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Aligning Political Options and Aggregated Personal Opinions on the Issues

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Synopsis

Much work has been done in studying how to aggregate voter opinions to decide a fair election. These models presuppose that each voter has a solid understanding of their choices and can express that opinion in the election process. We discuss why this is not always the case. Further, we explore some of the issues that arise when considering the multidimensional nature of both voter preference, with respect to the slate of issues in an election, and the platforms of the various candidates, with respect to the same slate of issues. In light of the complications we encounter and with full apologies to Jonathan Swift, a modest proposal is made for conducting future elections in a way that offers all of the voters a true chance to find a voice.

Keywords. decision theory, voter preferences, elections, multidimensional opinions

1. Introduction

The political system in the United States, and in many countries around the world, presents voters with choices so that they can express their individual opinions regarding policy by electing officials that will represent their perspectives when making decisions. There are two aspects of such votes, each with its own essential mathematical issues. One of these aspects, which I will call the “election resolution problem,” (ERP) is about aggregating the preferences of a population to determine a fair outcome for a given vote.

This has received a great deal of attention for quite a long time; Arrow's Theorem, the works of Donald Saari (e.g. [9]), and recent work by Siegenfeld and Bar-Yam [12] provide some highlights of this. The study of ERP involves voting systems, apportionment, and gerrymandering, and [13] succinctly illustrates how these lead to unfairness with near mathematical certainty. But, by and large, there are well-studied methods for resolving ERP. By contrast, the second aggregation problem receives much less attention. Resolving this problem requires aggregating an individual voter's various opinions and ideas in order to match them with options in an election. The implicit assumption is that if your preferences are not available, you need to expand the candidate pool, possibly by standing for election yourself. But that method of resolving what I will call the "preference matching problem" (PMP) merely sidesteps the issue. In what follows, we will explore PMP and some ways of resolving it.

In the end, the simplifications and models I introduce will lead us to an idea for PMP that exploits many aspects of the modern world, but at quite a severe cost. This foray into mathematically modeling voters is meant to give readers a framework for thinking about voting processes without representing a serious view of the situation. I hope you, Dear Reader, will indulge my satire as a personal catharsis in the aftermath of the political campaigning of 2020 in the United States. And the catharsis extends beyond the present work. Later in this issue of *Journal of Humanistic Mathematics* interested readers will find my first science fiction short story, which takes place in a political system built on the ideas here, where the costs of such a system can be explored freely.

We will frame PMP in the following way. Define S to be the slate of candidates running in an election. An election is about, ultimately, a set of issues. Each issue might relate to a position on a particular piece of legislation, or it might be more general opinions about taxes, education, social security, national security, etc. These might even be attitudes or more general statements, like "prefers to compromise" or "puts certain views first." Each candidate $C_i, i = 1, \dots, M$, will be represented as a vector that encodes their platform, an expression of their opinions, attitudes, and so forth, on each of the N issues at hand. Thus, $S = \{C_i | i = 1, \dots, M\}$ and $C_i = \{c_{ij} | j = 1, \dots, N\}$. Likewise, the set of voters is $V = \{V_i | i = 1, \dots, K\}$,

and each of the voters will express their preferences on the issues in a vector so that $V_i = \{v_{ij} | j = 1, \dots, N\}$. For now, we take the underlying vector space to be binary. That is, each of c_{ij} and v_{ij} are elements of the set $\{0, 1\}$ such that $c_{ij} = 0$ indicates that candidate i takes position 0 on issue j and $c_{ij} = 1$ indicates that the candidate takes position 1 on that issue. Voter preferences are defined analogously. It is also useful, at this point, to note that we will assume that all combinations of preference and platform are possible, which is really just a way to say that each issue is independent of the other issues. (If this is not the case, which is entirely likely, one can simply combine the linked issues into a single meta-issue.)

Visually, if such can be said in reference to n -dimensional vectors, this means that one can imagine the issue space as a hypercube, where the coordinates of each vertex defines a combination of opinions on the issues. Each voter and each candidate is positioned at one of these vertices, based on their preferences or platform, respectively. In a typical two-party system, then, each party would seek to define its platforms to accomplish two goals. The first is that the platform they endorse be one that is, in some sense, near the preferences of a large number of voters. The second goal is that they can find candidates whose platforms fall within this scope in order that they represent the party effectively. Thus, one expects that party platforms are evolving over time as both the issues (in number and specific content) and the voters' opinions on those issues change. While modeling the political parties over time on this hypercube might be interesting, we will focus on PMP, formulated as a series of questions:

1. PMP-1. How well does each voter actually understand the values of their preferences, v_{ij} ?
2. PMP-2. How well does each voter actually understand the platforms of each candidate, c_{ij} ?
3. PMP-3. What relative importance (weight) does each voter assign to the various issues?
4. PMP-4. How does each voter determine which of the C_i is "closest" to their V_j ?
5. PMP-5. How do we ensure that as many voters as possible can match their preferences "close enough" in any election?

