Journal of Humanistic Mathematics

Volume 12 | Issue 2 July 2022

The Fractal Geometry of Mathematics Classrooms: Navigating Classroom Environments in Factory-Model Schools

Cacey L. Wells
Appalachian State University

Recommended Citation

Cacey L. Wells, "The Fractal Geometry of Mathematics Classrooms: Navigating Classroom Environments in Factory-Model Schools," *Journal of Humanistic Mathematics*, Volume 12 Issue 2 (July 2022), pages 430-442. DOI: 10.5642/jhummath.XXVA3028. Available at: https://scholarship.claremont.edu/jhm/vol12/iss2/24

JHM is an open access bi-annual journal sponsored by the Claremont Center for the Mathematical Sciences and published by the Claremont Colleges Library | ISSN 2159-8118 | http://scholarship.claremont.edu/jhm/

The Fractal Geometry of Mathematics Classrooms: Navigating Classroom Environments in Factory-Model Schools

Cacey L. Wells

Department of Curriculum and Instruction, Appalachian State University, Boone, North Carolina, USA
wellscl1@appstate.edu

Synopsis

Mathematics education has a diverse and complex history. After decades of national reform, mathematics classrooms today are still in desperate need of change. This article examines the current state of factory-model schooling through the lens of Mandelbrot's *The Fractal Geometry of Nature* in order to rethink pedagogic practices.

Keywords: mathematics education, factory-model schools, fractal lens, mathematics education policy

1. Introduction

As with many institutions in the United States, mathematics education has a rich and complex story. Our field has been shaped, in large part, by major reform initiatives over several decades; but, despite reform, instruction in mathematics classrooms often looks similar to how it did shortly after the Industrial Revolution. At the turn of the twentieth century, progressive pedagogical ideals were beginning to gain popularity in North America. Constructivist educators, like Rousseau and Dewey, believed teachers should take students' interests into account, rather than serving as "taskmasters" [14, page 329]. These approaches to teaching and learning were in direct contrast to traditional, lecture-based disseminations of information during this period in history.

To worsen the situation, educational reforms driven by "back to basics" federal initiatives, like *A Nation at Risk* and *No Child Left Behind*, have emphasized efficiency in mathematics education, thus propagating classrooms that look strikingly similar to factories and are managed like businesses [14, 28]. This model of schooling has been in place for some time and the pendulum seems like it is unlikely to swing back to progressive ideals in the near future. Kilpatrick summarizes the progress in mathematics education this way:

> ... any change in the mathematics taught in school is likely to come rather slowly and that pressures on the public schools from politicians and the public may be as powerful in shaping such change as anything the professional mathematics education community proposes. [14, page 332]

It is easy to become discouraged when attempting to navigate the current climate of schools; however, many teachers are pushing against the norm and are beginning to navigate these spaces [13].

Despite national pushes for efficiency in schools, classrooms can still be thought of as interconnected systems, which include relationships between students, teachers, and their environments [5]. In the following paragraphs, my aim is to address issues found in navigating today's educational landscape by borrowing ideas from Benoit Mandelbrot's *The Fractal Geometry of Nature* [18], to shape my theoretical framework. My hope is to use fractal terminology to connect how educators can begin navigating complexities found in mathematics classroom environments.

2. Fractal Geometry as an Interpretive Lens

In *The Fractal Geometry of Nature* [18], Mandelbrot outlines the nature of fractals by providing nomenclature for and understanding of what humans had not be able to understand mathematically what they had observed in nature for centuries. When observing objects in nature, one may see interesting shapes found in clouds, self-similarity in branching patterns in plants, and varying degrees of roughness in rocks and stones. In order to think about geometry in the natural world, new terminology was needed to define what was being studied. Mandelbrot used the term *fractal* to describe fracturing found in objects that appeared to have reiterative patterns and degrees of irregularity [18]. No longer did mathematicians have to use inexact language to define the geometry of nature. In a similar sense, there are systemic issues found in mathematics classrooms that need new nomenclature and ways to understand deeply rooted issues associated with factory-model schools.

In the following, I would like to describe in more detail the status of today's schools and some ramifications of the current model of schooling before focusing my attention on mathematics classrooms, in particular. But to be able to do any of that, I will first need to say more about fractals.

