The Mystery of the Dancing Men

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Synopsis

In this paper I describe an activity based on a 1903 Sherlock Holmes murder mystery, in which a substitution cipher is used to encrypt secret messages. The story provides a fun and interesting way to talk about frequency analysis, and can be used as a segue into mathematical constructs such as modular arithmetic and computation. The activity is accessible to ages twelve and above, and has been successfully used in mathematics outreach and popularization efforts as well as in general education and mathematics courses.

Keywords: substitution cipher, Sherlock Holmes, mystery, activity, outreach, fun, humanities.

Preface

We all love a good mystery story. What if a math class includes a murder mystery in which students act as detectives, and at the same time learn quite a few mathematical concepts? We can perhaps convince our students and the larger community that mathematics is not all about busy work, and that a lot of mathematics is developed because of real world problems. Through the years, I have liberally used cryptologic and number-theoretic activities and puzzles in my outreach and popularization efforts, as they work well to get students excited about mathematics in general. In this paper I describe a particularly appealing activity involving ciphers that has been very successful with a wide range of students.

In particular, in this paper I describe an activity based on a 1903 story "The Dancing Men" written by Sir Arthur Conan Doyle. This is murder mystery, in which a substitution cipher is used by the criminal to encrypt secret messages, and once cryptanalyzed, the method is used by Sherlock Holmes to catch the criminal. The story is easily available online (see for example [3, 4]) and in print [1, 2]. The activity provides a fun and interesting way to talk about mathematical constructs such as frequency analysis and the mathematics behind encoding and decoding secret information. It is also a great way to talk about the essence and beauty of mathematical thinking and the fact that mathematics is so much more than "just" arithmetic.

This activity has been used in venues such as math circles, STEM days, open houses and talks for math majors at four-year institutions in and around Illinois. In my own teaching I used it as an ice-breaker and as a part of the curriculum in Discrete Math, Cryptology, and a general education mathematics course designed for students who do not have a mathematics requirement for their major. I have even used the activity in the new virtual modality forced upon us by the current pandemic; it is a flexible and rich activity that can be modified to suit the level of the audience and the available time.

This story about dancing stick figures is one of thirteen stories in the cycle published as *The Return of Sherlock Holmes* [1]. Doyle ranked "The Dancing Men" third in the list of his twelve favorite Holmes stories "on account of the originality of plot" [2, Introduction by Davies]. This is one of only two Sherlock Holmes short stories where Holmes' client dies after seeking his help. The other is "The Five Orange Pips", part of *The Adventures of Sherlock Holmes* [3]. In some collections the story is titled "The Adventure of the Dancing Men", but the original title was "The Dancing Men" when it was published as a short story in *The Strand Magazine* in December 1903 [3, 4].

The initial idea of this activity came from an example in a classical book on cryptography by Trapp and Washington [8]. The authors use this example to show how cryptography has been used in the literature and argue that a bunch of lucky coincidences can lead to easy cracking of the substitution cipher. The authors also use the example to stress the importance of authentication and error correction in cryptography. For instance, in early prints of the story, there is a typographical error which throws off the decryption scheme. Also in the original story, Sherlock Holmes uses the encryption

scheme to help catch the criminal. Had the criminal used authentication methods, he would not have fallen into the hands of the law enforcement. In their own words, the authors give a "cryptic, and cryptographic, summary of the plot" in this example.

While the original story is much more fun to read, it should be kept in mind that Arthur Conan Doyle aims to highlight the brilliance of Sherlock Holmes. Our goal, on the other hand, is to impress upon the students that mathematical thinking is fun and within their reach. Holmes cracks the cipher over a period of a few days. To fit the whole activity to within fifty minutes, the original story has been shortened and simplified. Reference to England has been completely removed and some other superfluous information that distracts the reader instead of helping solve the mystery have been omitted. In the original story Elriges is the name of an inn but we have taken the liberty to use it loosely as the name of a town. Hints and suggestions have been added to guide the students in the right direction.

Details about the implementation of the activity are described in a later section. The pictures of all stick figure messages except the fourth are from the collection *The Return of Sherlock Holmes* [3]. As mentioned above, the original story has a typographical error that throws off the decryption scheme. To remove this (intentional or unintentional) error, the fourth figure has been taken from [8]. The fourth message is meant to have a different handwriting, so this serves our purposes well.