PMP-4 and PMP-5 will be the primary focus of our analysis and discussion, but along the way some proposals for making progress on the first two will be explored. In fact, a single approach will be proposed that, in principle, resolves all PMP. As a caveat, I would like to acknowledge that many of these ideas are not completely new. At least, they are not new to political theorists or to those who study and develop models of elections and voting. For similar formulations of elections in an issue space, see [3] or [11]. But I hope to frame them in a new light and connect some ideas together in a way that leads to, with apologies to Jonathan Swift, a “modest proposal” for solving PMP that relies on various machine learning tools to help voters.

First, we will explore PMP by looking at how a failure to acknowledge the dimensionality of the issue space poses complications for current political practice. We will then investigate successively more complex ways to account for these dimensions in our voting, explicitly involving the electorate as we do. This starts with an extremely simplified representation of elections, and expands to allow more realistic portrayals of the issues. Everything culminates in a possible way of resolving all of PMP in one simple system, as the main barriers to resolving it are caused by not having enough candidates participate in an election to represent all distinct sets of voter preferences.

2. Linear Thinking is Not Enough

The set of options presented to voters in the United States is often presented as a binary choice between individual representatives of two political parties rather than a choice among a larger slate representing a variety of ways of thinking on the issues. This simplification means no matter how many important issues are on the table, we are projecting everything into a one-dimensional space with a dividing line to separate the two parties. This leads to a lot confusing hair-splitting to “define” our politicians. Is a particular candidate on the right or left? Is she a moderate conservative, a liberal conservative, or an extreme conservative? Is the other candidate a democratic socialist? A socialist? A capitalist? All of this confusion comes from attempting to line candidates up along a single axis. One of the most striking examples of this occurred in the first three of the 2019 Democratic Party presidential debates. Although the candidates were arranged on stage (over two nights for some of these debates, due to the sheer number of them) randomly, one could be forgiven for seeing a line of candidate options from “far left” to “moderate left” to just plain “left.”

Indeed, there is evidence that the human brain has co-opted the visual cortex that sees such lines into the processing center that we use for mathematics, so that such a mapping onto a one-dimensional space is inevitable [1, 6].

Most of us realize, hopefully, that the biggest problem with this is that political issues, and the world in general, are multi-dimensional. But we routinely treat them as one-dimensional, with extreme views of democrats on the left and extreme views of republicans on the right, for example. Then we try to force all the candidates to align somewhere on this spectrum, placing dividing line at some point and asking the voters on each side of this point to vote for the party in their “half” of the space. Projecting platforms and preferences this way ignores the richness of the perspectives possible, and it ultimately leaves people confused when a candidate fails to behave consistently with where we think they fall on this single axis.

Even worse is that, depending on the projection used, a different ordering of the candidates along the line of projection is possible. Figure 1 illustrates how easy it is to have quite different orderings of candidates by choosing different projections. Using a projection onto a vertical line (labeled Projection 2) places candidate *A* in between *B* and *C*; but projection onto the horizontal line (Projection 1) puts a lot of ideological space between *A* and *C*, leaving *B* as the middle ground.

Using projections in this way to simply a high-dimensional space may add necessary efficiencies to our thinking and communication [8] since treating each politician and each voter as a unique combination of factors and interests could be overwhelming. In many situations, such stereotyping is essential to let us function, as is the case when we approach an intersection in a moving car: We treat this as we would any other intersection, and assume that other drivers will follow the same rules observed and obeyed in the past. If we did not, the number of possible actions each driver could take would lead us to considering a near infinite tree of decisions, paralyzing us instead of allowing us to drive onward to our destination. I argue that in the political arena, however, we have been misusing stereotyping. No longer are the labels acting as a rough-but-convenient shorthand for talking about a candidate’s position. Instead, once they are assigned, we tend to think that these labels should apply to all of a candidate’s actions at all times. Rather than trying to fit a label to the actions and expressed platforms of the candidates, we are trying to force the candidates to fit the labels.

