

Could Raphael's School of Athens Contain Hidden Geometry?

Frode S. Larsen

UiT The Arctic University of Norway

Harald E. Moe

Østfold University College

Follow this and additional works at: <https://scholarship.claremont.edu/jhm>



Part of the [Arts and Humanities Commons](#), and the [Mathematics Commons](#)

Recommended Citation

Frode S. Larsen & Harald E. Moe, "Could Raphael's School of Athens Contain Hidden Geometry?," *Journal of Humanistic Mathematics*, Volume 13 Issue 2 (July 2023), pages 228-279. DOI: 10.5642/jhumath.SCYQ4586. Available at: <https://scholarship.claremont.edu/jhm/vol13/iss2/14>

©2023 by the authors. This work is licensed under a Creative Commons License.

JHM is an open access bi-annual journal sponsored by the Claremont Center for the Mathematical Sciences and published by the Claremont Colleges Library | ISSN 2159-8118 | <http://scholarship.claremont.edu/jhm/>

The editorial staff of JHM works hard to make sure the scholarship disseminated in JHM is accurate and upholds professional ethical guidelines. However the views and opinions expressed in each published manuscript belong exclusively to the individual contributor(s). The publisher and the editors do not endorse or accept responsibility for them. See <https://scholarship.claremont.edu/jhm/policies.html> for more information.

Could Raphael's School of Athens Contain Hidden Geometry?

Cover Page Footnote

We want to thank Kristin Emilie W. Bjørndal, Marianne Fjellingsdal, Anniken Greve, Nils Herman Hornnæss, Jonas Oskarsson and Eimund Sand for valuable suggestions in writing this paper.

Could Raphael's *School of Athens* Contain Hidden Geometry?

Frode Sirnes Larsen

Department of Education, UiT The Arctic University of Norway, NORWAY
frode.sirnes.larsen@uit.no

Harald Elvind Moe

*Department of Natural Sciences, Practical-Aesthetic, Social and Religious Studies,
Østfold University College, NORWAY*
harald.e.moe@hiof.no

Synopsis

In this article we argue that Raphael has hidden a geometric shape called a *vesica piscis* in his fresco *The School of Athens* (1510-1511). The *vesica piscis*, and several findings which can be interpreted as suggesting the presence of a *vesica piscis* in the fresco, are presented. Several of these suggestions relate to the *vesica piscis* drawn in the construction of an equilateral triangle in the first proposition of Euclid's *Elements*. Based on findings in the fresco, we suggest that the *vesica piscis* should be interpreted in light of a philosophical and theological controversy which took place in Italy in the decades around 1500, between the Catholic Church and philosophers at the University of Padua.

1. Introduction

Raphael's sympathetic depiction of pagan philosophers, scientists and mathematicians in *The School of Athens* (Figure 1) suggests that those in charge of decorating the Stanza della Segnatura, once a part of pope Julius II's private apartment, regarded classical philosophy as being compatible with Christianity, and more generally, regarded secular reasoning as being compatible with religious faith based on revelation and canonical scripture. Given the central role of Thomas Aquinas' interpretation of Aristotle to Catholic

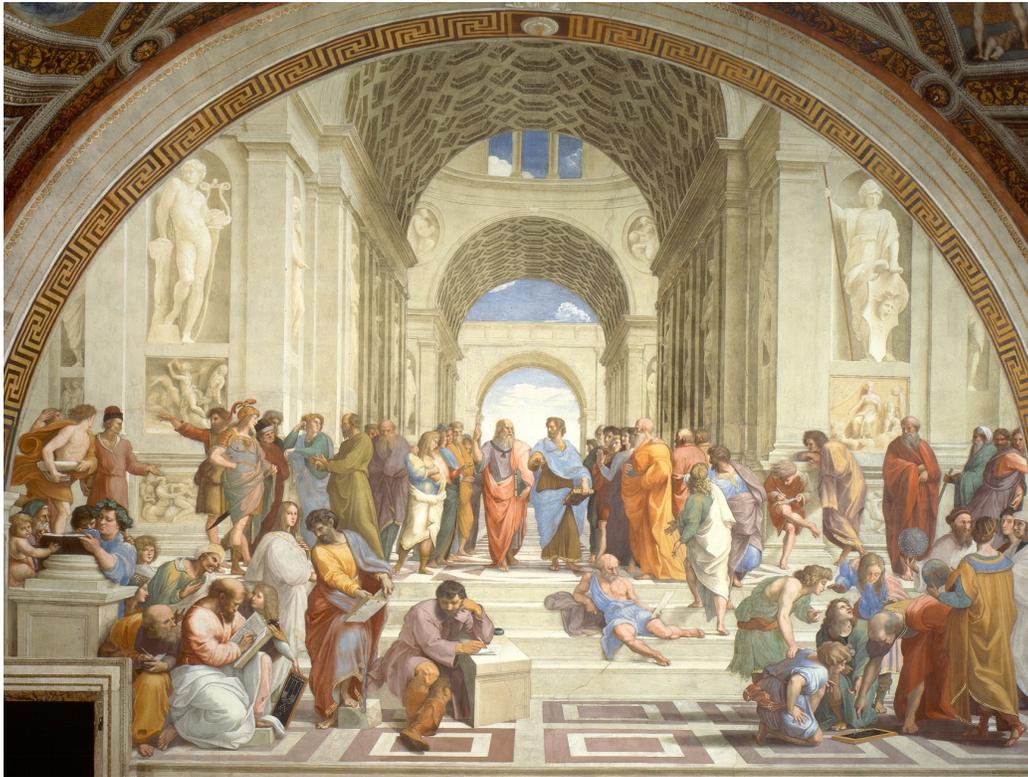


Figure 1: Raphael, *The School of Athens* (1510-1511). Public domain image from Wikipedia, available at https://en.wikipedia.org/wiki/The_School_of_Athens, last accessed on July 28, 2023.

theology, this should not be surprising. But when it comes to the layout of the Stanza della Segnatura, Raphael's supposedly favorable assessment of philosophy and secular reasoning is most frequently attributed to the humanist movement which held sway in the Roman Curia at the beginning of the 16th century. The philosophical interests of this movement were greatly influenced by the Neo-Platonism of the Italian scholar and Catholic priest Marsilio Ficino, and his idea of a *prisca theologia* (an “ancient theology”). Several scholars have suggested that Giles of Viterbo, a follower of Ficino, advised Raphael during the planning of *The School of Athens* (see [28, page 169-171] and [38, pages 147-148 and 158-159]).

At the time when Raphael was painting *The School of Athens*, the attitude of the Catholic Church towards philosophy was ambivalent, though.

For several decades, there had been a conflict between the Church and Aristotelian philosophers primarily located at the University of Padua.¹ An edict issued in 1489 by Pietro Barozzi, the Bishop of Padua, which forbade public disputations of an interpretation of Aristotle contradicting the immortality of the soul, was explicitly addressed to the Paduan philosophers. In 1513, two years after Raphael had finished the frescoes in Stanza della Segnatura, the Fifth Lateran Council issued a papal bull condemning philosophy in conflict with Christian doctrine. Also this bull is believed to be primarily motivated by the conflict with Aristotelian philosophers in Padua. John Monfasani puts it this way in his article “Humanism and the Fifth Lateran Council”:

The most famous decree to come out of the Council was the papal bull *Apostolici Regiminis*, which has traditionally been viewed as aimed at the secular Aristotelians in Padua who refused to make philosophy conform to the dictates of theology. [27, page 27]

All but two of the delegates to the council supported the decree, which instructed philosophers to conform to Christian doctrine.

In this article we will argue that Raphael has hidden a geometric figure in *The School of Athens*, which can be interpreted as an approval of the compatibility of secular reasoning and Christianity. Surprisingly, however, we shall see that the geometric figure leads us to a clue indicating sympathy with the Paduan philosophers.² This could explain why the geometric figure is not immediately visible, but only can be found by following a set of hints laid out in the fresco.³ Since Raphael depended on the Church for commissions, and criticism of the Church was a dangerous endeavor, we should not expect to find historic sources documenting that Raphael was critical of the Church, but an anecdote written by his friend Baldassare Castiglione, and published after Raphael's death, indicates at least a certain irreverence towards the church:

¹Presentations of the controversy can be found in [37], [25, chapter 3] and in [43, pages 50-92].

²If this interpretation is correct, Raphael's conception of how secular reasoning is compatible with Christian beliefs must have been more radical than that of the Roman Curia, possibly involving a less orthodox and dogmatic understanding of Christian beliefs.

³What Raphael has done here, according to our interpretation, can be understood as a visual version of the distinction between exoteric and esoteric writing expounded in books by Laurence Lampert [23] and Arthur Melzer [26] and before them by Leo Strauss [44].

In this manner also the painter Raphael replied to two cardinals with whom he was on familiar terms, and who (to make him talk) were finding fault in his presence with a picture that he had painted — in which St. Peter and St. Paul were represented — saying that these two figures were too red in the face.⁴ Then Raphael at once said: “My Lords, be not concerned; because I painted them so with full intention, since we have reason to believe that St. Peter and St. Paul are as red in Heaven as you see them here, for shame that their Church should be governed by such men as you.” [9, pages 146-147]

The veracity of the anecdote is not as significant, we think, as the fact that this is how his good friend chose to present Raphael.^{5,6}

2. The Vesica Piscis

An interesting feature of *The School of Athens* is that Raphael was bold enough to place himself in the company of the intellectual giants of ancient Greece.⁷ By locating Pythagoras and Euclid in the foreground, surrounded by pupils, and each with a tablet exemplifying their mathematics, Raphael has made mathematics prominent in the fresco, and it is in the group surrounding the geometer Euclid that we find Raphael’s self-portrait.⁸

⁴In Raphael’s fresco “The Meeting of Leo the Great and Attila”, the faces of St. Peter and St. Paul are painted markedly more red than the faces of the other figures in the fresco.

⁵It is worth noting that the anecdote is found in the same book [9], where Castiglione connects Raphael with Leonardo's arguments about the status of painting.

⁶Raphael’s attitude towards the Catholic Church might have been influenced by his teacher Pietro Perugino. Vasari writes in *The Lives of the Artists* [49, page 266] that “Pietro was a person of very little religion, and no one could ever make him believe in the immortality of the soul. On the contrary, with words suitable to that rock-hard brain of his, he most obstinately rejected every good reason.”

⁷Based in particular on the likeness with the self-portrait now in the Uffizi Gallery, it is generally agreed that the figure looking at us at the far right of *The School of Athens* represents Raphael himself.

⁸This figure was earlier thought to be Archimedes, but most scholars today identify him as Euclid [2, page 158].

Ingrid Alexander-Skipnes thinks that Raphael in this way is telling us something about himself and the status of painting:⁹

That Raphael included himself in this group indicates that the painter views his art as a mathematical activity governed by principles of geometry. [...] Raphael wants the viewer to see that the artist is not merely a craftsman but an inspired intellectual with knowledge of mathematical principles. [2, page 161]

Raphael's connection to Euclid is further emphasized by his signature. *The School of Athens* is the only fresco in the four *Raphael Rooms* in the Palace of the Vatican signed by Raphael, and the signature, consisting of the four letters RVSM denoting "Raphael Vrbinus Sua Mano" ("Raphael of Urbino, His Hand") is written with golden letters on the neck of Euclid's tunic.¹⁰ We will argue that there is more to this connection with Euclid, in that the geometric figure we think is hid in *The School of Athens* is closely associated with Euclid. The figure is often referred to as a *vesica piscis*, and it is found in the demonstration of the very first proposition in the first book of Euclid's *Elements*. *The Elements*, originally written in the third century BCE, still formed the basis of all teaching on geometry when Raphael painted his fresco. The first proposition in the first book of *The Elements* consists in the construction of an equilateral triangle, and should be familiar to any high school student, since it essentially is the standard construction of a 60° angle. The proposition and construction are shown in Figure 2, which is a facsimile from Erhard Ratdolt's edition of Euclid's *Elements*, published in Venice in 1482 [36].^{11,12}

⁹A similar interpretation is found in [22, page 51], where also the significance of Raphael's signature is commented.

¹⁰An overview of the signatures found in works by Raphael is found in [15].

¹¹The public demand for Ratdolt's book, the first printed edition of Euclid's *Elements*, prompted a re-edition published in Venice already the same year (1482) and to new editions in Ulm (1486) and Vicenza (1491) [3, page 710].