2.1. Need for Fracture

Fractal geometry was not a concept which had previously existed in mathematics prior to the mid-twentieth century. The term *fractal* was concocted in order to better describe the mathematics of nature and to better understand famous mathematical monsters of the nineteenth century. Mandelbrot states:

> I coined *fractal* from the Latin adjective *fractus*. The corresponding Latin verb *frangere* means "to break:" to create irregular fragments. It is therefore sensible — and how appropriate for our needs! — that, in addition to "fragmented" (as in *fraction* or *refraction*), *Fractus* should also mean "irregular," both meanings being preserved in *fragment*. [18, page 4]

In the words above, there are two parts that stand out and which I believe are relevant to the status of education today. The first is "to break" and the second is the idea of something being described as "irregular."

2.1.1. Frangere.

Fractus, when used a verb, means "to break." It is clear that our education system has systemic issues that need to be critically examined, even to the point of fracturing them to recognize the brokenness that exists within the system. When speaking about the predominant model of schooling in the United States today, Noddings describes “twin economic aims of education” [21, page 75]. These twin aims are to advance the economic supremacy of the United States and to produce students for economic success. In essence, this is similar to the “functional” model of schooling proposed by Bobbit [3] in the decade prior to the Great Depression. Functional, or factory-model, schools recognize students as potentialities on the path to maturity [27].

The idea of *fractus* imposed on our educational system is not to literally break it, but to recognize that the system is outdated. In many ways, society has progressed and mathematics as a field has evolved, yet our schools are still designed to produce students to fulfill needs within society determined decades ago. Furthermore, what we are producing through schools can sometimes be far less noble than what prosaic politicians paint through eloquent speeches. Anyon [1] points out that the needs we are trying to fill are often taught through a hidden curriculum, which can inevitably reinforce socioeconomic status. In other words, the children of the elites are being taught to be elites while the children of the working class are taught how to follow orders.

Additionally, unhealthy systems, like tracking in mathematics education, are consequences of factory-model schools. Tracking is essentially sorting students based on their intellectual abilities and prior achievements [23]. A major problem with tracking is the manner in which students are sorted. The process is often based on factors such as race and/or socioeconomic status, rather than one’s abilities. In a similar fashion to teaching a hidden curriculum, tracking students in mathematics often leads to the reproduction of social class and unequal access to resources [2].

2.1.2. Irregularity.

In his 2010 TED Talk [19], Mandelbrot spoke to the notion of irregularity and how irregularities can be described using fractal geometry. This idea takes seemingly ordinary, but irregular, objects found in nature (like rocks or clouds) and examines them at high resolutions. When we do that, more details are revealed about the object itself. Unlike Euclidean geometry, Mandelbrot's fractal geometry gives mathematicians the ability to measure irregular objects and allows us to describe them geometrically by dimension, length, and surface area [17].

In a similar way, today's education system tends to shy away from the irregularities found in schools. Since the dawn of the new millennium, schools have been subjected to reform-accountability and standards movements such as *No Child Left Behind* and *Common Core*—neither of which take into consideration the complexities that exist in schools. Accountability measures like these are simply quick-fix ideas set forth by politicians that fail to take into consideration the long term effects they have on schools and society. Schools tend to focus on atomized sets of standards that reduce the potentialities of meaningful content to "mush" [20, page 85]. When looking at school systems in the United States, it does not take long to see the irregularities when we start to increase our resolution. Rather than looking critically at schools as a whole, my intention is to now narrow the focus to look at complexities found in mathematics classroom environments and how teachers can begin navigating within their tangible spaces.

3. Classrooms as Fractals

Fractals are complex mathematical structures that involve various levels of irregularity and an emergent nature of self-similarity. The same can be said of classrooms found in the United States today. The manner in which teachers go about establishing their classroom environment can vary drastically. The closer one examines these environments, the more one will learn about teachers' philosophies and ideologies. While classrooms can vary significantly, there are aspects of many classrooms that remain remarkably similar. Classrooms generally involve teachers and students interacting, desks or tables to work on, a front of the room, a back of the room, and content in which to engage. Environments can shape and form students' and teachers' identities. Likewise, students' and teachers' lived experiences can also impact classroom environments [13]. This dialectic is important to remember as teachers build supportive places for students to learn.

Speaking in general terms, while teachers may be subjected to working within factory-like models of schooling, there are ways to navigate this landscape that effectively support students and their learning. In the following paragraphs, I outline human aspects within classrooms that teachers can implement with potentially lasting impacts on students and communities. These include fostering spaces for collaboration, building positive relationships, and constructing democratic classroom norms. My intention is not to impose a particular curriculum ideology or teaching philosophy onto teachers, but to provide a platform for teachers to begin thinking about the spaces in which they teach.