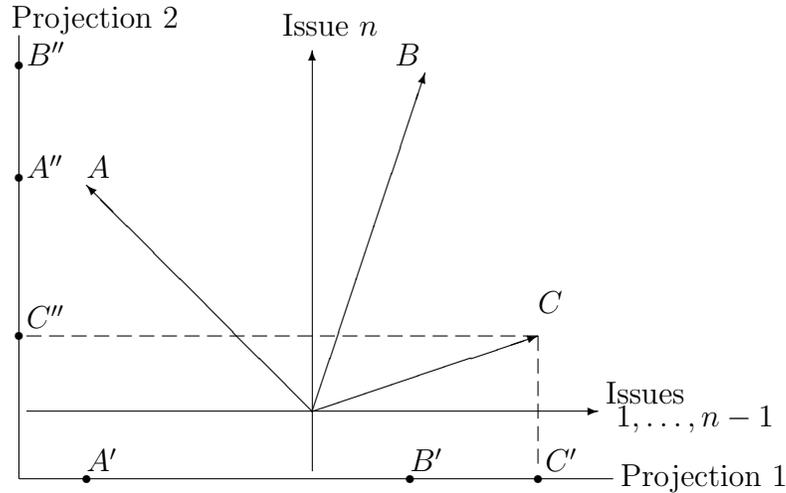


Figure 1: Schematic illustration of three candidates, A , B , and C , in n -dimensional issue space, showing two different lines of projection that give different orderings for the candidates. The dashed line shows the projections of candidate C for clarity. The projection of candidate C onto projection 1 is marked by C' and onto projection 2 by C'' . Note that for illustrative purposes, the axes for issues $1, \dots, n-1$ are shown as the edge of a hyperplane, and each candidate ranks the each issue on the interval $[-1, 1]$.

Using the hypercube visualization of an election, though, we can frame this projection in a way that is slightly easier to visualize. Start by assuming that each party has articulated a set of platforms that fall within their scope. This basically means that there is a hyperplane passing through the issue-hypercube such that all of the platforms and preferences on one side of the hyperplane are expected to align with party A , while the others are expected to align with party B . In general, we can allow for each party to specify its own hyperplane, which means some voters may be ignored by both parties and some voters are actively courted by both parties. Such a case is illustrated in Figure 2. Each election, then, is a way of asking voters to vote for a candidate that falls on the same side of the hyperplane as the party candidate. Of course, this resolution of PMP-4 presupposes that the voters have solid answers to PMP-1 and PMP-2.

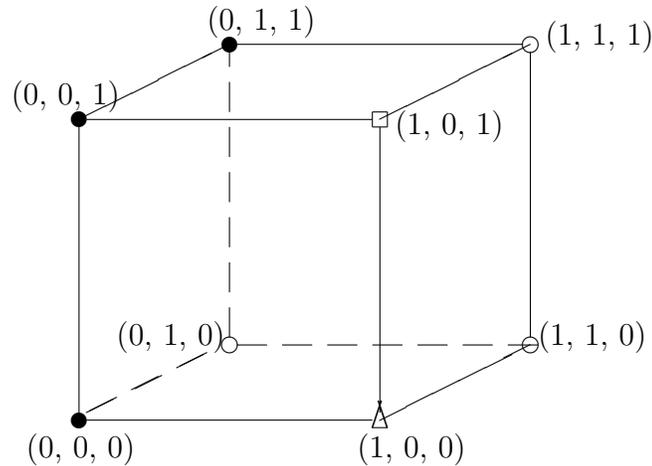


Figure 2: Illustration of issue hypercube with three issues. Preference sets (vertices) are hollow circles if they are aligned with candidate *A* and filled circles for those aligned with candidate *B*. Voter preference sets that are courted by both parties are shown as hollow squares, and hollow triangles mark voters ignored by both parties. It is left as an exercise for the reader to sketch in the two party planes which would lead to these divisions among the preference sets.

3. Binary Platforms Provide Too Much Freedom

From the start, PMP naturally deals with the dimensionality problem by embracing the messiness inherent to political decision-making and expressing the platforms of candidates and preferences of voters as vectors in some “issue space” of N dimensions. In principle, then, we are not tied to party choices that link opinions about the issues into single platforms about all the issues, tying opinions on education to those about health care. The possible upside is obvious: each voter may be able to find a candidate that better fits their individual preferences, removing the need to oversimplify the decision of which candidate to support.

To see some of the downsides to embracing higher dimensions, let us dig deeper into an N -issue space when our underlying platforms and preferences are described as above, with $c_{ij}, v_{ij} \in \{0, 1\}$. This gives 2^N different possible platforms for candidates, with the same number of possible voter preferences (equivalently, 2^N vertices on the issue hypercube.)

