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Tips for Undergraduate Research Supervisors

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Synopsis

I was recently asked to contribute a paper to a forthcoming *Foundations for Undergraduate Research in Mathematics* (FURM) volume [1]. The article contained, in addition to several lengthy case studies, a list of brief recommendations for undergraduate research supervisors. Several colleagues suggested developing these general principles into a separate, self-contained article. This is that article.

I was recently asked to contribute a paper to a forthcoming *Foundations for Undergraduate Research in Mathematics* (FURM) volume [1]. The article contained, in addition to several lengthy case studies, a list of brief recommendations for undergraduate research supervisors. Several colleagues suggested developing these general principles into a separate, self-contained article. What follows is an expanded version of the recommendations introduced in [1]. I focus here on how a supervisor of undergraduate research can direct students to fruitful research projects and shepherd the subsequent results through to publication.

This is not an exhaustive guide, simply a list of recommendations based upon my individual experience. It is not a scholarly endeavor, but rather a personal impression of tips, tricks, ideas, and perspectives that I have found useful in developing a sustainable undergraduate research pipeline. However, these recommendations are not necessarily universal. I am a pure mathematician whose experience supervising undergraduate research stems mostly from academic-year projects, often as part of a senior exercise. Thus, this advice is not precisely tailored for those running REUs. Moreover, I have generally focused on producing published research articles with my students. This may not correlate exactly with the reader's goals and expectations.

Nevertheless, I hope that the reader will find at least some of the following reflections useful when working with their own students.

1. **Time is a luxury you don't have.** There are major differences between the research undertaken by a graduate student in pursuit of a Ph.D. and an undergraduate student working on a senior thesis or summer research project. The Ph.D. student takes graduate-level courses and trains for several years in a specific area under the supervision of the advisor, who is a leading expert in the field. In contrast, the undergraduate may have little or no coursework related to the topic, especially if the subject matter falls outside of the standard curriculum. Think carefully whether your preferred topic is realistic for a student to engage with in the given time frame. The research supervisor must be nimble, able to adjust to circumstances and constraints. Finding projects that are accessible and realistic for your students takes time and thought.

2. **You are not an old dog.** You can learn new tricks. Undergraduate research might require you to take a plunge into unfamiliar territory. Learning new mathematics with students is one of the great personal benefits of undergraduate research. Use it as an opportunity to get out of your Ph.D.-thesis shell. The further down a narrow research avenue you travel, the more difficult, specialized, and potentially inaccessible the problems become. To gain the flexibility and perspective necessary to develop viable undergraduate research projects, you need to expose yourself to new ideas. Do not be afraid to wade into uncharted waters for a project. Embrace the challenge!

3. **You know more than they do.** Even the cleverest of your undergraduate students has a perspective that is largely limited by standard coursework. Students who have taken part in mathematical enrichment programs may have a larger repertoire of tricks and tools, but they are not up to date with current research. On the other hand, you have an advanced degree in the mathematical sciences and are trained to conduct independent research. You are perfectly qualified to lead students on an exciting mathematical adventure. With your training and experience, you can stay a few steps ahead of your students, even in unfamiliar territory. In fact, your ideas and greater perspective may be useful, even in an unrelated field. Trust yourself!

4. **Nobody knows everything.** You do not have to be a world expert on a topic to get your students started on a project in that area. Even in your own field of expertise you probably cannot answer every question that comes your way. When learning new mathematics there are bound to be points of confusion, unclear definitions, and seeming contradictions. Working through ambiguity and uncertainty are key traits of a good mathematician. Seeing a mathematician learning new material provides your students a valuable lesson.

5. **You can be human.** At some point, your students may discover that you are human. Even truly great mathematicians encounter problems that stump them, so there is no shame in admitting that you are stuck. Students have a tendency to doubt themselves when they hit a snag. They may have unrealistic impressions of how "real" mathematicians operate and judge themselves against an unattainable ideal. Seeing a professor deal with pitfalls and roadblocks is an excellent learning experience for students.