3.1. Strange Attractors

Prior to the seminal work published by Mandelbrot, strange attractors were being studied outside of mathematics. Strange attractors correspond to chaotic systems in which "strictly deterministic equations of motion" result in unpredictable phenomena [5]. When talking about the chaotic nature of fractals, Mandelbrot states:

> ... turbulent intermittency was the first major problem I attacked (starting in 1964) using early forms of fractal techniques, and (quite independently) the theory of strange attractors took off for earnest with the study of turbulence in Ruelle & Takens (1971) [26]. Thus far, the two approaches have not met, but they are bound to meet soon. [18, page 193]

In fact, the two approaches have now met and are known to be intricately connected. In order to analyze the chaotic nature of dynamic systems, it is paramount to be able to identify and classify attractors by creating phase portraits based on qualities within systems. Within chaotic systems there are often points of instability, called bifurcation points. These points "mark sudden changes in the system's phase portrait" and are often viewed as points of emergence [5, page 116]. Mandelbrot's geometry provided language to describe this chaotic attractors within phase portraits.

In many ways, traditional classrooms are quite Euclidean in nature and chaos is generally discouraged. When looking inside factory-model classrooms, desks are most likely arranged in rows, facing the front of the room. This is sometimes out of necessity because of overcrowding; and, other times, because teachers feel like they need to disseminate as much information as possible for success on exams. Mathematical metaphors like strange attractors and bifurcation points can be quite powerful in thinking differently about the dynamics that exist in classroom environments. To move away from factory-model schools, it is important to teachers to recognize the spaces they create in classrooms can produce rich conversations and collaboration between students.

Creating collaborative spaces and opportunities for substantive conversations in classrooms is more than increasing the amount of discussion-based activities and horizontal information flow [24]. Coupling this with the notion of bifurcations points and strange attractors can allow for meaningful conversations to take place in mathematics classrooms. As teachers allow for discussion to revolve around meaningful mathematics that allows students to critically examine information [16], there exist opportunities for turbulence to revolve around these topics. As chaos increases, conversational bifurcation points allow for dialogue to become emergent in nature and students can begin grappling with and making meaning about personally meaningful mathematical topics. This includes students making sense of "patterns of reasoning, justification, conjecturing, and abstraction" [15, page 309].

3.2. A Lesson from Hausdorff

Functional models of schools typically use prescribed curriculum with predictable and quantifiable learning outcomes. Classrooms, however, are made up of unique individuals with varying degrees of mathematical interest and prior knowledge. As individuals in classrooms interact with on another, impactful relationships between students and between students and teachers begin to emerge [6, 29]. As teachers create spaces that are more collaborative in nature, it is imperative to begin building positive relationships with students.

In fractal geometry, there is an interesting way of describing the dimensionality of objects with high levels of irregularity. When contrasting dimensions found in Euclidean space with dimensions of fractals, Mandelbrot was able to incorporate the work of Hausdorff into his new geometry. In referencing Hausdorff's dimensions he states:

> Euclid is limited to sets for which all the useful dimensions coincide, so that one may call them dimensionally concordant sets. On the other hand, the different dimensions of the sets to which the bulk of this Essay is devoted fail to coincide; these sets are dimensionally discordant. [18, page 14]

In a similar fashion, classrooms can be difficult to define, in general terms, because of the diversity that exists within them. When working within reform movements that emphasize test scores, teachers are often forced to see their classrooms as dimensionally concordant.

One way for teachers to begin building positive relationships with students is through care. According to Noddings [20], healthy, caring relationships between students and teachers are essential to establishing supportive classroom communities. Furthermore, "parents and taxpayers need to know that any content–no matter how important symbolically–is worthless in a classroom unless there are strong relationships between teacher and students as well as appropriate teaching practices" [7, page 185].

The integration of Hausdorff's work into defining fractals dimensionality was key to allowing Mandelbrot to describe the degrees of roughness a fractal had. In a similar sense, when teachers are able to build positive relationships with their students, classrooms become more dynamic and can have dimensions to them that do not fit the norm. Approaches to building positive relationships varies teacher to teachers, depending on how comfortable teachers feel sharing various aspects of their lives. Regardless of levels of comfort, deep care for students' academic, social, and emotional well-being is critical. As teachers navigate their practice in schools that promote the twin economic aims of education, building positive relationships with students is priority one. By doing so, teachers learn to work within dimensionally discordant spaces that can positively impact students.

3.3. The [In]finite

What students learn in classrooms in factory-model schools, is often orchestrated by political stakeholders outside classrooms. Curricula are often sterril and teachers' roles are reduced to that of a "technical-production manager who has the responsibility for monitoring the efficiency with which learning is being accomplished" [8, page 191]. As teachers move away from functional roles and begin to recognize the complexities that exist within their classroom environments, it does not take much to see the amount of fractal irregularity that exists.