Ideally, each voter would then choose to vote for the candidate whose platform is, in some sense, closest to their preference vector. Even assuming that PMP-1 and PMP-2 are resolved, so that each voter is able to make a fully informed choice, there are at least three major problems remaining. First, there is no guarantee a candidate espousing a particular platform during the election process will actually act in a way that is consistent with this platform once elected. Apparently, this is not as common as many of the voters think (c.f. [15] and [4]), and we cannot deal here with such misrepresentation, so we will leave it alone. Second, we are assuming that the issues are independent of one another, and that it is possible that all 2^N platforms make logical sense. Any correlations among the issues reduces the issues space considerably. But this actually works in our favor, given what follows.

The third issue we must address is PMP-4, which is clear from a basic counting argument: With only five issues, there are already 32 possible platforms or preferences. But in a two-party system with five issues, at most two of these 32 platforms are available for voters to select. (I say “at most” because it is possible, albeit unlikely, that both candidates have very similar political stances.) This means that voters are much less likely to find their exact preference expressed by either of the candidates. Voters left out by this then have to decide whether to skip the election or vote for a candidate whose platform does not fully align with their preferences. A popular method, if we are to believe the political theorists and pundits, is for voters to resolve this dilemma by focusing on only a single issue rather than all N of the issues. One of the dangers of this is that after several election cycles voters may find it simpler to change their own opinions to align more fully with the platform of the candidates (or their party). It is this author’s opinion that neither of these is a good outcome, and that it is not what the authors of the United States political system intended, but I will not delve into that here.

So how can voters make choices in an N -issue election when fewer than 2^N candidates are running for the position while avoiding either single-issue voting or changing their opinions? One method for decision making would be to have each voter to make a list of all the candidates and then compare their own preferences to each platform. Each simply writes out new vectors for each candidate computed by checking elementwise equality between the platforms and their preference. Basically, for each candidate-voter pairing, we create a new vector to measure the degree to which they match.

Define the vector comparing V_i to C_j to be $M_{ij} = \{m_{ijk} | k = 1, \dots, N\}$ with the components of the vector defined as follows: $m_{ijk} = 1$ if $v_{ik} = c_{jk}$ and 0 otherwise.

Thus, $m_{ijk} = 1$ represents agreement between voter i and candidate j on issue k , and 0 represents disagreement. Then $A_{ij} = \sum_{k=1}^N m_{ijk}$ results in a score that measures the number of issues on which voter i and candidate j agree. Each voter can then compute these and select the candidate with the highest agreement score. This is exactly what potential voters can try through various online quizzes, such as [7], where answering the questions truthfully could surprise you! Table 1 shows this comparison for a single voter with preference $[0, 1, 1]$ in a full election on three issues, in which each possible binary platform is espoused by one of the eight candidates.

Candidate, j	Platform, C_j	M_{ij}	A_{ij}
1	0, 0, 0	1, 0, 0	1
2	0, 0, 1	1, 0, 1	2
3	0, 1, 0	1, 1, 0	2
4	0, 1, 1	1, 1, 1	3
5	1, 0, 0	0, 0, 0	0
6	1, 0, 1	0, 0, 1	1
7	1, 1, 0	0, 1, 0	1
8	1, 1, 1	0, 1, 1	2

Table 1: Comparison of a voter with preference $[0, 1, 1]$ to each of the possible candidates.

In our example, the voter should select candidate 4, whose opinions align perfectly with the voter. But what if only candidates 1, 2, 3, and 7 were running in the election? The voter now has a dilemma: both candidates 2 and 3 have the same agreement score! And what if only candidates 1, 5, and 6 were running? None of these have a score above 1, leaving this voter feeling like none of the options presented is worth supporting. Another clear problem is that we are not guaranteed that each candidate has a unique platform, so that there could be ties for the most matched candidate.

4. Ranking the Issues Isn't Much Better

We can resolve some of these complications by abandoning one of our unstated assumptions: that all issues are of equal importance in an election to both the voters and candidates. This is a somewhat unlikely scenario anyway, as many of us weigh the issues differently. This would require each voter V_i to specify a weight vector, $W_i = \{w_{ij} | j = 1, \dots, N\}$ for the weights — the relative importance — they associate with the issues. Then, instead of a simple sum, the agreement score for voter i against candidate j is $A_{ij} = W_i \cdot M_{ij} = \sum_{k=1}^N w_{ik} m_{ijk}$ and then compare the weighted scores. Such a method is feasible with the right computational support, but without it, and without each voter carefully going through the analysis, it is very possible that voters might choose to vote for candidates that are not actually in line with their expressed preferences. Attempting to resolve this is why we need to include PMP-3, although it can lead to some interesting results.