¹²When the *vesica piscis* has been drawn, there are two ways to draw the equilateral triangle, and we should notice that Ratdolt has drawn both of these triangles, "pointing" in opposite directions. We find the same construction in Pacioli's *Summa de arithmetica, geometria, proportioni et proportionalita* from 1494 [29] and in Pacioli's edition of Euclid's *Elements*, published in 1509 [31]. It could be significant that the content of Euclid's tablet in Raphael's fresco contains two triangles "pointing" in opposite directions, although

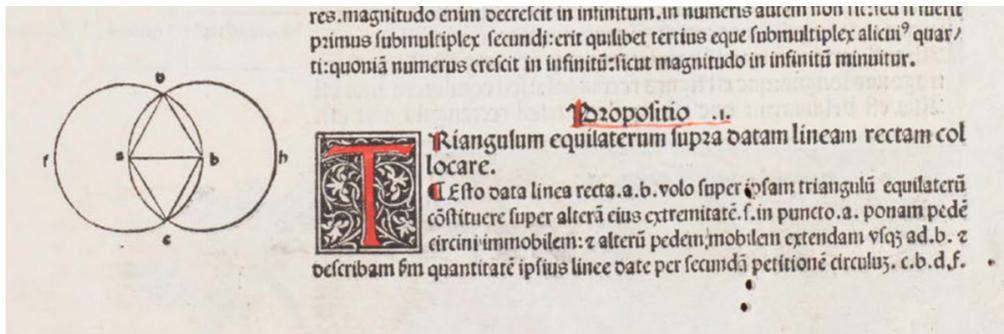


Figure 2: Euclid’s construction of an equilateral triangle in Ratdolt’s edition of Euclid’s *Elements*.

The vesica piscis is the shape made by two intersecting circles. (We will refer to the whole figure as the *double circle*.) Mathematically, a vesica piscis is a geometric figure constructed by drawing two circles, where the center of each circle is located on the perimeter of the other circle (Figure 3).

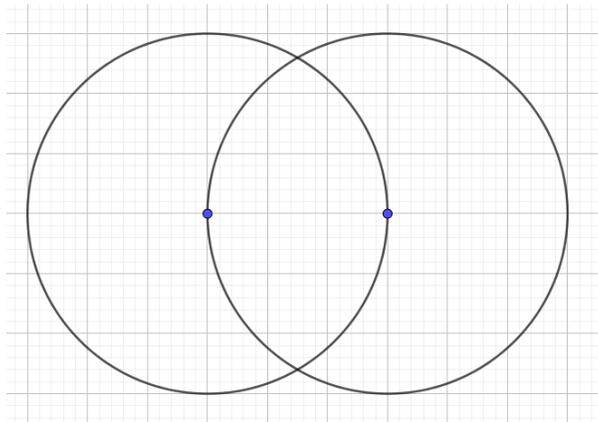


Figure 3: Double circle with a vesica piscis.

We think Raphael found this figure especially fascinating, since there already was a tradition making use of the figure in Christian art. Given the status of Euclid’s *Elements* as a peak in secular reasoning, the vesica piscis therefore has the potential of symbolically expressing a harmony between

the geometric construction on Euclid’s tablet does not seem to represent any particular proposition from the *Elements*. See [17] for a discussion of this.

Christianity and secular reasoning, fitting well with how the figure in itself often symbolizes a union of apparent opposites (see the two quotes below). In line with this, we will argue that Raphael in *The School of Athens* has created a context where a double circle and a vesica piscis found in his fresco are given a particular Christian interpretation, while simultaneously being connected with Euclid. This Christian interpretation was well established when Raphael made the painting. According to this interpretation, one of the intersecting circles represents heaven, while the other represents earth. In medieval and renaissance art, Jesus is often placed in the intersection, representing the unification of heaven and earth, being both God and man. Both John Baldock, in his book *The Elements of Christian Symbolism* [6] and Rachel Fletcher, in the article “Musings on the Vesica Piscis” [13], refer to this interpretation when they explain the use of the vesica piscis in Christian iconography:¹³

The circles may be taken to represent spirit and matter or heaven and earth. In this case the vesica piscis symbolises their meeting point. In Christian iconography it appears as the aureole enclosing the figures of the Virgin and Child or Christ. [6, page 119]

The vesica piscis signifies the mediation of two distinct entities; the complementariness of polar opposites, as when two extremes complete and depend upon one another to exist. [...] As a Christian symbol, the vesica piscis may signify Christ Incarnate, who mediates heaven and earth, or humanity and the divine. In Medieval Christian art, Christ is commonly depicted emerging from a vesica piscis to portray the entry of transcendent form into the physical world and made flesh. [13, page 96]

¹³Not all examples of this shape in Christian art conform with the exact geometry of the vesica piscis: “By what process Christ came to be depicted within the overlap of the two circles [...] cannot at present be stated. The evidence makes clear, however, that although a great many vesica piscis figures appear to be true circumscriptions of two equilateral triangles base to base, many others are drawn with a horizontal axis greater or smaller than this, suggesting their artists were either unaware of the underlying geometry, or were simply interested in the geometric figure for representing the intermediate realm shared by heaven and earth, regardless of its exact proportion” [20, chapter 3, section 5].

We know that Raphael was aware of this relationship between the geometric shape of the vesica piscis and Jesus, since he placed Jesus (together with God the Father and the Holy Spirit, in the shape of a dove) inside a mathematically exact vesica piscis in the *Holy Trinity Banner* at Città di Castello (ca. 1499) (Figure 4).

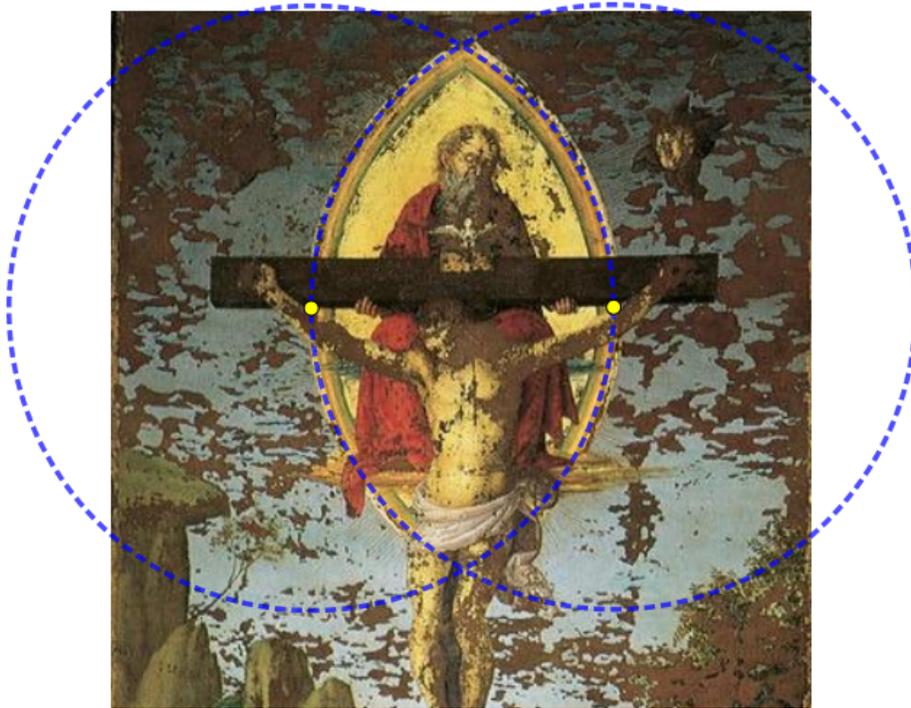


Figure 4: Raphael, *Banner of the Holy Trinity* (ca.1499) with a vesica piscis superimposed on it. The original is a public domain image from Wikipedia, available at https://en.wikipedia.org/wiki/List_of_paintings_by_Raphael, last accessed on July 28, 2023.

3. The Vesica Piscis in *The School of Athens*

In Raphael's *School of Athens* we find the theme of heaven and earth distinctly present at the center of the fresco, where Plato is pointing up to heaven, while Aristotle is making a gesture directed out towards the em-

pirical earth.¹⁴ But this duality finds an even more concrete expression, with a possible connection to the circles of the vesica piscis, in the celestial and terrestrial globes held up in the right part of the painting. Instead of painting the globes in mathematically correct perspective, with an elongated oval shape, Raphael has done what works best visually, and painted them as circles. We will present reasons to believe that these two circles, literally representing heaven and earth, are meant to hint at the drawing of two large circles also representing heaven and earth, and making a large vesica piscis. This is a vesica piscis which is not immediately perceptible in the fresco.



Figure 5: Right and left, details from *The School of Athens* (Figure 1).

The men holding the two globes are directing their gaze towards the figure scholars agree represents Raphael himself, and Raphael is looking at us (Figure 5). In the left part of the fresco, another figure, with a look and expression similar to Raphael, is also looking at us (Figure 5).¹⁵ This fig-

¹⁴We do not know for certain how freely Raphael chose the content of the fresco. Ingrid D. Rowland thinks Pope Julius II gave Raphael much freedom: “Julius seems to have given artists comparative freedom, not for lack of interest [...] but because he seems to have known how to delegate responsibility. [...] For the intricate details of the Stanza della Segnatura, the pope may have given significant responsibility for their actual design to the artists themselves; this certainly seems to be the case for Raphael, whose drawings for some of the Stanze frescoes show major changes as he thought through the room’s design.” [39, pages 97-98] It is also uncertain who might have helped Raphael with the design. [28, pages 169-170, note 60] contains a long list of possible candidates. See also [46, pages 103-105].

¹⁵In all, there are three figures in the fresco looking at the spectator. The third figure is the small child in the far left. We will return to him later in this article.

ure is often taken to represent Francesco Maria I della Rovere, the Duke of Urbino and the nephew of pope Julius II, but we side with those also identifying the female-looking figure as Hypatia. Hypatia was a mathematician and astronomer from Alexandria, and daughter of Theon of Alexandria, whose version of Euclid's *Elements* was the basis for the first renaissance editions of Euclid's work, and who also was thought to be responsible for the proofs in Euclid's work, Euclid himself only providing the propositions [16, page 150].¹⁶ The construction of the large vesica piscis makes use of these two figures, whose eyes are made particularly salient by being directed at the spectator of the fresco. The right half of Raphael's face is painted in a shadow, and it is the eye in the shadowed part of his face we shall make use of, possibly involving a symbolic association of the shadow with something hidden or secret. When we now construct the vesica piscis, it will be explained how by starting at this eye (the eye of Raphael closest to the center of the fresco) we are lead to the left eye of Hypatia (the eye closest to the center of the fresco).¹⁷

¹⁶The main argument against identifying the figure as Hypatia is the fact that she was murdered by a Christian mob (which later made her a symbol of opposition to the Catholic Church). Since we argue that *The School of Athens* contains criticism of the Catholic Church, this argument works the other way around from our perspective. Here, Abbott and Abbott present reasons why the identification as Hypatia must be discarded: “[...] a devoted group of mathematicians continue to lobby for Hypatia. Who else would it be, after all? In a room full of otherwise male philosophers, a beautiful woman glides in from the left, pulling her cloak across her shoulders. Given the overall compositional emphasis on mathematics in general, who other than Hypatia could hold her own in this gathering of the greatest of ancient Greek minds? The art historical retort is quite straight-forward: the walls of Julius II's library were no place for Hypatia. To argue for her in *The School of Athens* is to ignore entirely the cultural context of the time and place in which the fresco was conceived, and to forget the controversial accounts of Hypatia's death at the hands of an angry, Christian mob. It would have been a delicate maneuver, indeed, for Julius II to advance the achievements of certain ancient Greek philosophers while simultaneously drawing attention to a female mathematician whom many believed perished at the bidding of the church” [1, page 529] Specific arguments for the identification as Francesco Maria are found in [19, page 132-133].

¹⁷This line from eye to eye matches the focus on lines to the eye in 15th century theory of perspective. Kirsti Andersen writes about Leon Battista Alberti's *De pictura* (1435): “From optics Alberti took over the idea that an eye can be considered as a mathematical point, and the idea that any point in front of the eye is perceived through the straight line, called the visual ray, connecting the point and the eye.” [4, page 19].

A high-resolution image of *The School of Athens* with the vesica piscis, where one can zoom in on details in the painting, are found at <https://www.geogebra.org/m/bweqnzhd>, using the freely available software GeoGebra. The large vesica piscis can be constructed by a few elementary Euclidean operations (Figure 6).