6. **Follow their passions.** A student can fall in love with a particular topic and get obsessed. A theorem or example can pique their interest and spark their imagination. Students have more energy and enthusiasm for projects they love than for projects they are simply assigned by decree. It can sometimes be a good idea to permit students to follow their passion, as long as the journey is tempered by realism. Instead of focusing on a major open problem, you might steer students to variants or closely-related topics that still hold their interest. A student who is keenly interested in a subject, paper, or theorem can provide you with an important opportunity for exploration. Learning something new together can be a great experience. Your students might lead you into terrain that you never imagined exploring.

7. **Search for fertile ground.** Topics and questions that have been combed over by experts for decades are like dry, parched earth. With work, they might still yield fruit, although the field will be tough, perhaps too tough for an undergraduate to obtain results during the course of a summer or a senior thesis. You do not want your students competing head-to-head against leading experts who have dedicated their lives to a subject. Instead, you need to find different angles and

new questions that are adjacent or parallel to where the "big names" are working. You need to be close enough to established work that your project is respectable and cannot be easily dismissed, but not so close that your students will be constantly scooped by more experienced mathematicians.

8. **Andrew Wiles' approach is not for everyone.** Sir Andrew Wiles spent seven years in near isolation working on Fermat's Last Theorem before he completed his proof (and even then it contained a gap). Most mathematicians, myself included, are not capable of banging their heads against a difficult problem, day in and day out, for long periods of time before frustration and boredom set in. These feelings are nothing new for the professional mathematician. However, they can rapidly erode students' confidence and drive them to question whether they belong in mathematics at all. Students should not be given the impression that the Wiles-style approach is how most mathematics is done.

9. **Pivot when returns diminish.** Our students lack the experience and instinct of the professional mathematician, who has written a dissertation over the course of several years and many sleepless nights. They cannot always tell which problems are too difficult and which are realistic targets. Be prepared to pivot and shift to more feasible problems if necessary. Always make sure that your students are engaged. When mathematics stops being fun and exciting, it is time to change things up. There should be other options on the table, other questions and variations on the original theme that might prove more tractable.

10. **Focus more broadly.**[1] An intimidating and often unhelpful way to introduce students to a topic is to hand them a paper and say "read this thoroughly, understand it, and report back to me next week." One does not learn mathematics by reading – one learns by doing. Students need to hit the ground running. Have your students *skim* through a few papers in the area. Are there gentle expository papers on the topic? Those are ideal.

11. **Play twenty questions.** Ask your students to devise a list of twenty questions inspired by the reading. They come to a topic fresh and are

[1] An oxymoronic instruction from a reviewer who dismissed a colleague's grant proposal.

able to ask the naïve questions that we long ago learned to suppress. Sometimes these questions can be the starting point for an entire new line of research. Encourage your students to find interesting variants of existing questions. They need not be directly related to the topic, tangents should be especially appreciated. Even if only 10% of the questions that come up pan out, the exercise will have been a smashing success. Moreover, the students will feel ownership and pride in a project they develop themselves.

12. **Everything is negotiable.** Students should understand that every aspect of a question or problem is fluid and changeable. The original problem might prove too difficult. Perhaps there is a change of context that renders a tractable problem. What happens if some of the hypotheses of your conjecture are changed? What about looking at the same question in a different setting? Be open to sudden changes in direction and embrace the opportunity to pursue promising leads. The more questions that you and your group generate, the larger your interface with the unknown is. Instead of chipping away relentlessly at one specific problem, your students can investigate multiple points of entry, in the hope that one of them will give way. You need only gain traction on a single problem before new results start pouring in.

13. **Turn lemons into lemonade.** Sometimes things do not work out as planned. Your hunches may prove incorrect and your conjectures might be false. Do not think of this as a failure, but rather as an unusual flavor of opportunity. You must show your students how to turn things around and salvage something from the apparent disaster. Think of it as a challenge: you need to turn the negative into a positive, a failure into success. Perhaps a counterexample is more interesting than the conjecture itself. Perhaps the proof broke down in an interesting way and the counterexample points toward the correct result. Can you tighten the conjecture to make it correct? Can you count, predict, or characterize the counterexamples? There are many directions one can pivot to after an initial disappointment if one does not view a setback as a final roadblock, but rather as a detour down an unfamiliar road.