For example, suppose a voter's matching with candidate 1 is the vector $M_1 = (1, 1, 1, 0, 0, 1)$ and their matching with candidate 2 is $M_2 = (0, 0, 0, 1, 1, 1)$. Further, suppose the voter has weighted the issues $W = (5, 1, 1, 4, 4, 2)$. The voter would then have an unweighted agreement of 4 with the first candidate and 3 with the second. But the weightings would give a score of 9 to candidate 1 and 10 to candidate 2, thus indicating preference for the second candidate, even though the first candidate matches on more issues, including the issue that the voter has ranked most highly. This is only one such example, and it does require a particular collection of characteristics in the various weights and matchings. However, it illustrates how voters may need to accept a different way of viewing and ranking candidates in order to get more out of their votes. Furthermore, unless there is an agreed-upon (among voters) system for ranking the issues or some sort of standardized weighting, it would be impossible to compare one voter to another directly. This is not needed, however, if the only purpose is to aid an individual voter in making their decision, so long as each voter remains consistent in their process when applying the weights to each of the candidates.

Another, somewhat simpler, solution would be to use what we can call “ordered platforms.” For this, we assume that a candidate's opinion about issues is expressed by the order of importance they place on the issue. In order to allow candidates and voters to express their opinions both for and against each issue, one would need to include each issue twice, once as a “support

this issue” option, and once as a “work against this issue” option, so that a candidate strongly in favor of an issue can rank the “pro” side highly and the “con” side with slightly lower importance, or vice versa. At the same time, it opens up the ability for our system to allow candidates and voters to express their willingness to compromise by having very different rankings for the two sides. For example, if a candidate is willing to accept the legislation failing, in order to compromise and let something else pass, then their con position might be more highly ranked than the pro position for an issue.

A ranked system, in effect, expresses how the candidates are going to spend their energy and resources, once elected. For a first analysis, we will ignore the candidates’ particular views. We assume that by prioritizing an issue, the candidate will work to their best ability to find a solution to the issue that addresses the common good, so that while the outcome may not be one that a voter would have initially been in favor of, the voter respects the process by which the candidate worked to resolve the issue.

Unfortunately, there are far more ordered platforms than binary platforms when N issues are considered, since there are $N!$ ways to order N issues. For example, a five-issue election results in 120 possible ($1 \times 2 \times 3 \times 4 \times 5 = 120$) platforms but only 32 (2^5) binary platforms. And placing the dummy issues to represent the opposing sides, we see the situation is much worse: for an N issue election, we would need $(2N)!$ platforms. So a given voter’s likelihood of finding their opinion perfectly matched by a candidate in a limited election decreases dramatically with the number of issues. Table 2 compares the number of platforms for various values of N ; note that the number of platforms grows so quickly that even with only 8 issues, more candidates would need to run for election than the current population of the entire planet in order to guarantee all possible voter preferences are presented!

Issues, N	4	6	8	10
Binary Platforms, 2^N	16	64	256	1024
Ordered Platforms, $(2N)!$	40,320	$4.79 \cdot 10^8$	$2.09 \cdot 10^{13}$	$2.43 \cdot 10^{18}$

Table 2: Candidates needed to guarantee that all possible voter preferences are expressed in an election based on the number of issues, N , and the method of expressing the platform.

Given the variety of possible platforms in this case, and the low probability that any given voter finds their preferences expressed, it seems that many voters will be ignored. There is some hope here though, since voters can look

at partial orderings for agreement, and there are far more partial ordering-matchings possible. For example, if a voter orders the issues in a five-issue election from most to least importance as 12345, a candidate for whom the issues are ranked 13245 is quite similar (only one swap is needed to match perfectly). This gives rise to questions about whether 52341 is a better match than 14325, for example, but there are many ways to compare the similarity of two orderings.

5. Other Options Just Muddy the Process

This is all getting rather complicated, and seems to be requiring more and more from the voters in order to make a rational selection, so we will take a different path in a search for a simple solution. For example, voters could focus on only matching the rankings of the top m issues. This dramatically reduces the number of candidates required in the field in order to guarantee full representations. Table 3 shows this effect. For example, with a four-issue election, attempting to match all four issues for priority would require 24 candidates, but only four candidates are needed if the first issue is the only one considered important. Note that we could have multiple candidates with similar platforms, which would force voters to consider additional aspects of the candidates, such as personal behavior, family connections, and so forth. However, it might be useful, as we will discuss later, to include those features in issue-space from the start.

k	$n = 4$	$n = 6$	$n = 8$	$n = 10$	$n = 12$
1	4	6	8	10	12
2	12	30	56	90	132
3	24	120	336	720	1320
4	24	360	1680	5040	11880
5		720	6720	30240	95040

Table 3: Number of candidates needed for full representation in an n -issue election where only the top k issues are considered critical.