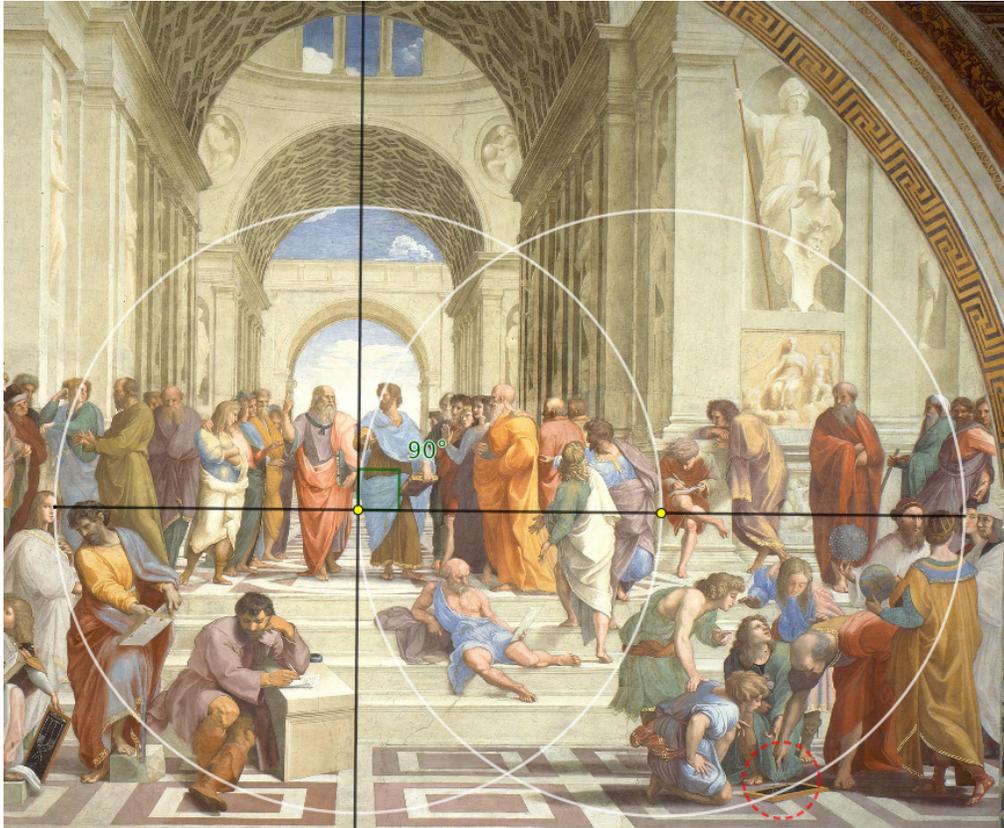


Figure 6: *The School of Athens* with a vesica piscis (Figure 1). The yellow dots mark the centers of the circles.

The center of the left circle, representing heaven, is determined by first drawing a vertical line exactly through the center of the building pictured in the fresco.¹⁸ When this vertical line has been drawn, a perpendicular to the line is drawn from Raphael's shadowed eye. This horizontal line hits the eyes of

¹⁸We let this center be determined by the two columns in the window above Plato and Aristotle, and the squares on the ground floor.

Hypatia. The intersection of the vertical and horizontal lines is the center of the left circle in the double circle we now shall draw. Drawing this circle through the left eye of Hypatia gives us one more intersection, halfway between the center of the circle and Raphael's right eye. This is the center of the right circle of the double circle. Making a vesica piscis, by drawing this circle through the center of the left circle, we find that the right circle intersects the horizontal line exactly at Raphael's right eye.^{19,20}

We should notice the aptness of using eyes to guide us to the vesica piscis in the fresco, due to the rather obvious similarity between the shape of the human eye and the shape of a vesica piscis (Figure 7).

In addition to how the proportions of the lengths between the eyes and the center of the building in the fresco fit the proportions of a double circle with a vesica piscis, we will later in this article present further reasons to believe that the large hidden vesica piscis was intentionally placed in the fresco. For now, we can notice that the right circle passes through the geometric construction

¹⁹The many half circles of the arcs and barrel vaults above Plato and Aristotle could be taken as indicating the significance of circles in the fresco.

²⁰In our GeoGebra reconstruction, we used an image from Wikipedia available at: https://en.wikipedia.org/wiki/The_School_of_Athens#/media/File:%22The_School_of_Athens%22_by_Raffaello_Sanzio_da_Urbino.jpg. Using this image gives an almost perfect match with the construction described. There are a lot of other images of *The School of Athens* available on the internet. In many of them proportions deviate from what is found in the image we have used, but in all the images we have checked, it is possible to draw a double circle through the heads of Hypatia and Raphael, with a vertical line through the center of the building and through the center of the left circle, which is perpendicular to a horizontal line through the heads of Hypatia and Raphael. In these images, the right circle also passes through Euclid's tablet. Since we have not checked against measurements of the actual fresco in Rome, we are not sure whether the precision seen in our construction is coincidental or intended. We do believe, however, that the image from Wikipedia we have used gives a relatively accurate representation of the fresco. In contrast to most other available images, this image is composed of two photos that have been fused together to reduce distortions of the original proportions. Lines that should be straight are straight, lines that should be parallel are parallel and lines that should be perpendicular to each other are so (except the slight irregularity mentioned below). In the image we have used, the line between the eyes of Raphael and Hypatia are parallel to the horizontal lines in the stairs, and exactly perpendicular to the vertical lines in the building. The proportions do seem to be a bit distorted in the lower part of the image, though, where the horizontal lines in the squares on the ground are slightly curved.

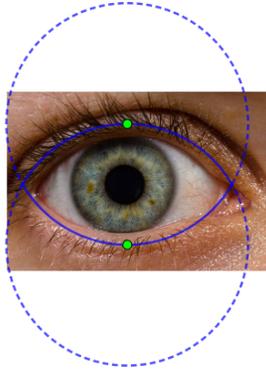


Figure 7: Human eye and a vesica piscis. Picture of human eye available at https://commons.wikimedia.org/wiki/File:Human_eye_with_blood_vessels.jpg, last accessed on July 28, 2023.

Euclid is making (or explaining) while he holds a pair of compasses, an instrument used to draw circles (Figure 8). This feature of the double circle fits well with the presence of a double circle in the first proposition of *Euclid's Elements*, while the celestial and terrestrial globes held up immediately above Euclid suggest the connection between the first proposition of the *Elements* and the Christian interpretation of the vesica piscis.

The School of Athens is most often interpreted as containing a twofold structure, with philosophers associated with Plato on the left side, and philosophers and scientists associated with Aristotle on the right side. The presence of the double circle and the vesica piscis in the fresco indicates a threefold structure, which also could be reflected in the tripartite window above Plato and Socrates. Plato and the figure representing Socrates (dressed in green) are placed in the left circle, presumably the circle representing heaven.²¹ In addition to Plato's upward pointing finger, this accords well with Plato's focus on the realm of the ideal forms, and Socrates' dialogical exploration of abstract concepts with less weight on the testimony of experience. Given the obvious association between 'heaven' and 'sky' (both called "caelum" in Latin), we should notice that most of the visible sky in the fresco is located within the circle we think represents heaven.

²¹According to the interpretation presented in [24], the Neoplatonist Boethius can also be found inside this circle.



Figure 8: Circle through Euclid's construction in *The School of Athens* (Figure 1).

In the right circle, presumably representing earth, we find the two figures holding up the celestial and terrestrial globes. Without going into details of exactly who they might represent, it is reasonable to assume that they represent an astronomer and a geographer, both what we today would call empirical scientists.²² The geometer Euclid is also placed in the right circle. He fits well in the “earth-circle” when we remember that the Greek etymological root meaning of ‘geometry’ (‘geometria’ in both Latin and Italian) is ‘earth-measuring’ (γεωμετρία; geo- “earth”, -metron “measure” [48, page 1]). Correspondingly, Euclid’s tablet is placed flat on the ground.

Aristotle is placed inside the vesica piscis, the intersection of the two circles. This matches the fact that Aristotle can be viewed as representing a fusion of the conceptual and metaphysical investigations of Plato and Socrates, with

²²Their identity is discussed in [21].

the empiricism of natural philosophy. Also Aristotle's gesture with his right hand matches his location inside the vesica piscis. Although it is natural to interpret Aristotle's gesture as directed at the perceptible world, contrary to what is sometimes claimed, he is not pointing downward, diametrically opposite to the upward pointing of Plato, but is holding out his hand, and making a gesture midway between pointing upwards and pointing downwards. Balancing Plato's upward pointing in the left circle, the large man dressed in red in the right circle looks like a mirror image of Plato and seems to be pointing downward. Aristotle's clothing is another indication that he represents a fusion of heaven and earth, wearing clothes in blue and brown reminiscent of the sky and earth. The upper part of his body is painted against the sky, while his feet are firmly planted on the ground, in contrast to Plato who is depicted with his heels raised. In line with the Christian interpretation of the vesica piscis as uniting heaven and earth, Aristotle's placement in the vesica piscis will later in this article be given an interpretation associating him with Jesus.

4. A Right Angle

We shall now take a closer look at several features strengthening the hypothesis that Raphael intentionally hid a large vesica piscis in *The School of Athens*. To find the center of the circle representing heaven, a vertical line should be drawn through the center of the building. This vertical line is rather exactly aligned with the right edge of the obtrusive block of marble Michelangelo/Heraclitus is leaning on (Figure 9).²³

Michelangelo/Heraclitus is usually described by scholars as sitting on the bottom step of the stairs, while there are, as Glenn W. Most puts it "[...] three stone blocks of increasing size along the floor from left to right [...]" [28, page 157, note 25]; Pythagoras and Boethius have placed their left feet on a block each, and Michelangelo is leaning on one.²⁴ But looking closer, it is rather easily seen that there are in fact four blocks of stone here, since

²³Michelangelo/Heraclitus and the block of marble he is leaning on was a late addition to Raphael's composition. It is not found in the cartoon (a full-scale preparatory drawing) now at the Ambrosiana Gallery in Milan.

²⁴The man standing to the right of the slate is identified as Boethius in [24, pages 219-227]. The most direct reference to Boethius in the fresco is given by the similarity between the figures on the slate and figures in Boethius' *De Institutione Musica*.

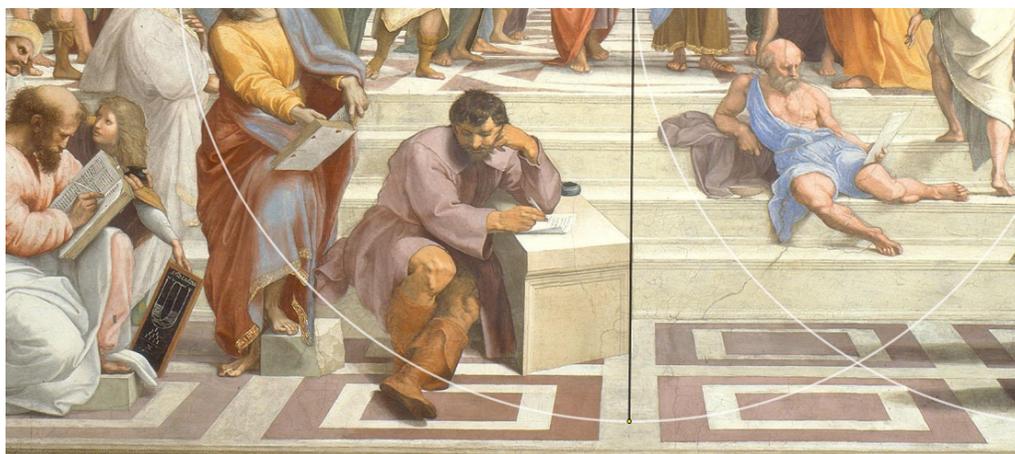


Figure 9: Vertical line along the right edge of a block of marble in *The School of Athens* (Figure 1).

Michelangelo is also sitting on one, and not on the bottom step. Comparing the squares on the ground floor, we see that Michelangelo is sitting on a stone block covering almost half of the square beneath (Figure 10).

In [24, pages 208-229] Larsen has shown how Raphael in the fresco makes use of the Boethian concept of the *Quadrivium* (arithmetic, music, geometry and astronomy) as a stairway leading up to philosophy. Representatives of the four disciplines of the quadrivium are depicted on the ground floor,

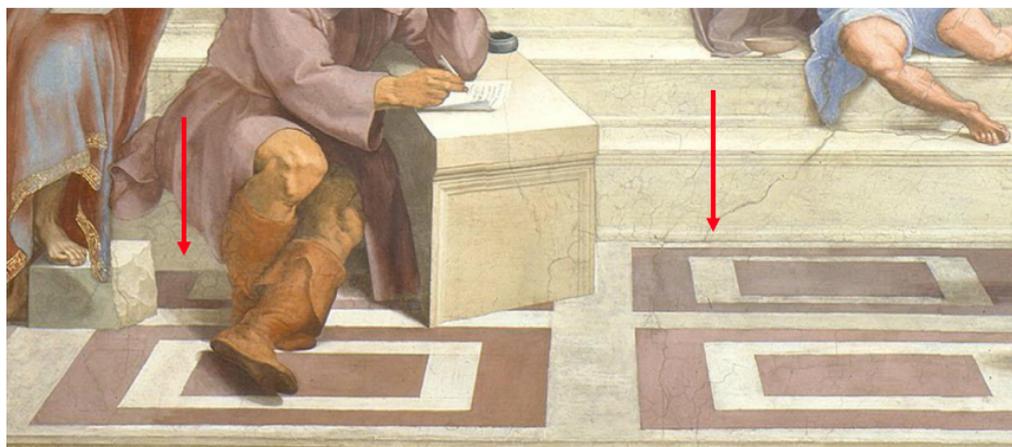


Figure 10: Detail from *The School of Athens* (Figure 1).

while the four steps of the stairway up to Plato and Aristotle correspond to the four Quadrivian steps up to philosophy. According to this interpretation, the left feet of Pythagoras and Boethius are placed on a stone each, to tell us that they represent two of the steps in the quadrivium. The figures on the slate between Pythagoras and Boethius tell us that they represent arithmetic and music. According to Boethius, who introduced the concept of the quadrivium, the order of the disciplines was arithmetic first, then music, geometry and astronomy. With four blocks of stone on the ground floor, increasing in size from left to right, it is natural also to associate the two largest stones with disciplines of the quadrivium, which then would have to be associated with geometry and astronomy (Figure 11).

