical Rigor From Within

Lowell Abrams
The George Washington University

Recommended Citation

Lowell Abrams, "Mathematical Rigor From Within," *Journal of Humanistic Mathematics*, Volume 11 Issue 2 (July 2021), pages 477-484. DOI: 10.5642/jhummath.202102.29. Available at: https://scholarship.claremont.edu/jhm/vol11/iss2/29

Mathematical Rigor From Within

Cover Page Footnote

Lowell Abrams is an Associate Professor of Writing and of Mathematics at the George Washington University.

POETRY FOLDER

◊

Mathematical Rigor From Within

Lowell Abrams

The George Washington University
abrams@gwu.edu

Whether engaged in defining new terms, assembling and applying a conceptual framework, or simply enjoying a clever idea, there is a certain feel that is unique to the rarefied context of rigorous mathematics. The three poems below constitute an exploration of my experience of mathematical rigor when I am in the midst of exercising my skills as a research mathematician.

The poems also contain a measure of philosophizing. This is not an additional layer intended to "deepen" the poems but, for me, part and parcel of what it means to study and create abstract mathematics. I have often wondered whether I am a mathematician who thinks philosophically or a philosopher who thinks mathematically, but I believe the constant interplay of these points of view is what fuels the poetics of the whole endeavor.

"The Proof May Begin" was inspired by a thought I heard Israel M. Gelfand share multiple times, that the definition is the most creative act in mathematics. It is more than this, of course, since the nature of mathematical definition itself plays a key role in defining what we think of as rigorous mathematics. The concrete subject matter of the poem—different notions of polyhedron—draws from a recently published article of myself and Landon Elkind [1]. The line "Euler applied the knife" echoes Euler's wording when he refers to the process of cutting solid angles [4, page 4]. The stanza beginning "In the new reign" harks back to Old Testament Isaiah 28:13; I invite the reader to compare this portion of the poem with the context there.

I have often dreamed of what "lyricism in mathematical proofs" might mean, and how it might be achieved, but I've come to no satisfying conclusion. The impressionistic "Spanning Trees" is my attempt to convey the feel of the proofs I actually produce. It draws on my joint work with Daniel Slilaty in topological graph theory [2, 3], where we often use the voltage graph construction to build covering spaces for embedded graphs, and use spanning trees to maintain control over what that construction produces.

"Euclid's Whisper" is a poetic recounting of Euclid's famous proof of the infinitude of the set of prime integers. About twenty years ago, while Héctor J. Sussman (of Rutgers University) and I were sharing notes on our respective proof-writing courses, he described proof by contradiction as placing oneself in a "richer universe." This has been percolating in me since, and finally made its way into this poem.

References

[1] Lowell Abrams and Landon DC Elkind. "Word choice in mathematical practice: a case study in polyhedra." *Synthese*, Volume **198** (2021), pages 3413–3441.

[2] Lowell Abrams and Daniel Slilaty. "Cellular automorphisms and self-duality." *Transactions of the American Mathematical Society*, Volume **367** Number 11 (2015), pages 7695–7773, 2015.

[3] Lowell Abrams and Daniel Slilaty. "The minimal $\mathbb{Z}_n$-symmetric graphs that are not $\mathbb{Z}_n$-spherical." *European Journal of Combinatorics*, Volume **46** (2015), pages 95–114.

[4] Leonhard Euler, "Demonstratio nonnullarum insignium proprieatatum, quibus solida hedris planis inclusa sunt praedita," *Novi commentarii academiae scientiarum Petropolitanae*, **4** (1758), pages 72-93; also in *Opera Omnia*, series 1, vol. 26, Birkhäuser, Leibzig, pages 94-108. (The pagination refers to the translation, by Christopher Francese and David Richeson, titled "Proof of some notable properties with which solids encased by plane faces are endowed," available at http://eulerarchive.maa.org//docs/translations/E231.pdf, last accessed on July 13, 2021.)

The Proof May Begin

Polyhedral facets catching, flashing,
ancient elegance, beauty, warmth,
embodied wonder, symmetry
breathed into life in the hand.

Euler applied the knife
and flashing facets dulled,
delicate vertices reduced
to marks where substance returns
to the space containing it,
and out of which it had been carved.

Now we
    still have cuts,
    but do no cutting.
Now it,
    weightless, needs
        no hand, takes
            no breath, and warms
                none.

Now it is Precision
that fancies
            vertices
to anchor edges,
                edges
to frame faces,
                a complex
to coronate shape
enthroned in study.

In the new reign,
    Part matches part, and part matches part.
    Symmetry sits, symmetry is
    sets of points
    relating to sets of points,
    preserved forever
    in black and white.

Stiffened by its own definition,
stern faced and drawing no breath,
the polyhedron, gazing down,
                finally,
                slowly,

                nods.

Spanning Trees

Facing each point,
The mind fixed.
This case,
this point.

Climb the branch, graph the branch,

Thus.

Point at the branch
Contained in both faces.

Between the edges,
Deleting what one can.
The trunk below,
Deleted.

The leaves above,
Embedding edges in their orbit.

If any vertex,
Any single point,
Were only a graph,
Thus,

The tree would be a contracting canopy,
One can see from below.

Then, contracted,
Edges of an orbit, if any,
Grow accordingly.

What will surface?

Minors correspond to minors,
Orbiting an elusive surface,
Observing first,
Recalling the proof you never knew.
Facing the orientation of and for
every trunk, branch, leaf.

Then, point,
The mind fixed with orderly growth.

If.

This is the proposition.

Euclid's Whisper

Suppose
for the sake of a bigger, richer, universe,
in which we can prove
more than is true,
in which the possibility
of impossibility
lends us logical leverage.

Suppose
we can hold the periodic table of numbers
in our hands,
        a finite list of primes,
        p_1 and p_2 and up to p_n
that we can build,
    choosing and combining,
        any and every number.

If
we build,
choosing one of each,
$\mathrm{P} = p_1 \times p_2 \times$ (up to) p_n,
then
each pi is a factor of P,
 cleanly and evenly divides.

But Euclid taught a better idea,
        set P to be
$1 + (p_1 \times p_2 \times$ (up to) $p_n)$

we listen close and we see,
that none evenly divides,
no p_i is a factor of P.

So what can be true?
Our rich universe cradles a lie!
either
        P itself is a missed prime,
or
    P factors with a prime
        missed by our list

In either case,
the universe we built is not
one in which our math may live;

our finite list of primes
was too short,

and always will be.