Activity

Hilton Cubitt of Elriges visits you and gives you a paper with the following mysterious sequence of stick figures that he found lying on the sun-dial in his mansion.
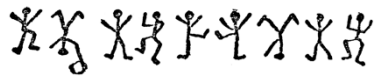
Message 1:

Cubitt explains that he recently married a Chicago woman named Elsie Patrick. Before the wedding, she had asked him never to ask about her past,

as she had had some "very disagreeable associations" in her life, although she said that there was nothing that she was personally ashamed of. Their marriage had been a happy one until the messages began to arrive, first mailed from Chicago and then appearing in the garden of his mansion.

The messages had made Elsie very afraid but she did not explain the reasons for her fear, and Cubitt insisted on honoring his promise not to ask about Elsie's life in Chicago. You look at the figures closely to understand them a little better and notice that some of the figures are holding flags. What could the flags mean? Perhaps the end of words?

The next morning Cubitt finds "a fresh crop of dancing men drawn in chalk upon the black wooden door of the tool-house":

Message 2:

Two mornings later, "a fresh inscription had appeared":

Message 3:

Three days later, "a message was left scrawled upon paper, and placed under a pebble upon the sun-dial":

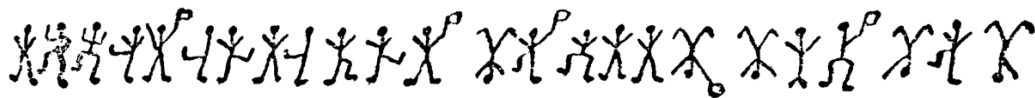**Message 4:**

Cubitt gives copies of all these messages to you. Your task is to help him understand what is going on. You call your friend in the Chicago Police Department and ask her to find background information on Elsie Patrick. You learn that Elsie is the daughter of a Chicago crime boss, and was engaged to Abe Slaney, who worked for her dad, and that she had fled to escape her old life.

You examine all the occurrences of the dancing figures. Message 4 is in a different handwriting, so you guess that it is from a different person, most likely, Elsie, while messages 1, 2 and 3 are from the unknown person (the criminal). You spend the next two days trying to make some sense of the stick figures. You are now sure that the flags on some of the figures indicate the end of words. You also know that a simple substitution cipher is being used for the encryption, and that frequency analysis is the way to solve these ciphers.

1. What is the most common figure? This would likely be 'E'.
2. The most common figure appears twice in the last message (Message 4). What could this message be?
3. The most common figure appears thrice in Message 3, once in the first word and twice in the second word. So the first word is _ _ _ E, and the second word is E _ _ _ E. The message is from the criminal to Elsie. What could this message be?
4. Message 2 is from the criminal, has two words, the first being two letters. What is Message 2?
5. Using what you know so far, what is Message 1? What is the key for this substitution cipher?
6. Three days later, another message appears.

Message 5:

This message causes you to fear that the Cubitts are in immediate danger. You rush to Elriges and find Cubitt dead of a bullet to the heart and his wife gravely wounded from a gunshot to the head. What is Message 5?

7. Inspector Martin of the Norfolk Constabulary believes that it is a murder-suicide attempt; Elsie is the prime suspect. But you, after noting some inconsistencies in that theory, know that there is a third person involved. How will you prove to Inspector Martin that a third person is involved?

8. How will you catch the criminal?

Figure 1: Illustration by Sydney Paget, 1903. Public domain image from Wikipedia, *available at* https://en.wikipedia.org/wiki/The_Adventure_of_the_Dancing_Men, *last accessed on July 26, 2021.*

The key and helpful hints:

1. The most common figure is . So it must be 'e'.

2. Message 4 is NEVER (compare with LEVER, SEVER, FEVER, NEVER).

3. COME ELSIE (the first word could be LOVE, DOVE, SAME, COME, LIKE, ... and the second word could be ELOPE, ELATE, ELSIE, ...)

4. _ _ ELRI_ES Possible two letter words are IS, AN, AT, AM, ON, OF, GO, IN,... The second word is easier (ELRIGES). Answer: AT ELRIGES.

5. A_ _ERE A_E SLANE_ (AM HERE ABE SLANEY).

6. ELSIE _RE_ARE TO MEET TH_ GO_ (ELSIE PREPARE TO MEET THY GOD).

7. Based on the decrypted messages, ABE SLANEY is the criminal.

8. Different answers expected.

Postscript

The original story is based in England and United States and meanders along with many more details. Riding Thorpe Manor is a country home in rural Norfolk, England, and is owned by the Cubitt family who have lived there for five centuries, and is the current home of Mr. Hilton Cubitt, while Elsie is the daughter of a crime boss in Chicago. Elriges is the name of the hotel where Abe Slaney stays when he comes to visit Elsie.