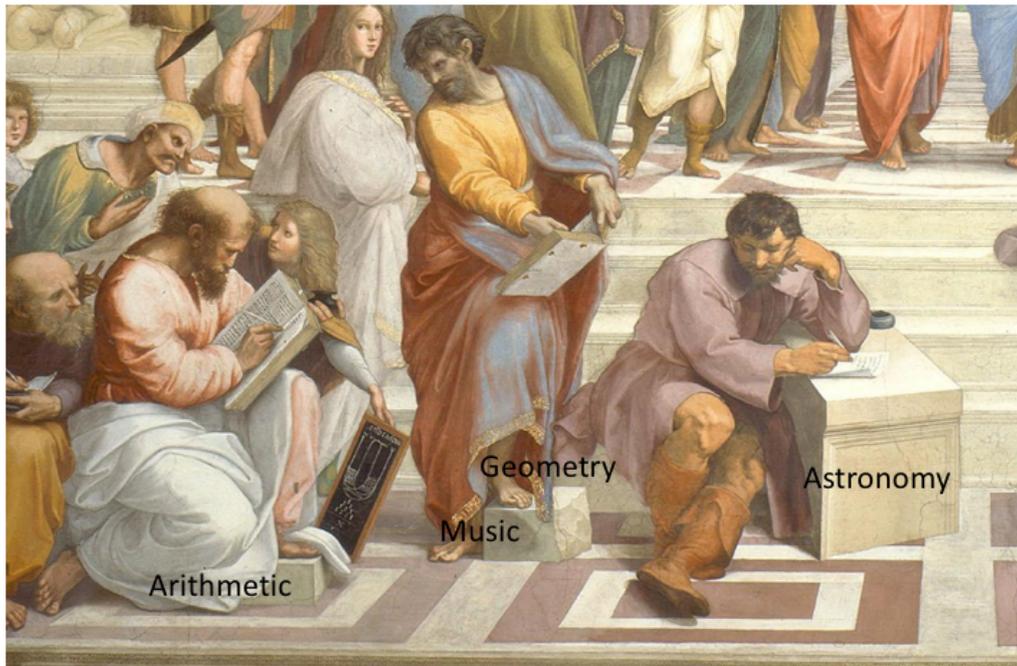


Figure 11: Stones representing the four disciplines of the Quadrivium in *The School of Athens* (Figure 1).

This interpretation fits well with how the right edge of the block of stone, now associated with astronomy, helps us in finding the center of the circle representing heaven (Figure 12).

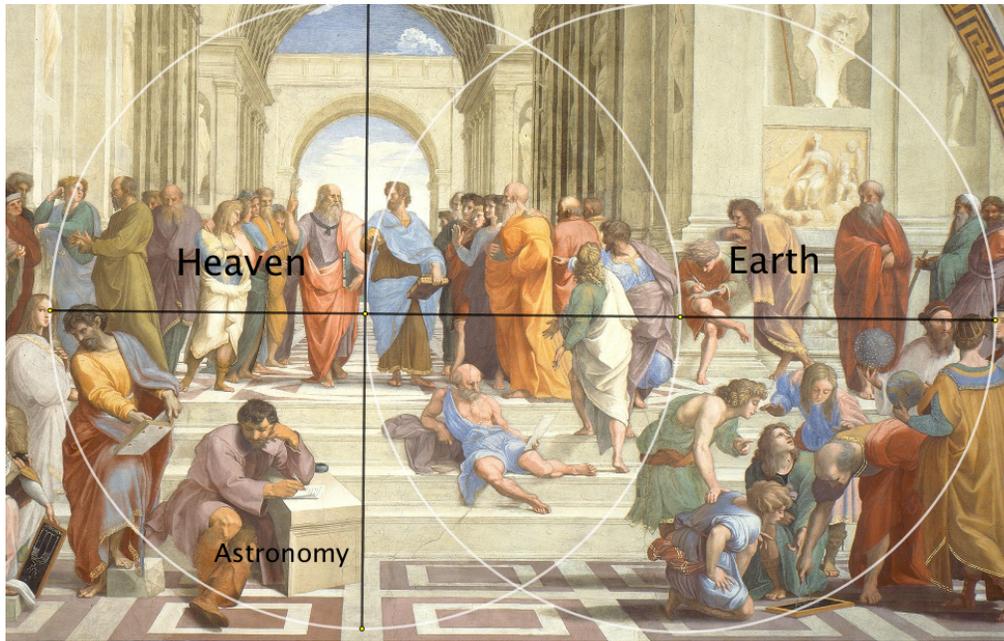


Figure 12: *The School of Athens* with a vesica piscis (Figure 1).

We shall next look at two visual hints to the effect that a line perpendicular to the vertical line should be drawn from Raphael's right eye. Extending the vertical line along the right edge of the "astronomy-block" past the edge of the fresco, we hit a personification of Philosophy in the ceiling above (Figure 13). (Notice that Plato's upward pointing finger can be interpreted as indicating that we should look for hints "above.")

Below the personification of philosophy, this vertical line first hits a putto holding a sphere painted as a circle, which could be interpreted as strengthening the hypothesis that this line participates in determining the center of a circle representing a sphere (Figure 14).

The personification of philosophy is holding a book labeled MORALIS perpendicular to a book labeled NATURALIS (Figure 15). We think Raphael is doing two things here: First, he is illustrating a double meaning found in many languages, including English, Italian, Latin and Greek, where the same word can express both that something is morally correct and that something is perpendicular. In English the word "right" has this property (something can be "morally right" or "right angled"), while we have "retto" in Italian,



Figure 13: *The School of Athens* and ceiling above. Photo: Frode Sirnes Larsen.

“ὀρθός” (orthós) in Greek and “rectus” in Latin. The double meaning is illustrated by holding a book about what is morally right at right angles to the book about natural philosophy. But secondly, since the extended vertical line hits the woman making the right angle, we also think this is meant to hint at the perpendicular we are supposed to find/draw from Raphael's eye on this line, to determine the center of the circle representing heaven.²⁵

All of the figures in the painting above are looking towards their right (Figure 15). Thus we think it is reasonable to assume that they are meant to draw our attention towards the painting they can be interpreted as looking at.

²⁵In [53], Edgar Wind presents a fascinating interpretation of the design of the ceiling of the Stanza della Segnatura, with hidden allusions similar in nature to some of those presented in this article: “The allusions are extremely remote and reveal the playfulness of a humanist mind which rejoices in making itself understood only to a select and erudite circle.” [53, page 76]

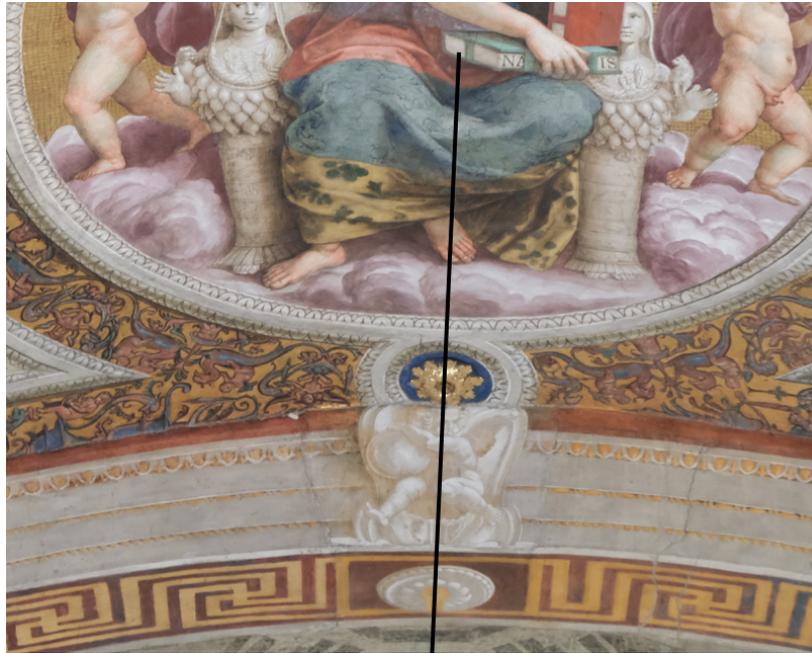


Figure 14: Putto holding a sphere beneath the personification of philosophy. Photo: Frode Sirnes Larsen.



Figure 15: Personification of philosophy in the ceiling above *The School of Athens*. Photo: Frode Sirnes Larsen.



Figure 16: Celestial globe and personification of philosophy in the ceiling above *The School of Athens*. Photo: Frode Sirnes Larsen.

This is a painting containing a large celestial globe (with a smaller terrestrial globe inside), supporting the idea that the woman making the right angle is giving us a hint related to the construction of a circle representing heaven (Figures 16-17).



Figure 17: Celestial globe in the ceiling above *The School of Athens*. Photo: Frode Sirnes Larsen.

A hint suggesting the drawing of a perpendicular from Raphael's eye, can also be found immediately above Raphael's head, where Raphael has painted an index finger making a right angle (Figure 18).



Figure 18: Right angle made by an index finger above Raphael's head. Detail from *The School of Athens* (Figure 1).

5. Beginning of Books

Raphael's fresco is filled with books, and the room where it was painted was most probably once the private library of Julius II [42]. The possible reference to Euclid's first proposition follows an apparent pattern in *The School of Athens*, of references to the beginning of books. According to the interpretation presented by Larsen in [24], Plato with Leonardo da Vinci's face involved in a public debate at the center of *The School of Athens* can be interpreted as referring to the beginning of Luca Pacioli's *Divina Proportione*, where Pacioli describes Leonardo da Vinci participating in a public debate in Milan.²⁶ The focus on Boethius' concept of the quadrivium can be interpreted as referring to the first paragraph of Boethius' *De arithmetica*, where the first known presentation of the concept of the quadrivium is found.

²⁶The most direct connection between Stanza della Segnatura and Pacioli's *Divina Proportione* are the two Platonic solids painted beneath the fresco representing poetry, near the North-West corner of the room. They are copied from Leonardo da Vinci's illustrations at the end of *Divina Proportione*. See [24, page 206-207].

We think this is a real pattern, and that the main motivation to focus on the beginning of books is what appears to be a fascination with a correspondence between the beginnings of three of the most significant books to a Renaissance intellectual: Plato's *Timaeus*, *The Bible* and Euclid's *Elements*.

The two standing figures looking at us were a key to find the large vesica piscis in the fresco. But the fresco contains one more figure looking out at the spectator: the small child sitting in the left corner (Figure 19).



Figure 19: Small child looking at us in *The School of Athens* (Figure 1).

As with the figures of Raphael and Hypatia, the small child's look at us can be interpreted as a way of bringing attention to him. The child is sitting on the base of an unfinished column, where only a small part of the shaft above the base of the column is finished. We believe that both the small child and the unfinished column are meant to represent the idea of 'a beginning'. The child is touching a book with his right hand, and this book is also resting on the unfinished column. We think this ensemble, taken together, is meant to symbolize the beginning of books.

Let us now take a closer look at the correspondence between the beginning of the Bible and the beginning of Euclid's *Elements*. The first proposition of Euclid's *Elements* describes the drawing of two circles, which, given the Christian interpretation of the double circle and the vesica piscis, can be interpreted as representing heaven and earth, and an equilateral triangle,

which with its three sides (or vertices) can be described as a trinity. Now, since God according to Christian theology is a trinity, the first verse of the Bible also presents us with heaven, earth, and a trinity:

In the beginning God created the heaven and the earth.

It seems like Raphael was playing with this correspondence already when he painted the vesica piscis on a processional banner around 1499. In this composition he has placed all three members of the Holy Trinity within the vesica piscis, corresponding to the triangle (trinity) within the vesica piscis in Euclid's construction.²⁷ Euclid's construction of an equilateral triangle in the first proposition of the *Elements* starts with the line segment AB, where A and B will be the centers of the two circles (Figure 20). In Raphael's painting, this line segment coincides with the lower edge of the horizontal bar of the cross (dashed red line, Figure 20). All the three vertices of an equilateral triangle are thereby marked in the composition. We should also notice the approximate equilateral triangle made by the head, shoulders, arms and hands of God the Father.

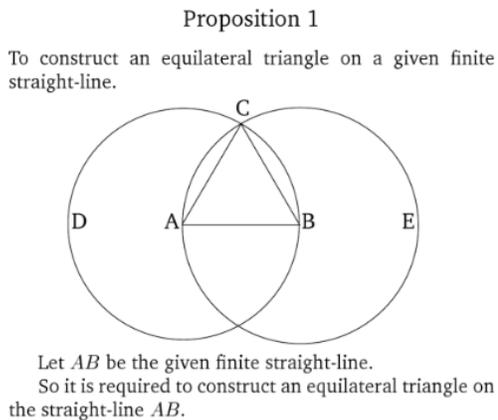


Figure 20: Left: Image from [12, page 8] about Proposition 1. Right: Raphael, *Holy Trinity Banner* (see Figure 4).

²⁷See [14, page 124] on the association between the Trinity and the triangle in the Renaissance.