14. **Complement your research.** "Undergraduate research" and "research" are not necessarily distinct things. There may be parts of

your own research that could benefit from student involvement. Perhaps there is a tedious example that you have not had the patience to work through, even though you suspect that it will be straightforward. Maybe you have a lot of numerical evidence to gather, but have not had time to write the program and run the code. Perhaps you feel that a certain result is true and that you could prove it if only you had a few hours to grind through the details. These are perfect tasks for student researchers. What might take you hours could take a student weeks, to the benefit of both parties. What you find tedious, a student might find new and enlightening. Your personal research problems might have simpler versions suitable for students. Conversely, student research might inspire and inform your own work. It might suggest questions that you have not considered before or require you to learn new material that opens up novel avenues in your work.

15. **The computer is your friend.** Mathematical software is an excellent tool for undergraduate research. Software can help you visualize data and see patterns that would otherwise escape human notice. Students can jump right into experimentation and conjecture-making, even before they fully understand the theoretical underpinnings of the project. This is a lot more fun than the traditional "read this paper and report back on what you learned" approach. Students catch on quickly to technology, especially these days when many mathematics majors take at least an introductory computer science course. They can often learn what they need to know on the fly, since there is no shortage of how-to websites and instructional videos for `Mathematica`, `Matlab`, `Maple`, `Sage`, and so forth. Moreover, websites like StackExchange answer many rookie questions and often provide snippets of code for common tasks. Software lets your students hit the ground running.

16. **Build upon previous success.** If an earlier project worked out, you might as well continue it. Are there variations, generalizations, or extensions worth pursuing? The roadmap provided by a previous project can suggest a series of discrete tasks for the next generation of students. However, even apparently straightforward generalizations may lead to unexpected results. It almost always pays to investigate leads even if they look simple and unassuming. Often there are additional compli-

cations that turn up once one works through the details. Variations are often less transparent and more difficult than one suspects. Perhaps the obvious generalization fails because of a subtle counterexample? Complications make for more interesting follow-up projects.

17. **Feel free to hand-wave.** Proofs are not written on the first attempt. Rather, they are the result of many abortive tries, lots of scratch paper, and much hand wringing. However, students do not always see the labor and long hours that go into a result. Standard coursework suggests that each theorem follows from the previous one, in logical progression and in bite-sized chunks. How do we professors establish our research results? We usually have a general idea, informed by experience and intuition. We do not typically see all of the details at the beginning, but have only a rough idea of an approach. Students need to learn how we go about proving things. Heuristic arguments, informal reasoning, and numerical evidence can get a project started. The skeleton of an argument can be fleshed out later. A hand-wavy argument can help to parcel out different pieces of the project to multiple students. Perhaps a student could prove an important sub-case or a crucial lemma, or develop an instructive example.

18. **You don't have to go it alone!** Students should learn early on that mathematics is a social endeavor. Contrary to popular opinion, most mathematicians do not pursue difficult problems in total isolation, emerging only years later to declare their triumph. We communicate through networks of collaborators, online message boards like MathOverflow, and attend conferences, seminars, and colloquia. We are not solitary animals, although students do not always realize it. Most mathematicians love to talk about their work. In fact, they are often grateful for any attention their research generates and they will typically respond to a sincere inquiry about one of their papers. Feel free to write to a mathematician out of the blue if their work is relevant to the project. There is much to gain, and nothing to lose. In the worst case scenario, you have wasted a few minutes of your time. In the best-case scenario, you might get a crucial insight into your problem, understand a confusing point from a paper, or even gain a new collaborator.

19. **Know your audience.** There are many suitable outlets for undergraduate research. Although *Involve* is probably the most well-known example, there are many other options, such as the *Rose-Hulman Undergraduate Mathematics Journal*, *SIAM Undergraduate Research Online*, the *Pi Mu Epsilon Journal*, the *PUMP Journal of Undergraduate Research*, and the *Minnesota Journal of Undergraduate Mathematics*. On the other hand, getting undergraduate research published in mainstream research journals requires some strategy. One should have an idea of the audience before completing the final writeup since the introduction and style of the paper will depend upon the target journal. Hopefully, your project involves some relatively mainstream ideas and buzzwords, even if the work is not at the bustling center of the field. You must make some effort to relate your work to the existing literature, hopefully to papers published in the sort of journals you are considering. If your paper is more of an isolated curiosity or an oddball result, the chances of acceptance are lower.