One way to implement this is by using the weighted issue system above, and simply assigning weights of 0 to the $(N-m)$ issues we do not want to consider. Of course, the number of important issues will vary from voter to voter, as will the rankings of those issues. And now we are much more likely to have multiple candidates with equivalent matching scores for a particular voter.

Also, while the equally-ranked candidates may indeed be isomorphic in the views of some voters, one would probably need to make use of uncoded data (past history, for example) to break any ties. We could, of course, combine several of these methods into an “ordered-binary platform” or a “weighted-ordered platform.”

A relatively simple way to do this would be to move from an underlying binary vector space of $\{0, 1\}$ for ranking preferences and platforms. Instead, we can allow $v_{ij} \in [-1, 1]$. Negative values would indicate support for the opposing side of the issue, and values near zero would indicate indifference, eliminating the complexity of doubling the dimensions to include separate pro and con sides of each issue. Once the voters and candidates have ranked or weighted the issues, these can be rescaled so that each entry is the original value divided by the square root of the sum of the squares of the of entries: the scaled weights

$$\tilde{w}_{ij} = w_{ij} / \sqrt{\sum w_{ij}^2}$$

then guarantee that each vector has unit length. If we also do this for each candidate’s platform, we can visualize each voter and candidate as a point on a unit hypersphere in the issue space. Once all the platforms are determined, an individual voter can then use any of several methods to select the candidate that is “closest” to them, such as using the standard relationship that the cosine of the angle between vectors is given by their dot product divided by the products of their magnitudes. This then allows each voter to find the the platform of the candidate having the smallest angle with their preference vector by choosing the one having the largest dot product to their weighted preferences.

On the other hand, mapping an entire collection of voters V onto this hypersphere would allow a political party or candidate to use clustering methods to design a platform that would appeal to the largest number of voters while also being clearly distinguished from other platforms. So the weighted issues formulation here would, given sufficient data on the electorate, allow the parties to divide up the issue-space. Effectively, though, we are now merely in a continuous analogue of the hypercube discussed earlier. So the approach brings with it many the potential difficulties of the original approach. However, if voters could all be mapped onto this hypersphere, it might be possible to employ machine learning to develop policies without directly involving representatives in the government to make decisions.

For example, a nearest neighbors or clustering algorithm could identify large groupings of opinions and employ evolutionary algorithms to develop policies that would match the preferences of these groups. Such policies could be vetted by teams of policy workers and made public, so that voters outside the clusters would have a chance to review them and revise their own opinions prior to voting on the policies.

A real concern with all of these methods to resolve aspects of PMP is not even stated in our formulation of the problem: Do the candidates actually know their position on each issue? To see why this could be a problem, depending on our underlying vector space, consider using a scale from 0 to 1 indicating “percent of agreement.” Each candidate can clearly determine this, one would hope, but remember that they will, once in office, be asked to make decisions that are new to them and about which they had no previous knowledge. So what actions will they take? One can imagine an experienced candidate filling out a survey on the issues and drawing upon their experiences in office to help nuance their answers, while a new candidate with no experience merely uses an educated guess. Thus, each candidate is actually working on a different scale! And while we could focus on objective voting records rather than self-reported agreements to build our scale, new candidates will not have previous voting records.

Thus, the system might need to expand to viewing other records for candidates in order to determine where they stand on various issues. One could imagine potential candidates being asked to opt in to allow artificial intelligence bots to comb their social media and other public statements for data. A.I. could then perform semantic analysis to compute each candidate’s “fraction of statements made consistent with an opinion on the issue” or use banking records and other data to produce the scores for each candidate. Again, those new to the scene will have less data available to inform the system, and candidates who have changed their opinions over time would need to have more recent data weighed more heavily. And none of this addresses the ethical issues associated with mining a person’s background. For example, most social data mining requires branching out from the target individual’s data to those most closely associated with them, who might not be aware of this scrutiny nor have been asked to participate. Nor does this address the vast quantity of information on a given person that has virtually no relevance to their political opinions or position. Even more troubling is the need to train the A.I. to do this mapping from personal data into political opinion.

As we are learning all too often, the algorithms may start as unbiased code, but the training data used to prepare them for use introduce bias (see [2] for an analysis of one prominent example).