If this is an idea Raphael also is playing with in *The School of Athens*, the members of the holy trinity should also be represented at the center of this fresco. We will argue that Raphael makes Plato and Aristotle represent God the Father and Jesus, respectively, and that he has also included a shape representing the holy spirit.

We will start with the Holy Spirit. Both in his *Disputation of the Holy Sacrament*, on the west wall of the Stanza della Segnatura, opposite to *The School of Athens*, and on the holy trinity banner, the Holy Spirit is represented as a dove, following a long-standing artistic convention motivated by the representation of the Holy Spirit as a dove in all the four Gospels of the New Testament. At the center of the *School of Athens*, two of the men looking most affectionately at Aristotle (Jesus) are holding their hands in a way that makes a shape resembling a dove (Figure 21).



Figure 21: Left: Dove (ID 12019743 © Irochka | Dreamstime.com). Right: “Disciples” making the shape of a dove in *The School of Athens* (Figure 1).

Associating Plato with God the Father accords with the tradition of representing God the Father as an old, bearded man, as Raphael did both on the processional banner from 1499 and on the wall opposite to *The School of Athens* (in *The Disputation of the Sacrament*) (Figure 22). Plato is not positioned inside the vesica piscis, but the position in the circle representing heaven, outside the circle representing earth, accords with how God the Father represents the transcendent aspect of God (see Figure 12).

The idea that Aristotle should be associated with Jesus was suggested by Larsen in [24], based on similarities between the center of *The School of Athens* and the center of Leonardo's *Last Supper*:



Figure 22: Left: Plato from *The School of Athens* (Figure 1). Middle: God the Father from *Holy Trinity Banner* (Figure 4). Right: God the Father from *Disputation of the Holy Sacrament*. Public domain image from Wikipedia, available at https://en.wikipedia.org/wiki/Disputation_of_the_Holy_Sacrament, last accessed on July 28, 2023.

To further support the thesis that Raphael intentionally placed Plato/Leonardo's left hand at the vanishing point of his fresco, and that this is connected to Leonardo and Pacioli's arguments about painting and the quadrivium, we observe that Raphael seems to have copied Plato/Leonardo's left arm from a left arm at the center of Leonardo's *The Last Supper*. (Recall that Pacioli ended his argument about painting and the quadrivium with a praise of this masterpiece.) Comparing the left arm of Plato/Leonardo with the left arm of John from the *The Last Supper*, we can see that both arms are covered by a red cloak over the left shoulder, with darker clothes sticking out beneath, near the hand (Figure 23). Next to these left arms, the right hand of Aristotle is held in a position similar to the right hand of Jesus. Moreover, Plato/Leonardo's pointing right hand is similar to the pointing right hand next to Jesus. [24, page 236]

The vesica piscis in *The School of Athens* provides further reasons to connect the center of the fresco with Pacioli and Leonardo's *Last Supper*, and thereby with Jesus. As we have seen, Jesus was often placed inside a vesica piscis in Christian art. Counting, we find that there are thirteen men inside the vesica piscis in *The School of Athens* (Figure 24). This, of course, could be the number of Jesus and his twelve disciples, a number that plays a significant role in Luca Pacioli's *Divina Proportione*. In the passage below, from chapter XXIII, Pacioli tells us that he describes exactly thirteen “effects” of the divine proportion since this corresponds to the number of Jesus and his disciples, before he explicitly connects this with Leonardo's *Last Supper*:



Figure 23: Top: Detail from Giampietrino's 1520 copy of *The Last Supper*. Public domain image from Wikipedia (<https://commons.wikimedia.org/wiki/File:Giampietrino-Last-Supper-ca-1520.jpg>). Bottom: Detail from *The School of Athens* (Figure 1). We use Giampietrino's 1520 copy, the main source for the 1978–1998 restoration of the original, since the details are more distinct than in Leonardo's original.

How, through reverence for our salvation, our discussion of the effects is ended.

It does not seem to me, excellent Duke, that I should go on any further about its infinite effects, because the paper would not suffice for the ink to express all of them. But we have selected only these 13 from among the others, in reverence of the group of twelve and its most holy Leader, Our Redeemer Jesus Christ, since having attributed to them the name divine, we should also end their study with the number of our Redemption, the 12 Articles [of the Creed], and the 12 Apostles with Our Savior.

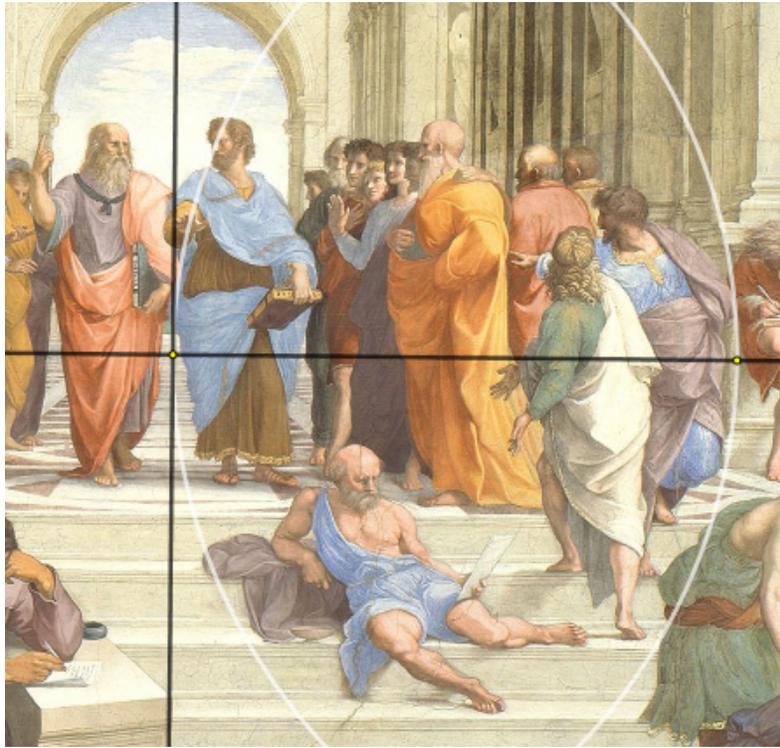


Figure 24: Detail from *The School of Athens* (Figure 1).

And I understand that your Excellence the Duke, has a singular devotion to that assembly and commissioned our aforementioned Leonardo, with his graceful paintbrush to paint them in the place cited above, the most sacred temple, Santa Maria delle Grazie in Milan. [32, page 27]

The association of Plato and Aristotle with God the Father and Jesus, supports our interpretation that Raphael makes use of the vesica piscis as a symbol of compatibility between secular reasoning and Christian beliefs, in line with how the vesica piscis provides a link between the first proposition of the *Elements* and the first verse of the Bible. Part 7 of this article contains what we regard as the most compelling evidence that the depiction of Aristotle in *The School of Athens* also represents Jesus. In that part it will be explained who the large man dressed in yellow at the center of the vesica piscis is supposed to represent, and we will present a possible explanation as to why this man, and not Aristotle/Jesus, is placed at the center of the vesica piscis.

As we shall see, this seems to involve an intimate connection with the Aristotelian philosophers in Padua, who at the time were caught up in a conflict with the Catholic Church.

6. The Beginning of *Timaeus*

Raphael signals the significance of *Timaeus* not only by placing it in Plato's hand at the center of his fresco. As Nicholas Temple has pointed out, the hand carrying *Timaeus* is also placed at the vanishing point in the fresco:

Given the importance of the *Timaeus* in the Renaissance, it is perhaps not surprising that Plato's left hand should coincide with the vanishing point of the fresco. While it is generally the case that vanishing points in Renaissance paintings were not the locations of important symbolic elements, the example of the *School of Athens* would seem to suggest otherwise. [47, page 139-140]

The correspondence connecting the beginning of Euclid's *Elements*, the beginning of *The Bible* and the beginning of Plato's *Timaeus* consists in the fact that the first verse of the Bible mentions the trinity (God), the first proposition in Euclid involves a triangle, while Plato's *Timaeus* begins with Socrates counting to three:

One, two, three . . . but now where's our fourth, my dear Timaeus, of yesterday's feasters and hosts of today? [35]

Since triangles play a vital part in Plato's dialogue, the first three words ("one, two, three") are naturally interpreted as alluding to the theme of triangles.²⁸ That, at the least, is how we think Raphael is interpreting this line

²⁸Triangles, and in particular equilateral triangles, play a vital part in Plato's book. *Timaeus*, the main character in the dialogue, is a Pythagorean philosopher. Towards the end of the dialogue he describes how three of the four basic elements (water, air, and fire) are composed of what is now called *Platonic solids*, constructed entirely of equilateral triangles. After describing a rather tedious way to make an equilateral triangle, we are told, without use of the technical terms, that fire consists of tetrahedrons made of four equilateral triangles, water consists of octahedrons made of eight equilateral triangles and air consists of icosahedrons made of twenty equilateral triangles. Equilateral triangles are thus literally the fundamental building blocks of reality, according to Plato's dialogue.

in his depiction of Socrates in the fresco. The possibility of a correspondence between the opening line of *Timaeus* and Raphael's depiction of Socrates has also been pointed out by other scholars:

[...] Socrates' gesture of ticking off his fingers could also be a way of illustrating the introductory words "One, two, three" of Plato's *Timaios*; for at the beginning of that book Plato represents his teacher Socrates as counting those of his partners in dialogue who were already present: One, two, three. [54, page 301]

By looking closely at details in the painting, we will argue that there *is* a correspondence, and that Raphael is presenting Socrates as pointing out, and counting, the three vertices of an equilateral triangle (Figure 25).



Figure 25: Socrates counting in *The School of Athens* (Figure 1).

On Socrates' left hand, three fingers are visible; the thumb, the index finger, and the middle finger. The fingers of his right hand are holding the index finger of his left hand. We believe Raphael is depicting him as he is "ticking off" the three fingers shown on his left hand, and that he now has come to finger number two, the index finger which he is holding, after ticking off the thumb as finger number one. We should now notice that the person to the left of Socrates is indicating the shape of a triangle with his arms and fingers.

(Since the identity of this figure has not been agreed upon by scholars, and he is showing Socrates a triangle, we will refer to him as Timaeus). The two forearms of Timaeus make two sides in this triangle, while the bent fingers at the top indicate the beginning of the third side. His pose indicates that the triangle should be understood as an equilateral triangle; the two visible sides of the triangle are both represented by a forearm, indicating that they should be of equal length, and the angle between the forearms is approximately the 60° of an equilateral triangle.

We should next notice that the index finger on Socrates' left hand, which is ticked off as finger number two, is pointing at the hand of Timaeus where *two* fingers are visible. Socrates will presumably next tick off his middle finger as finger number three, and this middle finger is pointing upwards towards the hand of Timaeus where *three* fingers are accentuated.

In this way, the portrait of Socrates in the fresco can be interpreted as representing the pointing out, and counting, of vertices in an equilateral triangle. The thumb, presumably ticked off as "one" before ticking off the index finger, is the first vertex. Timaeus' hand showing two fingers, pointed to by Socrates index finger, is the second vertex, while the hand showing three fingers, pointed to by Socrates' middle finger, is the third vertex (Figure 26).



Figure 26: Socrates counting in *The School of Athens* (Figure 1).

If we now draw an equilateral triangle with one vertex at the tip of Socrates thumb and one vertex where the fingers of Timaeus' right hand bends,

we find that the third vertex ends up on one of the two visible fingers Socrates is pointing to with his index finger (Figure 27). In this equilateral triangle we can see that the line from Socrates' thumb up to the apex of the triangle follows the direction of the thumb. To the left we see that the line from Timaeus' left hand up to the apex of the triangle correspondingly follows the direction of the fingers on this hand. At the apex of the triangle we see that the lines meeting at the vertex coincide with Timaeus' bent little finger.

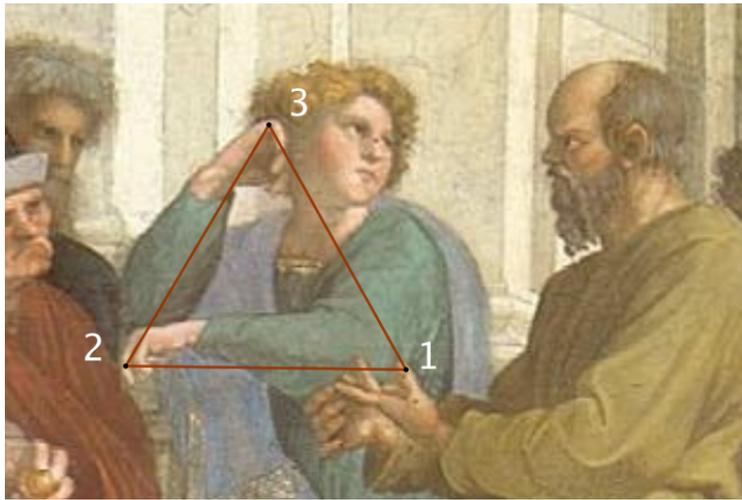


Figure 27: Equilateral triangle in *The School of Athens* (Figure 1).