One of Mandelbrot's most famous problems focuses on measuring the coastline of Great Britain. He makes the argument that coastlines of islands have infinite perimeter, depending on the method chosen to measure the coast. Paradoxically, as the perimeter of a coastline diverges towards infinity, its area converges to a finite number.[1] Mandelbrot extends this idea further in saying:

> To begin, let us echo "How Long Is the Coast of Britain" and ask how many islands surround Britain's coast? Surely, their number is both very large and very ill-determined. As increasingly small rock piles become listed as islands, the overall list lengthens, and the total number of islands is practically infinite. [18, page 116]

In fractal geometry, measuring something differently has potential to bring about infinite possibilities. This notion can be captured in mathematics classrooms through the establishment of democratic norms.

According to Giroux, "transformative intellectuals take seriously the need to give students an active voice in their learning experiences" [12, page 379]. An alternative to functional classroom pedagogies is for teachers to engage in dialogue with students in order to establish democratic norms. These serve to guide behavior and curriculum, as opposed to managing behavior and prescribing curricular tasks [4]. As teachers and students work collaboratively to establish agreed-upon normative behaviors, there begins to exist potential for substantive conversation and meaningful learning. In essence, when teachers and students work together to agree on what they need from each other to be successful opportunities for meaningful learning begin to increase (like the perimeter of a coastline). Additionally, as opportunities increase, students become more invested in what they are learning because it connects

[1] Editors' Note: The *Journal of Humanistic Mathematics* has recently published a short story inspired by this observation. See [9].

to their lives. This allows for students to make meaningful connections and learning tends to become more solidified (like the convergent area within the infinite coastline).

4. Conclusion

4.1. Of Monsters and Boogie Men

Fleener (as cited in [25]) says that in "exploring the possibilities of and creating curriculum futures, we must address our own boogie man; those ideas, practices, and goals that have constrained our ability to change, adapt or create a new reality for schooling" [page 12]. For mathematicians of the early twentieth century, their boogie men, so to speak, were what were considered to be the mathematical monsters of the day. These monsters plagued mathematicians for decades. Mandelbrot describes the situation:

> In a letter to Dedekind, at the very beginning of the 1875-1925 crisis in mathematics, Cantor is overwhelmed by amazement at his own findings, and slips from German to French to exclaim that "to see is not to believe" ("Je le vois, mais je ne le crois pas"). And, as if on cue, mathematics seeks to avoid being misled by the graven images of monsters. What a contrast between the rococo exuberance of pre- or counterrevolutionary geometry, and the near-total visual barrenness of the works of Weierstrass, Cantor, or Peano! [18, page 21]

It was not until the 1960s and 70s that Mandelbrot's work would uncover "that some of the most austerely formal chapters of mathematics had a hidden face: a world of pure plastic beauty" [18, page 4].

Similarly, the boogie man that has "constrained our ability to change" within mathematics education is an oppressive, factory-model of schooling. The complex nature of schools today can tend to be overwhelming, and opportunities for a single teacher to change the entire system seems unlikely. However, a major theme of chaos theory we can learn from is the idea that "small perturbations over time lead to major changes" [10, page 90].

As teachers navigate realms in which their boogie man lurks, there are specific areas that can shine proverbial light to reveal its weaknesses. The way we can "adapt and create new reality for schooling" is by rethinking the way teachers structure their classroom environments to integrate spaces for meaningful dialogue, continuing to build positive relationships with students, and by establishing democratic ideals within mandated spaces.

References

[1] Anyon, J., "Social class and the hidden curriculum of work," *Journal of Education*, Volume **162** Issue 1 (1979), pages 67Ð92.

[2] Battey, D., "Access to mathematics: A possessive investment in whiteness," *Curriculum Inquiry*, Volume **43** Issue 3 (2013), pages 332–359.

[3] Bobbitt, F., "The new technique of curriculum-making," *The Elementary School Journal*, Volume **25** Issue 1 (1924), pages 45–54.

[4] Bovill, C., Cook-Sather, A., Felten, P., Millard, L., & Moore-Cherry, N., "Addressing potential challenges in co-creating learning and teaching: Overcoming resistance, navigating institutional norms and ensuring inclusivity in student-staff partnerships," *Higher Education*, Volume **71** (2015), pages 195–208.

[5] Capra, F., & Luisi, P. L., *The systems view of life: A unifying vision* Cambridge University Press, 2014.