Even more concerning is that the candidates may not even agree on what the list of issues is. So our set of possible issues would need to be the union of the sets of issues as perceived by each candidate and each voter. Depending on the level of detail we use to specify the issues, this could lead to quite a large value of N . For example, three candidates could all be in favor of similar legislation related to climate change. But the first candidate may want changes implemented within three years, a second candidate may want those same changes over five years, and the third might care less about the timeline and more about the particular level of emissions set in the legislation. This would have to be encoded as three separate issues, depending on exactly how closely we expect to model each candidate's platform.

There are many other approaches to generating the similarity scores needed to help each voter identify their best-matched candidate. And in reality, many issues are not independent. The connectedness results in a smaller set of issues to consider, and a correspondingly smaller slate of candidates. Using cluster analysis would allow us to group similar issues and positions, greatly reducing the dimensionality of the issue space. Then we would see that, perhaps position 0 on issue A and position 1 on issue B are always expressed together by voters. So we no longer need A and B as separate issues, but only as a composite. However, this clustering is essentially how we got into the two party mess in which the United States finds itself currently: we cluster one set of ideas as "Republican" and another set as "Democratic." The same approaches to those above may be used to analyze the situation, but in this case, each issue is really a meta-issue, expressing a collection of opinions. One potential danger here is that clustering in this way would get you back into a polarized electorate, with a very limited set of options presented to each voter and with little potential for compromise. In fact, recent work shows that this polarized state, once achieved, may be very difficult to break from [12, 16]. Once again, we all wind up bundling the issues into a single meta-issue with two sides, Republican or Democrat, rather than looking deeper. And in reality, political parties are always shifting their platforms in response to events and voter opinions, although different parties tend to do this in different ways and for different reasons [10].

All of the complexity above is caused by one common problem: Assuming that there are fewer candidates standing for election than there are voter preferences in the electorate. This could be solved by just having more candidates each election, with platforms more distributed in the issue space, and by having everyone much clearer on all their own preferences. But there is an inherent impracticality to large slates of candidates, aside from the difficulty of getting clear information to the public and the unwieldiness of really long ballots: Once the number of candidates gets too large, the number of members of the electorate who exactly match any one preference will be quite small, assuming that all combinations of preferences are equally likely. Thus, achieving a plurality vote becomes increasingly hard if we insist on perfect matchings, even for small slate of issues.

Table 4 illustrates this problem by showing how the number of issues affects the plurality of a vote.

Issues	Number of possible platforms (approximate)	Percent of electorate sharing any one preference (%)	Percent of electorate sharing all but one issue (%)
N	2^N	2^{-N}	$N2^{-N}$
5	32	3.125	15.625
10	1024	0.0977	0.97
20	$1.05 \cdot 10^6$	$9.54 \cdot 10^{-5}$	0.0019
40	$1.1 \cdot 10^{12}$	$9.09 \cdot 10^{-11}$	$3.64 \cdot 10^{-9}$

Table 4: Portion of electorate sharing a particular preference set (expressed as percent of population) for different numbers of critical issues, assuming support for each platform is uniformly distributed.

The table assumes that the voter preferences are uniformly spread over the possible options and shows the percentage of the electorate who would share any one particular preference. As you can see, with just ten issues involved, much less than 1% of the voters share a common preference. However, the hypercube visualization reminds us that there would be a large number of preferences adjacent to any particular platform. For each preference, represented by a vertex on the N -dimensional hypercube, there would be N preferences (vertices), adjacent to it, agreeing on all but one issue. Thus, while only a small portion of the population would be in complete agreement, there would be a larger fraction (proportional to the number of issues, N) of the population who agree on all but one issue, although the

single disagreement would vary over the entire issue space. Again, that is part of ERP, and there are some ways to resolve this, independent of the political system to be proposed below.

6. A “Modest Proposal” for Resolving PMP

So what are we to do? We cannot easily increase the candidate pool. And even if we did such a thing, many voters would still struggle to find their preferences matched perfectly by one of the candidates. It seems that we are at an impasse as long as we stick to volunteers entering the political arena and seeking an aggregate vote from the public that selects their platform over others. There is, however, a solution: Stop requiring humans to serve as elected officials and remove the candidates from the system completely.

We now have enough computing power to simulate how a decision-maker would implement their platform while in office. And if we can simulate one possible platform, there is nothing to prevent us from simulating *all* possible platforms. We can thus envision the creation of a “political algorithmic logic” unit (PAL) with artificial intelligence and tunable characteristics so that it could make decisions and vote as if it were a person holding whatever platform of opinions was desired.