In our interpretation of the fresco, the significance of this equilateral triangle is given by its connection to the vesica piscis in Euclid's construction of such a triangle. (Notice that the left circle of the large vesica piscis is passing through this triangle (Figure 6)). In Euclid's construction, the point of the two circles is that the three sides of the triangle all are radiuses in circles of equal size. We think Raphael makes use of a Latin pun to indicate that we correspondingly should draw circles around the triangle Socrates is pointing out. This is signaled by using two forearms as sides in the triangle, which functions as a pun since the word "radius" in Latin, in addition to referring to a line from the center of a circle to its perimeter, is also the anatomical name of one of the two long bones in the human forearm. In Figure 28, the radius in each arm is colored red. By drawing the two circles around the triangle Timaeus is indicating with his forearms, the radiuses in Timaeus' forearms will at the same time be radiuses in the two circles.

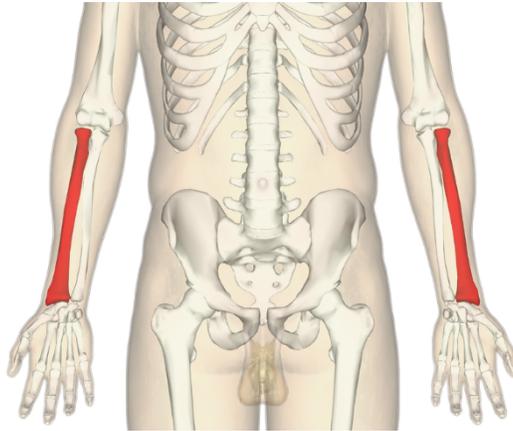


Figure 28: The radius in each arm colored red. Public domain image from Wikipedia, available at [https://en.wikipedia.org/wiki/Radius_\(bone\)](https://en.wikipedia.org/wiki/Radius_(bone)), last accessed on July 28, 2023.

According to William S. Haubrich's *Medical Meanings: A Glossary of Word Origins*, this connection between the human forearm and the Latin word "radius" was established already in the first century:²⁹

radius is the name of the smaller [shorter] bone in the forearm and was so called because it was thought to resemble the spoke of a wheel. At least that is what it looked like to Celsus, the 1st-century Roman writer who introduced the term to anatomy. The spoke of a wheel in Latin is *radius*. [18, page 202]

The two circles with Timaeus' forehands as radiuses (also resembling spokes of a wheel) give us the construction in Euclid's *Elements* I.1, as seen in Figure 29 below.³⁰

In the 1482 edition of Euclid's *Elements* [36], we saw that Ratdolt painted two triangles inside the vesica piscis illustrating *Elements* I.1. (The same construction is found also in books by Pacioli from 1494 [29] and 1509 [31].)

²⁹In his article "On the geometrical term radius in ancient Latin", Erik Bohlin shows that the geometrical meaning of 'radius' is found at least as early as in Calcidius' Commentary on *Timaeus* from the 4th century AD [8].

³⁰The left circle passes through a Gorgon on the breast of the soldier, while the right circle of the large vesica piscis passes through the Gorgon (Medusa) on Athena's shield. We are unsure of the eventual significance of this.

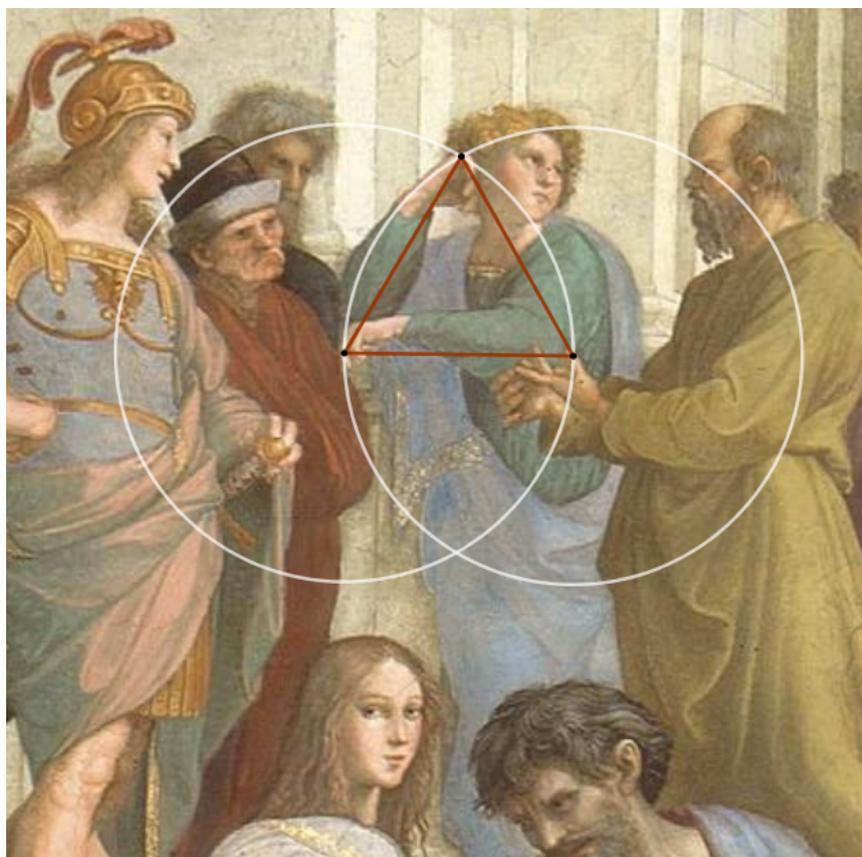


Figure 29: Equilateral triangle and vesica piscis in *The School of Athens* (Figure 1).

In the vesica piscis drawn to the left of Socrates, we can see that the bottom half of the vesica piscis, where another downward pointing triangle can be drawn, contains a triangular shape on Timaeus' robe pointing down at Hypatia. We find it reasonable to assume that this should be interpreted as suggesting the construction of the large vesica piscis involving the left eye of Hypatia.

We also find an indication of the significance of the first proposition in book I of *The Elements* below Hypatia, on the tablet in front of Pythagoras. Above the equilateral triangle of the tetractys, Raphael has placed a diagram illustrating Pythagorean harmonics. The diagram has the proportions of the double circle of a vesica piscis, and we see below that it contains one half of such a double circle (Figure 30).

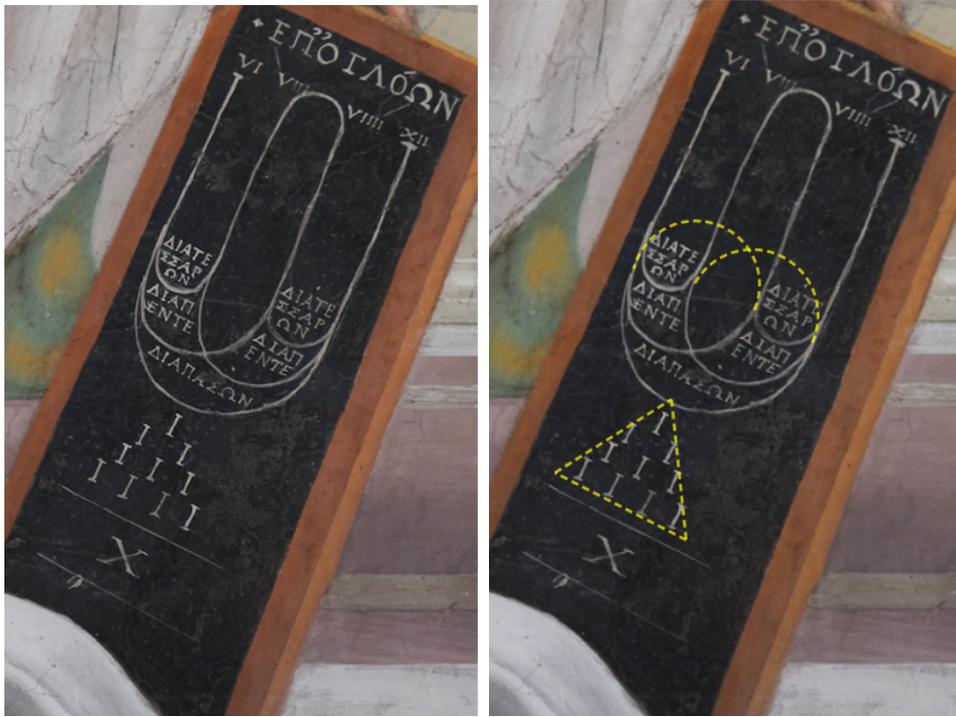


Figure 30: Slate in *The School of Athens*. Photo: Frode Sirnes Larsen.

These possible references to the triangle and vesica piscis from Euclid's first proposition, located immediately below and above Hypatia, could be interpreted as suggesting the construction of the large vesica piscis involving the left eye of Hypatia.³¹ (Figure 31).

7. Jesus and Judas in Padua

A large man dressed in yellow is placed at the center of the large vesica piscis in *The School of Athens*. We now argue that this figure represents Judas, where Raphael conforms to the artistic convention of painting Judas dressed

³¹Notice also that the oak tree emblem of Julius II, found in the corner panels flanking the fresco, has been placed inside triangles. The emblem contains an almond shape made by branches bent such that they make intersecting half circles, with a similarity to the intersecting double circle of a vesica piscis. The corner panels can be seen at: <https://www.geogebra.org/m/bweqzhd>.



Figure 31: Hypatia in *The School of Athens* (Figure 1).

in yellow [33, page 107]. The main reason we think that Raphael wants us to associate the man dressed in yellow with Judas is given by a large number of details connecting the group inside the vesica piscis with Giotto's famous portrayal of the *Kiss of Judas*, located in the Scrovegni Chapel in Padua (part of a fresco cycle completed around 1305). Given the identification of Aristotle as Jesus, both the man in yellow in *The School of Athens* and Giotto's yellow Judas are painted in profile, looking leftwards at Jesus. The references to this fresco, soon to be detailed, therefore also support the identification of Aristotle as Jesus, as well as the hypothesis that the large vesica piscis was intentionally placed in the fresco.

An apparent problem with the many details we shall see are reproduced in Raphael's fresco is that there is no documentation that Raphael ever visited Padua. The problem is only apparent, though, since it is generally accepted that Raphael made detailed use of artworks located in Padua in his own paintings [11, page 38]. Before we look at the correspondences between *The School of Athens* and Giotto's fresco, we shall look at another example where Raphael has copied details from a work of art located in a church in Padua. The example is particularly significant since it is found in *The School of Athens*.

Wilhelm Vöge was the first to discover that the three men in the far right of Raphael's fresco, above the self-portrait, are copied from Donatello's relief *Miracle of the Miser's Heart* located in the Basilica of Saint Anthony in Padua [51, page 11]. The figures are seen below in Figure 32.



Figure 32: Left: Detail from Donatello. *Miracle of the Miser's Heart*, (ca. 1386). Detail of image from [10, page 1073]. Right: Detail from *The School of Athens* (Figure 1).

This proves that Raphael either studied art in Padua himself, or that someone made detailed copies of the art from there, which Raphael subsequently made use of. In addition, the presence of these figures in *The School of Athens* establishes a connection between the painting and Padua.

To demonstrate the correspondences between the group of men inside the vesica piscis in Raphael's fresco, and Giotto's fresco of the *Judas kiss* in Padua, we will give a rather long and detailed description which fits both paintings equally well (see Figure 33 and Figure 34):

A man wearing a light pink robe, with dark hair and beard is painted in profile, looking and pointing towards left. The profile of his face is painted against the yellow clothes of a man standing behind him, who also is painted in profile, looking towards left. We do not see the hands of the man standing behind the pointing man. The pointing man's neck and chest are decorated with golden broidery. His left hand is covered in his robe.



Figure 33: Detail from Giotto. *The Kiss of Judas*, (ca. 1305). Public domain image from Wikipedia, available at https://commons.wikimedia.org/wiki/File:Giotto_-_Scrovegni_-_31_-_Kiss_of_Judas.jpg, last accessed on July 29, 2023.

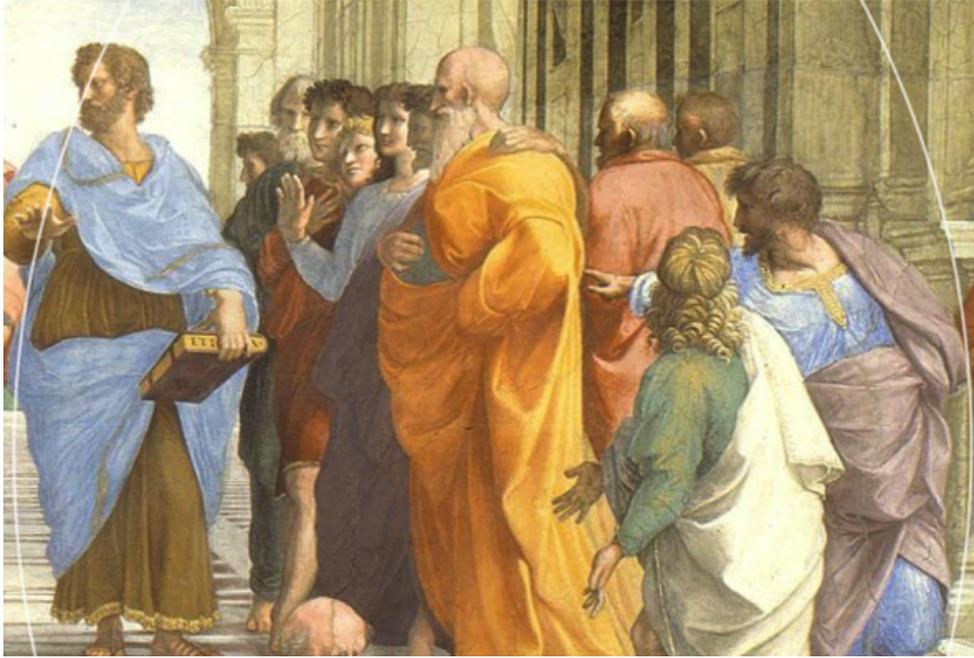


Figure 34: Group inside vesica piscis in *The School of Athens* (Figure 1).