20. **Use the modularity principle.** Not every student project is publishable. With care and foresight, however, many student projects can contribute meaningfully to a publication. For example, a student might have worked out the details of an instructive example, amassed a significant amount of computational evidence, or proved a small improvement on a known result. None of these might be publishable in and of themselves. However, all three could form a "module" in a larger work. A substantial article can be built up over the course of several undergraduate research projects. It may take a few "generations" of students before enough compelling material is collected to form a publishable research paper. You might have to hold on to a clever example, minor theorem, or a partial result for a while. Keep in touch with former students in case you need to contact them later. Most of them will be delighted to learn that their long-forgotten senior thesis project will form a crucial part of a research paper.

21. **Do not drag your feet.** Time is of the essence for undergraduate research students. Some of them might be applying to graduate school. A good publication can enhance an otherwise lackluster graduate school application. Other students may be looking for employment in the "real world" soon and a math paper might just spice up their resumé.

If you have something publishable, do not sit on it! For a tenured professor, there is little consequence in waiting a month or two to get around to writing something up. However, students (and untenured professors) are on a different clock. It is your duty to help complete the writeup and, if the results warrant it, submit the paper to a journal in a timely fashion. You should be the one responsible for posting it on the `arXiv`, ultimately deciding upon a journal, and submitting it. Only you can prevent your own foot dragging. Your students did a lot of hard work. You owe it to them to see things through in a timely manner.

22. **Is there an opportunity for exposition?** The literature on a particular topic develops organically. Important results might be strewn throughout dozens of articles spread over several decades. In many cases, there is not a standard reference that you can refer your students to. This is an opportunity to write an expository article, or a long survey article, on the subject. You and your students will probably conduct a literature review early in the project and take copious notes about the fundamentals anyway. It makes sense for this background work to pay off. Proper exposition may require detailed examples, fleshed-out proofs, extensive computations, and so on. Each of these pieces can provide a small project for a student. But where could such an article be published? In pure mathematics, conference proceedings or book chapters are common options. One might consider the journal *Expositiones Mathematicae*. You can aim high and shoot for the *Notices of the American Mathematical Society* or the *American Mathematical Monthly*, although one should always have a fallback plan in case of rejection.[2]

23. **Reach out to new communities.** Projects can pull you in new directions. You might need to learn a bit of new mathematics in order to supervise your research group. Make an effort to meet people in the field. Are there seminars nearby? Attend conferences and meet a few new people. Most mathematicians are welcoming to newcomers in an area. After all, new blood guarantees the vitality and long-term

[2] `https://mathoverflow.net/questions/15366/which-journals-publish-expository-work` discusses a few other options.

viability of the subject. Ask to give a short talk in a special session at an AMS conference. Ask to be put on the mailing list for annual conferences in your new subfield. Urge your students to give poster presentations at local or national meetings. You and your students need to get known, and this takes conscious effort.

24. **Get them recognized.** Suppose that you and your students have just completed a successful project. Perhaps it is a fine senior thesis, an excellent REU project, or a published research paper. Now comes the final, and possibly most important step, in promoting your student's research. Does your institution or department have a research prize for undergraduate students? If so, nominate them! It is up to you, since likely nobody else knows about the project or cares enough to nominate your student. Your institution probably has a large and well-funded communications staff that handles media relations, Twitter feeds, Facebook profiles, and so forth. If there is a compelling story to tell, your institution's communications staff wants to know. They are always on the lookout for good stories that highlight student research. This is part of their job! So get to know your communications staff. There will be a fancy webpage that lists the contact information of the key communications personnel. Get to know them and clue them in to any exciting work done by your students.

The preceding list is not a fool-proof recipe for successful undergraduate research mentoring. It is a reflection of my personal experiences and preferences. However, I hope that these general ideas have broader merit and that at least some of these points will apply to anyone in the mathematical sciences who is considering supervising an undergraduate research project.

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References

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