To implement the system, we poll the electorate to allow each voter to express their preferences. Then we select a Decision Body composed of a multitude of these artificially intelligent PAL systems, each adjusted to represent different voter preferences. Below, we will explore how this resolves PMP, but it is worth briefly noting that this solution also resolves ERP for free! We only need to weight each PAL’s contribution to the actions taken by the Decision Body by weighing their votes in proportion to the number of voters matching with them. Then the aggregate of voter preferences is directly put into office to make decisions.

Note that if the platforms/preferences contain only specific issues, then we are effectively talking about a direct democracy where the outcome of any proposal is predetermined by the composition of the Decision Body. To avoid this, instead, the issue space should also include attitudes like “willing to compromise on issue X,” “uncompromising,” or “prioritizes education.” Then the PALs can meaningfully work to convince each other and reach a collective decision that accounts for all the preferences and the interactions among the issues and attitudes, without being a direct democracy.

This system would also allow the Decision Body to interface and collate the different PAL positions, and factor in information that is unavailable to the public due to security or similar concerns. I would argue that this is critical, since it would be impossible to anticipate all future scenarios up front for matching with voter preferences. Indeed, while issues are critical, it is the way we anticipate a candidate will meet the unknown that often influences our vote or the way we feel about him or her (as explored by thermometers in [15]). In essence, legislative and other decisions would undergo a sort of evolutionary algorithm before emerging from the decision body of PALs. And given the nature of the senators in this model, we would not have to deal with political action committees or lobbyists influencing the system, although there are a host of other dangers to consider.

The PAL-based system would automatically resolve PMP-4 and PMP-5. Mining social media, life history information, and so forth, would help determine each voter's preferences on the issues. If necessary, direct surveys of voter awareness, knowledge, and opinion on the issues could be implemented, leading to the resolution — by not directly requiring the voters themselves to even be aware of it — of PMP-1. And since the system provides opportunities for every platform to participate, PMP-2 is also resolved. It is also not hard to envision a method that would simultaneously weight each issue for the voters, resolving PMP-3 as well. Voting computers could quickly determine matchings for each voter with the possible platforms. To be fair, one could also allow multiple points of intervention, where the voter can adjust what the system has determined about their preferences. This would be essential, since people often change their views, leading to different desirable candidates and different rankings of the issues. And if one's entire social media and other history were factored in, recent changes in one's opinion could be lost.

7. Conclusion

Clearly, the preceding has ignored much. For example, voting can be treated as an exercise in economic game theory: If there is a candidate that you like, but feel will not win enough votes, are you better off selecting that candidate anyway, or should you vote for a more popular second choice candidate in order to try and prevent a candidate from your “non-preferred” list from winning? We have also been completely ignoring the publicly available

polling data and the role of popularity in the process. There are several well-studied examples in which election results were influenced by voter decisions made from polling data rather than their preferences, since many chose not to spend time voting when the predictions made public during the election process appeared to already be strongly consistent with their preferences.

And the entire formulation here is based on a the idea of the candidates and voters having a common knowledge of the issues. If different voters have different lists of “the issues” then it would be difficult to generate a consensus on even what the PALs should look like for a given election.

But electing (or selecting) a mix of artificially intelligent PAL units to make decisions would restore the importance of each individual voter in the system. And it would still allow for a representational system. After all, we cannot have everyone in the country spending all of their time running the country; someone has to teach the children, build the homes, and grow the food. Everyone’s preferences now factor in the decision process. And it turns out that there is already work in using AI to match voter preferences to platforms, as in [14].

Of course, there are bound to be complications in letting a group of AI run the country. Mathematically, it is not hard to envision instances where the combination of the PAL would fail to reach resolution on a particular issue. The system could be manipulated by hackers during the decision-making process. The choices for which issues to include in the platforms could be manipulated; this has deep consequences explored in [5], for example. I’m sure there are a host of additional complications that I have not raised. But here’s the secret: Most of us believe these problems already exist in the human-centered election process.

Compared to the interminable and contentious 2020 election year in the United States, I, for one, welcome our robot overlords... er, decision-makers. And I welcome you, Dear Reader (apologies to the late Isaac Asimov) to explore a fictional future filled with PALs who want the best for you in my short story “The Deterministic Republic.”¹

¹ Green, K. H. “Deterministic Republic,” *Journal of Humanistic Mathematics*, Volume 11 Issue 1 (January 2021), pages 445–484. Available at <https://scholarship.claremont.edu/jhm/vol11/iss1/29>.

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