With his right hand he is pointing with his index finger and thumb towards left in the direction of a man dressed all in yellow. The pointing fingers are painted against the red clothes of a man standing between the pointing man and the man all in yellow. Also the man in red is painted in profile, looking towards left. The hands of this man are not visible. Between the pointing man and the man all in yellow, there is also a man wearing green. Of the men described so far, the man wearing green is the only one showing both of his hands. The man all in yellow is painted in profile, looking towards left. Immediately to the left of his profile, there are faces painted tightly together. He is looking at a man with dark hair and a beard who is wearing a blue robe.

The fact that our long description matches both paintings equally well lends strong support for an intended connection between Raphael's fresco and Giotto's depiction of Jesus and Judas. It therefore serves as evidence that the man dressed all in yellow is meant to represent Judas, and that Aristotle is meant to represent Jesus.

Given the centrality of the vesica piscis in our interpretation of Raphael's fresco, we want to point out one more connection between the vesica piscis and Giotto's fresco in Padua, which might have inspired Raphael. Giotto's fresco is located in the Scrovegni Chapel. This chapel is also called the *Arena Chapel* since it is built in conjunction with an old Roman arena. The connection with the vesica piscis consists in the fact that the shape of the arena is a mathematically exact vesica piscis, a fact contemporaries of Raphael easily could have checked by using a rope. The chapel is the building up to the right in these pictures. The two yellow pins mark the centers of the two circles (Figure 35).

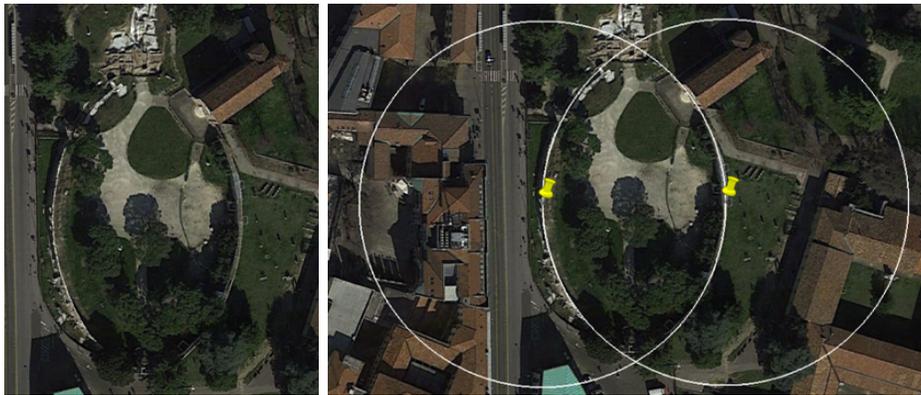


Figure 35: Roman arena outside of Scrovegni Chapel with the shape of a vesica piscis. Yellow pins mark the centers of the circles (Google Earth).

8. Criticism of the Church

Assuming that the group inside the vesica piscis contains references to Giotto's *Kiss of Judas*, it stands out as somewhat bewildering that Judas is placed at the center, while Jesus is placed at the left edge. Both the use of a pagan philosopher to represent Jesus, and the placement of Judas at the center of a vesica piscis tend towards blasphemy, and indicate some sort of betrayal. We believe Raphael hid a critique of the Catholic Church on the wall of the Pope's own apartment. Given how, as we have argued, the double circle and the vesica piscis in *The School of Athens* seem to express a compatibility of secular reasoning and Christianity, we believe that placing Judas, the arch-betrayer, at the center of the vesica piscis involves a criticism of the Catholic

Church concerning that relation. Since Aristotle is involved, as well as a reference to Padua, we find it reasonable to suggest that Raphael is pointing to the conflict between the Catholic Church and the philosophers in Padua concerning the independence of philosophy from theology, focused on how Aristotle should be interpreted.³²

In medieval Scholasticism, Aristotle was the most frequently read and discussed of the ancient Greek philosophers, and Averroes (1126-1198) was one of his most read commentators.³³ In the thirteenth century, Thomas Aquinas had seen that Averroes' interpretation of Aristotle presented problems for his project of reconciling Aristotle with Christian thought, and wrote several books criticizing Averroes.³⁴ To some extent Thomas Aquinas succeeded in bringing the thinking of Aristotle in agreement with Christian doctrines, but the problem concerning Averroes did not disappear. Averroes was continuously read and discussed, and in the decades around 1500 his ideas became a particularly hot topic in Italy, partly motivated by the availability of new translations of both Aristotle and Averroes. Most of Kara Richardson's chapter on *Averroism* in *Routledge Companion to Sixteenth Century Philosophy* [37] is dedicated to this controversy, and we will here follow her exposition of it.³⁵ The controversy was in a large part focused on Averroes' interpretation of Aristotle concerning the immortality of the human soul, a view referred to as "the unicity doctrine"³⁶.

³²Raphael's close friend Pietro Bembo forms a possible biographical link between Raphael and the philosophers in Padua. It is generally thought that the 1513 bull of the Fifth Lateran Council primarily was aimed at the Aristotelian Pietro Pomponazzi (1462-1525), who except for three years in Ferrara taught in Padua from 1488 to 1509 [34, page 39] and [45, page 48]. Pietro Bembo, whose family had a villa near Padua, and Andrea Navagero (another friend of Raphael) studied under Pomponazzi at Padua in the mid-1490s [41, page 112] and [52, page 162]. When Pomponazzi in 1518 was charged with heresy for his naturalist interpretation of Aristotle in *Tractatus de Immortalitate Animae* (1516), Pietro Bembo, then secretary to Pope Leo X, intervened and had the charges dismissed [45, page 48].

³³It has been suggested that the man in a green robe to the left of Pythagoras in *The School of Athens* represents Averroes. See for instance [7].

³⁴In *De unitate intellectus, contra Averroistas* (1270) Thomas Aquinas criticized Averroes on the issues at the center of the controversy in Italy around 1500.

³⁵Similar treatments of the controversy can be found in [25, chapter 3] and in [43, pages 50-92].

³⁶It also involved other issues, such as whether the world is eternal, and whether happiness can be attained in this life [37, page 141].

At the center of the storm was Averroes' account of the nature of intellect and its role in human thinking. According to Averroes, Aristotle held that there is numerically one material or possible intellect; this view, customarily called "the unicity doctrine," denies that the individual human soul has an incorporeal part or power. Church authorities censured the unicity doctrine in 1489 and in 1513. Their main objection was that discussion of it undermined faith, especially belief in personal immortality [37, page 137].

A group of philosophers, today referred to as 'Radical Averroists' and mainly located in Padua, was arguing that Averroes' interpretation of Aristotle was correct, and in doing that they were promoting a conception of philosophy as independent of theology:

Radical Averroists did follow Averroes in advocating the independence of philosophy from theology, arguing that the meaning of Aristotle's texts should be sought without regard to its confluence with Christian belief. More generally, philosophers should investigate all manner of things by means of empirical evidence and reason, without regard to faith. This commitment to the autonomy of philosophy challenged the view that philosophy is the handmaid of theology. At the center of Radical Averroism in the sixteenth century was the University of Padua. [37, page 143]

By adopting this strategy, the Radical Averroists at Padua did not themselves endorse heretical beliefs. Still, their philosophical activity was seen as threatening to Church authorities:

Even though Radical Averroist arts masters did not assert heretical positions as true, their views were still considered threatening, for they emphasized the tension between Christianity and Aristotle, whom many held in high regard, and they insisted that, when we leave faith out of our deliberations, and rely solely on experience and reason, we will draw conclusions opposed to Christian doctrine. Their views were, therefore, seen to promote unbelief. Radical Averroism at Padua was twice the target of formal sanction by Church authorities around the turn of the sixteenth century. Their prohibitions focused on Averroes' unicity doctrine. [37, page 145]

According to Richardson, the first of these sanctions was most probably aimed particularly at the Paduan scholar Nicoletto Vernia:

Vernia is thought to be the chief target of the first formal sanction of Radical Averroism at Padua: Bishop Barozzi's 1489 Edict against Disputants on the Unity of the Intellect [...]. The decree, which was addressed to professors of philosophy at the University of Padua and their students, forbade public disputations on Averroes' unicity doctrine. [37, page 147]

After the formal sanction in 1489, several of the Radical Averroists in Padua changed their views to less radical interpretations of Aristotle, more in accordance with Christian thoughts. It is not clear to what extent this change was caused by the sanction:

The role of Bishop Barozzi's decree in Vernia's and Nifo's reversals of opinion on the unicity doctrine is unclear. It seems likely that the decree encouraged them to develop views more in accord with Christianity. [37, page 148]

But the sanction did not succeed in obliterating Radical Averroism. As already mentioned, two decades later, in 1513, two years after Raphael had finished *The School of Athens*, Radical Averroism was once more condemned by Church authorities, this time in a bull issued from Rome:

The general effectiveness of Bishop Barozzi's decree is also unclear, for Radical Averroism persisted at Padua despite the decree, attracting censure from higher Church authorities some 20 years later in the form of a 1513 bull produced by the Fifth Lateran Council. The bull condemned those who held the intellectual soul to be mortal or maintained its numerical unity. [37, page 148]

The hostile attitude of the Catholic Church, condemning such exercise of rationality, is explicit in the 1513 bull, which clearly expresses the sentiment we believe Raphael was opposing:

[...] since truth cannot contradict truth, we define that every statement contrary to the enlightened truth of the faith is totally false and we strictly forbid teaching otherwise to be permitted. We decree that all those who cling to erroneous statements of this kind, thus sowing heresies which are wholly condemned, should

be avoided in every way and punished as detestable and odious heretics and infidels who are undermining the catholic faith. Moreover we strictly enjoin on each and every philosopher who teaches publicly in the universities or elsewhere, that when they explain or address to their audience the principles or conclusions of philosophers, where these are known to deviate from the true faith — as in the assertion of the soul's mortality or of there being only one soul or of the eternity of the world and other topics of this kind — they are obliged to devote their every effort to clarify for their listeners the truth of the Christian religion, to teach it by convincing arguments, so far as this is possible, and to apply themselves to the full extent of their energies to refuting and disposing of the philosophers' opposing arguments, since all the solutions are available. [50]

We have argued that the large vesica piscis, with its connections both to Euclid, Christianity and the Bible, was intended to express a compatibility of secular reasoning and Christianity, a compatibility fleshed out by letting Plato and Aristotle represent God the Father and Jesus. We find it plausible that Raphael, by copying details from artworks in Padua, and letting Aristotle represent Jesus, is referring to the conflict between the Paduan philosophers and the Catholic Church concerning how Aristotle should be read. By placing Judas at the center of the vesica piscis, we believe he is expressing sympathy with the Paduan philosophers' opposition to the church. This can be understood as support of a free pursuit of truth that follows reasoning wherever it leads, regardless of theology and religious doctrine, in opposition to the anti-intellectual stance exemplified in the 1513 papal bull, imposing blatant restrictions on what to say and think.³⁷ Raphael's sympathy with this opposition seems to be expressed by how a friendly arm around Judas' shoulder includes him into the group, in sharp contrast to the many depictions of Judas where a variety of pictorial means are used to separate him from the rest of the disciples.³⁸

³⁷The main point of the interpretation of *The School of Athens* found in [24] is that Raphael in the fresco presents himself as an intellectual, and not just a craftsman.

³⁸Erasmus of Rotterdam (1466-1536), a contemporary of Raphael, testifies to the possibility of being a critic of the papacy, while remaining a devoted Catholic. After having spent three years in Italy (1506-09) he wrote a satirical attack on Pope Julius II, the pope

If this is the message Raphael wanted to express, the group inside the vesica piscis, paying attention to Aristotle, could be interpreted as representing the Paduan philosophers with their controversial views on Aristotle, while Judas in the center accentuates the opposition to the catholic church.

The hypothesis that *The School of Athens* contains a hidden message concerning the attitude of the Roman Church towards secular reasoning, and that a hidden vesica piscis is used as a vehicle to convey that message, seems to be confirmed by a monochrome fresco painted by Raphael's pupil and assistant Perino del Vaga in the 1540s, beneath Euclid in *The School of Athens* [2, page 158]. The fresco depicts Archimedes as he is killed by a Roman soldier while making a geometric construction with a compass (Figure 36).