A passage from the original story in which Holmes explains his elegant analysis is included in the appendix. This is a much more interesting read and it is an interesting exercise to work through this cryptanalysis, especially since Holmes does not have the hints as provided to our readers. To conclude the activity, parts of this passage may be shared with the audience.

Question 8 of the activity is answered in the story as follows: Holmes writes a message "Come immediately," in stick figures, and has it delivered to Abe Slaney. Holmes knows that Slaney would believe the message is from Elsie. Slaney, unaware that Elsie is gravely wounded, rushes to Riding Thorpe Manor and is seized as he comes through the door (as shown in Figure 2.) He pretends innocence, but the stick figures give him away, and he confesses. Slaney had come to England to get Elsie back. When Slaney and Elsie were speaking through a window, Cubitt had appeared and shots were exchanged; Cubitt was killed and Slaney had fled. Apparently, Elsie then shot herself.

Figure 2: Illustration by Sydney Paget, 1903. Public domain image from `Wikipedia`*, available at* `https://en.wikipedia.org/wiki/The_Adventure_of_the_Dancing_Men`*, last accessed on July 26, 2021.*

Epilogue (from the original story)

> "Only one word of epilogue. The American, Abe Slaney, was
> condemned to death at the winter assizes at Norwich; but his
> penalty was changed to penal servitude in consideration of mit-
> igating circumstances, and the certainty that Hilton Cubitt had
> fired the first shot. Of Mrs. Hilton Cubitt I only know that I
> have heard she recovered entirely, and that she still remains a
> widow, devoting her whole life to the care of the poor and to the
> administration of her husband's estate." [4]

Implementation

This activity is accessible to students as early as in middle school, and is in-
teresting to kids and adults alike. Fifty minutes is sufficient time to complete
the activity, but if needed, it can be completed in fifteen minutes by giving
hints along the way. The activity can be adapted to suit the audience — the
younger students can do the stick figure dances as an ice-breaker activity,
while the older students can solve the mystery without much help.

The activity has been used in many venues. In my Discrete Mathematics
course, I use it towards the end of the semester after the discussion on logical
arguments and probability theory. In a course on Cryptology, I have used it to
introduce substitution ciphers and their cryptanalysis. In a general education
mathematics course, I used it to discuss how mathematical thinking is used
to send and receive secret messages, taking the discussion then to modular
arithmetic and to other ciphers that can be cracked by frequency analysis.
The activity can be used as an ice-breaker in any lower-level mathematics
course to get students excited about mathematics in general. Of all the topics
covered in class over the course of a semester, students tend to remember the
'stick figures mystery' best in their end of the semester course evaluations.

At a STEM Day at our institution, high school students were surprised to
see that the seemingly boring math professors love mystery books as much
as they do, and that doing mathematics is similar to solving mysteries. This
activity can help relay the excitement involved in mathematical thinking and
a discussion of the non-teaching career options available to math majors.

Most students think that the only career option for math majors is teaching. However, the industry values the ability to think mathematically — logical deduction, interpolation, extrapolation, solving problems that seem unsolvable, tackling a large problem by breaking it into smaller parts, etc. The fact that most math majors go into careers outside academia comes as a surprise to these students.

I have shared this activity at a session for new mathematics faculty [7] and the feedback has been positive. But I am most impressed by how students view this exercise. In a seminar talk [6] at a local undergraduate university which was primarily attended by mathematics and computer science majors, I used the activity to catch students' interest and followed it up with a discussion of public key cryptography and its use in e-banking and e-commerce. From there ensued a discussion of how useful and applicable mathematics is to our modern day-to-day life and what really is the essence of mathematics. While most students think of mathematics as involving remembering and understanding, I believe and want to convince students that it actually involves higher forms of thinking such as creating and analyzing, and that the mathematical way of thinking is exciting and beautiful in itself [5]. As such, I conclude this article with a comment from a student who attended the talk mentioned above:

> "At first the talk was very intriguing because of the short mystery game ... I didn't realize what (was the) conclusion from playing this game, until asked What is math? I thought of it as fundamental basis of understanding the world ... Then (I suddenly realized that) Math is so much more than just a fundamental basis — it is valuable, (it) plays an important role throughout history, and it occurs in nature and architecture. To change the perspective that math is not at the bottom of the triangle (of Blooms Taxonomy) rather it is on top ..."