[6] Çengel, M., & Türkoglu, A., "Analysis through hidden curriculum of peer relations in two different classes with positive and negative classroom climates," *Educational Sciences: Theory And Practice*, Volume **16** Issue 6 (2016), pages 1893–1919.

[7] Cuban, L., "The lure of curricular reform and its pitiful history," *The Phi Delta Kappan*, Volume **75** Issue 2 (1993), pages 182–185.

[8] Lampert, M., "How do teachers manage to teach? Perspectives on problems in practice," *Harvard Educational Review*, Volume **55** Issue 2 (1985), pages 178–195.

[9] Culik, Nora E., "How to Measure a Coastline," *Journal of Humanistic Mathematics*, Volume **10** Issue 2 (July 2020), pages 567–568. doi:10.5642/jhummath.202002.33. Available at https://scholarship.claremont.edu/jhm/vol10/iss2/33, last accessed on July 29, 2022.

[10] Doll, Jr., W.E., *A Post-Modern Perspective on Curriculum*, Teachers College Press, New York, 1993.

[11] Fleener, M. J., *Curriculum dynamics: Recreating heart*, Peter Lang, 2002.

[12] Giroux, H., "Teachers as transformative intellectuals," *Social Education*, Volume **49** Issue 5 (1985), pages 376Ð379.

[13] Houser, N., Krutka, D., Province Roberts, R., Pennington, K., & Faili Coerver, N., "Navigating the reform-accountability culture in Oklahoma social studies," *Theory & Research in Social Education*, Volume **45** Issue 1 (2017), pages 7–42.

[14] Kilpatrick, J., "Mathematics education in the United States and Canada," pages 323–334 in *Handbook on the History of Mathematics Education* edited by A. Karp & G Schubring (Springer, New York, 2014).

[15] Krussel, L, Springer, GT, and Edwards, B., "The teacher's discourse moves: A framework for analyzing discourse in mathematics classrooms," *School Science and Mathematics*, Volume **104** Issue 7 (2004), pages 307.

[16] le Roux, K and Adler, J., "A Critical Discourse Analysis of Practical Problems in a Foundation Mathematics Course at a South African University," *Educational Studies In Mathematics*, Volume **91** Issue 2 (2016), pages 227–246.

[17] Losa, G. A., & Nonnenmacher, T. F., "Self-similarity and fractal irregularity in pathologic tissues," *Modern pathology: an official journal of the United States and Canadian Academy of Pathology, Inc*, Volume **9** Issue 3 (1996), pages 174–182.

[18] Mandelbrot, B., *The Fractal Geometry of Nature*, WH Freeman, New York, 1983.

[19] Mandelbrot, B., *Fractals and the art of roughness*, TED talk, February 2010. Video recording available at https://www.ted.com/talks/benoit_mandelbrot_fractals_the_art_of_roughness, last accessed on July 29, 2022.

[20] Noddings, N., *The challenge to care in school*, Teachers College Press, New York, 1992.

[21] Noddings, N., "Curriculum for the 21st century," pages 75–81 in *Educational Studies in Japan: International Yearbook* Volume 2 (2007).

[22] Oakes, J., *Keeping track: How schools structure inequality* (2nd edition), Yale University Press, New Haven, 2005.

[23] Oakes, J., & Guiton, G., "Matchmaking: The dynamics of high school tracking decisions," *American Educational Research Journal*, Volume **32** Issue 1 (1995), pages 3–33.

[24] Otten, S, Engledowl, C, and Spain, V., "Univocal and dialogic discourse in secondary mathematics classrooms: The case of attending to precision," *ZDM: The International Journal On Mathematics Education*, Volume **47** Issue 7 (2015), pages 1285–1298.

[25] Reeder, S., "Classroom dynamics and emergent curriculum," pages 247–260 in *Chaos, complexity, curriculum and culture: A conversation*, edited by W. E. Doll, M. J. Fleener, and J. S. Julien (Peter Lang, 2002).

[26] Ruelle, D. & Takens, F., "On the nature of turbulence," *Communications in Mathematical Physics*, Volume **20** (1971), pages 167–192. doi:10.1007/BF01646553

[27] Schiro, M., *Curriculum theory: Conflicting visions and enduring concerns* (2nd edition), Sage Publications, Thousand Oaks, 2013.

[28] Teitelbaum, M., *Falling behind?: Boom, bust, and the global race for scientific talent*, Princeton University Press, Princeton, 2014.

[29] Uslu, F., & Gizir, S., "School belonging of adolescents: The role of teacher-student relationships, peer relationships and family involvement," *Educational Sciences – Theory & Practice*, Volume **17** Issue 1 (2017), pages 63–82.