Figure 36: Perino del Vaga, Monochrome fresco representing the death of Archimedes (1540s) beneath *The School of Athens* (Figure 1).

who commissioned *The School of Athens*, called *Julius Excluded from Heaven* (*Iulius exclusus e coelis*, 1517) [40, pages 216–238], where St. Peter explains why Julius II is denied entry to heaven. The satire was published anonymously, but is unanimously attributed to Erasmus. It focuses on Julius' warmongering (he is often called the “warrior pope”), his briberies, extravagance, self-aggrandizement and the obvious discrepancy between his behavior and how Jesus wanted people to behave.

The famous mathematician being killed by a representative of Roman power is well suited as a symbolic illustration of the conflict between intellectuals and the Roman Church, which we have argued is a hidden theme in Raphael's fresco. A closer look at Archimedes' construction in the monochrome fresco reveals an interesting detail. As a possible reference to the large vesica piscis which is essential to uncover the hidden theme in Raphael's fresco, Perino del Vaga has depicted Archimedes as he is constructing a vesica piscis on the ground (the right leg of the compass is placed at the center of the left circle, Figure 37).



Figure 37: Archimedes constructing a vesica piscis. Detail from monochrome by Perino del Vaga beneath *The School of Athens*. Photo: Frode Sirnes Larsen.

Historically and mathematically, there are no obvious connections between Archimedes and the vesica piscis, so the interpretation of this small vesica piscis as hinting at the large hidden vesica piscis makes sense of what otherwise would be an odd choice by del Vaga.³⁹ The vesica piscis Archimedes

³⁹Contrary to a modern fringe myth, Archimedes does not relate his approximation of $\sqrt{3}$ as $\frac{153}{265}$ to the vesica piscis in his *Measurement of a Circle*.

is constructing is presumably also meant as a reference to the first proposition of Euclid's *Elements*. This connection seems to be underscored by how del Vaga's depiction of Archimedes' hand and compass bears a close resemblance to Raphael's depiction of Euclid's hand and compass, which can be seen immediately above del Vaga's monochrome fresco (Figure 38).



Figure 38: Left: Archimedes' hand holding a pair of compasses in monochrome fresco by Perino del Vaga beneath *The School of Athens*. Right: Euclid's hand holding a pair of compasses in *The School of Athens*. Photos: Frode Sirnes Larsen.

Although the choice of depicting Archimedes as he is drawing a vesica piscis seems odd without relating it to the hidden theme in *The School of Athens*, the two circles of the vesica piscis do fit well with what supposedly were Archimedes' last words before he was killed: "Do not disturb my circles!" (*Noli turbare circulos meos!*).

9. Conclusion

In this article we have argued that Raphael hid a large vesica piscis in his fresco *The School of Athens*, and that this geometric shape should be given a Christian interpretation, as well as being related to the vesica piscis we find in the first proposition of the first book of Euclid's *Elements*.

We have shown how this geometric shape can help us discover that Raphael's fresco contains details copied from Giotto's *Kiss of Judas*, located in Padua (in addition to details copied from another work of art also located in Padua). Based on how Raphael's figure of Aristotle is involved in this reference to Padua, we have argued that Raphael here is referring to a conflict between the Catholic Church and philosophers in Padua concerning how Aristotle should be interpreted. In a broader picture this conflict concerned intellectual freedom, and we have argued that Raphael in his fresco seems to express support for the Paduan philosophers.

References

- [1] Katherine Smith Abbott and Stephen Abbott, "Conjecture and Proof: A Case of Shifting Identities in Raphael's School of Athens," in *Proceedings of Bridges 2011: Mathematics, Music, Art, Architecture, Culture, Tessellations Publishing*, Phoenix AZ, 2011, pages 527-530
- [2] Ingrid Alexander-Skipnes, "Mathematical Imagination in Raphael's School of Athens," in *Visual Culture and Mathematics in the Early Modern Period*, Ingrid Alexander-Skipnes, ed., Routledge, New York NY, 2017, pages 150-176.
- [3] Kirsti Andersen and Henk J. M. Bos, "Pure mathematics," in *The Cambridge History of Science: Volume 3, Early Modern Science*, David C. Lindberg, Roy Porter, Katharine Park, Lorraine Daston, Mary Jo Nye, Theodore M. Porter, and Dorothy Ross, eds. Vol. **3**, Cambridge University Press, Cambridge, 2003, pages 696-723.
- [4] Kirsti Andersen, *The geometry of an art: the history of the mathematical theory of perspective from Alberti to Monge*, Springer, New York NY, 2007.
- [5] Thomas Aquinas, *Summa Theologica, Volume 5 (Part III, Second Section & Supplement)*, Cosimo Classics, 2013.
- [6] John Baldock, *The elements of Christian symbolism*, Element Books, Limited, Shaftesbury, Dorset, 1997.

- [7] Deniz Belen and Hayrunnisa Bolay, "Averroës in The school of Athens: a Renaissance man and his contribution to Western thought and neuroscience," *Neurosurgery*, Volume **64** Number 2 (2009), pages 374-381.
- [8] Erik Bohlin, "On the Geometrical Term radius in Ancient Latin," *Philologus*, Volume **157** Number 1 (2013), pages 141-153.
- [9] Baldassare Castiglione, *The book of the courtier* (trans. L. E. Opdycke), Courier Corporation, Mineola NY, 2003.
- [10] Giovanni Cecchetto et al. "Back to the Future-Part 1. The medico-legal autopsy from ancient civilization to the post-genomic era," *International journal of legal medicine*, Volume **131** Number 4 (2017), pages 1069-1083.
- [11] Jan De Jong, "Universals and Particulars: History Painting in the Sala di Costantino in the Vatican Palace," in *Recreating Ancient History: Episodes from the Greek and Roman Past in the Arts and Literature of the Early Modern Period*, Karl A. E. Enenkel, Jan de Jong, and Jeanine De Landtsheer ed., Brill, Leiden, 2002, pages 27-56.
- [12] Euclid, *Euclid's Elements* (trans. Richard Fitzpatrick), Lulu.com, 2007.
- [13] Rachel Fletcher, "Musings on the Vesica Piscis," *Nexus Network Journal*, Volume **6** Number 2 (2004), pages 95-110.
- [14] John Gage, *Color and meaning: Art, science, and symbolism*, University of California Press, Berkeley CA, 1999.
- [15] Rona Goffen, "Raphael's Designer Labels: From the Virgin Mary to La Fornarina," *Artibus et Historiae*, Volume **24** Number 48 (2003), pages 123-142.
- [16] Robert Goulding, *Defending Hypatia*, Springer, Dordrecht, 2010.
- [17] Robert Haas, "Raphael's School of Athens: A Theorem in a Painting?," *Journal of Humanistic Mathematics*, Volume **2** Number 2 (2012), pages 2-26.
- [18] William S. Haubrich, *Medical meanings: a glossary of word origins*, ACP Press, Philadelphia PA, 2003.

- [19] George L. Hersey, *High Renaissance art in St. Peter's and the Vatican: an interpretive guide*, University of Chicago Press, Chicago IL, 1993.
- [20] Nigel Hiscock, *The symbol at Your Door: Number and Geometry in Religious Architecture of the Greek and Latin Middle Ages*, Kindle Edition, Routledge, New York NY, 2016.
- [21] Christiane L. Joost-Gaugier, "Ptolemy and Strabo and Their Conversation with Appelles and Protogenes: Cosmography and Painting in Raphael's School of Athens," *Renaissance Quarterly*, Volume **51** Number 3 (1998), pages 761-787.
- [22] Christian K. Kleinbub, *Vision and the Visionary in Raphael*, Penn State Press, Philadelphia PA, 2011.
- [23] Laurence Lampert, *The enduring importance of Leo Strauss*, University of Chicago Press, Chicago IL, 2013.
- [24] Frode Sirnes Larsen, "Leonardo da Vinci in Raphael's School of Athens," *Journal of Humanistic Mathematics*, Volume **11** Issue 2 (July 2021), pages 196–243. Available at <https://scholarship.claremont.edu/jhm/vol11/iss2/9>
- [25] Craig Martin, *Subverting Aristotle: Religion, history, and philosophy in early modern science*, The Johns Hopkins University Press, Baltimore MD, 2014.
- [26] Arthur M. Melzer, *Philosophy between the lines: the lost history of esoteric writing*, University of Chicago Press, Chicago IL, 2014.
- [27] John Monfasani, "Humanism and the Fifth Lateran Council," *Review of philosophy and theology of Fribourg*, Volume **64** Number 1 (2017), pages 27-44.
- [28] Glenn W. Most, "Reading Raphael: "The school of Athens" and its pre-text." *Critical Inquiry*, Volume **23** Number 1 (1996), pages 145-182.
- [29] Luca Pacioli, *Su[m]ma de arithmetica geometria proportioni [et] proportionalita*, Paganino Paganini, Venice, 1494.
- [30] Luca Pacioli, *De divina proportione* [...], Paganino Paganini, Venice, 1509.

- [31] Luca Pacioli and Euclid. *Euclidis Megarensis philosophi acutissimi mathematicorumq[ue] omnium sine controuersia principis op[er]a*, Paganino Paganini, Venice, 1509.
- [32] Luca Pacioli, *Luca Pacioli's 1498 On the Divine Proportion* (trans. Jonathan Tennenbaum, John P. Scialdone and Richard Sanders), 2005, available at <https://www.scribd.com/document/244035060/tennenbaum-pacioli-divine-proportion-pdf>, accessed on November 17, 2022.
- [33] Michel Pastoureau, *Red: The History of a Color*, Princeton University Press, Princeton NJ, 2017.
- [34] Martin L. Pine, "Pietro Pomponazzi: Radical Philosopher of the Renaissance," Antenore, Padua, 1986.
- [35] Plato. *Timaeus*, Transl. Peter Kalkavage, Hackett Publishing, Indianapolis IN, 2015
- [36] Erhard Ratdolt and Euclid. *Preclarissimus liber elementorum Euclidis perspicacissimi in artem geometrie incipit qua[m] foelicissime*, Venice, 1482.
- [37] Kara Richardson, "Averroism," In *Routledge Companion to Sixteenth Century Philosophy*, Benjamin Hill and Henrik Lagerlund ed., Routledge, New York NY, 2017, pages 151-170.
- [38] Ingrid D. Rowland, *The Intellectual Background of the School of Athens: Tracking divine Wisdom in the Rome of Julius II*, in Marcia Hall ed., *Raphael's School of Athens*, Cambridge University Press, Cambridge, 1997, pages 131-170.
- [39] Ingrid D. Rowland, "The Vatican Stanze," in *The Cambridge Companion to Raphael*, Marcia B. Hall ed., Cambridge University Press, Cambridge, 2005, pages 95-119.
- [40] Erika Rummel, ed. *The Erasmus Reader*, University of Toronto Press, Toronto, 1990.
- [41] John Edwin Sandys, *A history of classical scholarship. 2. From the revival of learning to the end of the 18th century in Italy, France, England and the Netherlands*. Cambridge University Press, Cambridge, 2011.

- [42] John Shearman, *The Vatican Stanze: Functions and Decoration*, in *Proceedings of the British Academy*, Volume **57**, (1971), pages 369-424.
- [43] Leen Spruit, *Species intelligibilis: From Perception to Knowledge*, Brill, Leiden, 1995.
- [44] Leo Strauss, *Persecution and the Art of Writing*, University of Chicago Press, Chicago IL, 2013.
- [45] Michael Tavuzzi, "Silvestro da Prierio and the Pomponazzi affair," *Renaissance and Reformation/Renaissance et Réforme*, Volume **19** Number 2 (1995), pages 47-61.
- [46] Paul Taylor, "Julius II and the Stanza della Segnatura," *Journal of the Warburg and Courtauld Institutes*, Volume **72** (2009), pages 103-141.
- [47] Nicholas Temple, "Gesture and perspective in Raphael's School of Athens," In *Renaissance Theories of Vision*, Charles H. Carman and John Shannon Hendrix ed., Routledge, New York NY, 2016, pages 147-160.
- [48] Elias C. Tonia and Constantine N. Tonia, *Geometric Procedures for Civil Engineers*, Springer, New York NY, 2016.
- [49] Giorgio Vasari, *The Lives of the Artists*, Julia Conaway Bondanella and Peter Bondanella transl., Oxford University Press, New York NY, 1998.
- [50] V Lateran Council, Session 8, 19 December 1513, available at http://www.intratext.com/IXT/ENG0067/_P9.HTM, last accessed on November 17, 2022.
- [51] Wilhelm Vöge, *Raffael und Donatello: Ein Beitrage zur Entwicklungsgeschichte der italienischen Kunst*, Heitz, Strassburg, 1896.
- [52] Gareth D. Williams, *Pietro Bembo on Etna: The Ascent of a Venetian Humanist*, Oxford University Press, New York NY, 2017.
- [53] Edgar Wind, "The Four Elements in Raphael's' Stanza della Segnatura'," *Journal of the Warburg Institute* Volume **2** Number 1 (1938), pages 75-79.
- [54] Matthias Winner, "The Mathematical Sciences in Raphael's School of Athens," in *The Power of Images in Early Modern Science*, W. Lefèvre,