References

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[5] Manmohan Kaur, "Increasing Math Appreciation using Upper Levels of Bloom's Taxonomy," TSG-62 Popularization of Mathematics, *14th International Congress on Mathematical Education*, Shanghai, July 2021 (preprint).

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[8] Wade Trappe and Lawrence C. Washington, *Introduction to Cryptography with Coding Theory* 2nd Edition, Pearson, 2005.

Appendix: Holmes' cryptanalysis as described in the original story [4]

"Having once recognised, however, that the symbols stood for letters, and having applied the rules which guide us in all forms of secret writings, the solution was easy enough. The first message submitted to me was so short that it was impossible for me to do

more than to say with some confidence that the symbol ⚡ stood for E. As you are aware, E is the most common letter in the English alphabet, and it predominates to so marked an extent that even in a short sentence one would expect to find it most often.

Out of fifteen symbols in the first message four were the same, so it was reasonable to set this down as E. It is true that in some cases the figure was bearing a flag and in some cases not, but it was probable from the way in which the flags were distributed that they were used to break the sentence up into words. I accepted this as a hypothesis, and noted that E was represented by 𝍢 .

"But now came the real difficulty of the inquiry. The order of the English letters after E is by no means well marked, and any preponderance which may be shown in an average of a printed sheet may be reversed in a single short sentence. Speaking roughly, T, A, O, I, N, S, H, R, D, and L are the numerical order in which letters occur; but T, A, O, and I are very nearly abreast of each other, and it would be an endless task to try each combination until a meaning was arrived at. I, therefore, waited for fresh material. In my second interview with Mr. Hilton Cubitt he was able to give me two other short sentences and one message, which appeared—since there was no flag— to be a single word. Here are the symbols. Now, in the single word I have already got the two E's coming second and fourth in a word of five letters. It might be 'sever,' or 'lever,' or 'never.' There can be no question that the latter as a reply to an appeal is far the most probable, and the circumstances pointed to its being a reply written by the lady. Accepting it as correct, we are now able to say that the symbols 𝍢𝍣𝍤 stand respectively for N, V, and R.

"Even now I was in considerable difficulty, but a happy thought put me in possession of several other letters. It occurred to me that if these appeals came, as I expected, from someone who had been intimate with the lady in her early life, a combination which contained two E's with three letters between might very well stand for the name 'ELSIE.' On examination I found that such a combination formed the termination of the message which was three times repeated. It was certainly some appeal to 'Elsie.' In this way I had got my L, S, and I. But what appeal could it be?

There were only four letters in the word which preceded 'Elsie,' and it ended in E. Surely the word must be 'COME.' I tried all other four letters ending in E, but could find none to fit the case. So now I was in possession of C, O, and M, and I was in a position to attack the first message once more, dividing it into words and putting dots for each symbol which was still . So treated it worked out in this fashion:— .M .ERE ..E SL.NE.

"Now the first letter CAN only be A, which is a most useful discovery, since it occurs no fewer than three times in this short sentence, and the H is also apparent in the second word. Now it becomes:— AM HERE A.E SLANE. Or, filling in the obvious vacancies in the name:— AM HERE ABE SLANEY. I had so many letters now that I could proceed with considerable confidence to the second message, which worked out in this fashion:— A. ELRI.ES. Here I could only make sense by putting T and G for the missing letters, and supposing that the name was that of some house or inn at which the writer was staying." Inspector Martin and I had listened with the utmost interest to the full and clear account of how my friend had produced results which had led to so complete a command over our difficulties.

"What did you do then, sir?" asked the inspector.

"I had every reason to suppose that this Abe Slaney was an American, since Abe is an American contraction, and since a letter from America had been the starting-point of all the trouble. I had also every cause to think that there was some criminal secret in the matter. The lady's allusions to her past and her refusal to take her husband into her confidence both pointed in that direction. I therefore cabled to my friend, Wilson Hargreave, of the New York Police Bureau, who has more than once made use of my knowledge of London crime. I asked him whether the name of Abe Slaney was known to him. Here is his reply: 'The most dangerous crook in Chicago.' On the very evening upon which I had his answer Hilton Cubitt sent me the last message from Slaney. Working with known letters it took this form:— ELSIE .RE.ARE TO MEET THY GO.

The addition of a P and a D completed a message which showed me that the rascal was proceeding from persuasion to threats, and my knowledge of the crooks of Chicago prepared me to find that he might very rapidly put his words into action. I at once came to Norfolk with my friend and colleague, Dr. Watson, but, unhappily, only in time to find that the worst had already occurred."