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Mathematics for the Masses: Door-to-Door Missionaries of Math and Twelve-Step Recovery Programs

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Synopsis

Conversion of strangers, or proselytizing, is a feature of a range of groups for religious, organizational and other aims. In twelve-step recovery programs, such as Alcoholics Anonymous, belief in a higher power is a requirement for working the steps to recovery. People are encouraged to find a higher power of their own understanding. This paper presents a model for using mathematics as a higher power, and shows how recovery works with mathematics in that role instead of a more traditional higher power such as God. A contemplative definition of math is given along with a description of a three-categorized epistemology: mathematics, science, and the personal. This epistemology is shown to be sufficient to work the Twelve Steps, with mathematics as a higher power. Proselytizing is not required.

Keywords: *mathematics education; Alcoholics Anonymous; twelve-step recovery programs; addiction; mental health*

1. Introduction and Layout of the Exploration

Imagine the missionary work that is carried out in the USA and other countries where proselytizing is legal or common [2] — wherein a group of two or a few people smartly dressed will carry their religious literature from door-to-door — and now imagine replacing their religious literature with books of mathematics. Replace their greeting as well with “Do you want to study mathematics?” The proselyte will doubtless be surprised.

The idea of proselytizing mathematics occurred to me as I was studying for a master's degree in geology in my middle age, a third career. I was frequently beset by a range of religious people coming to the door where I lived near the university. I recall one group whose main point was that God is female, and that their church is unique in using female iconography and language. These women who came to visit spoke with strong accents, and it was with some difficulty that I was able to work out what the tenets of their religious affiliation were, and then to praise them and agree with them. My habit is to invite people in and find out what they are promoting. I will suggest prayers for their safety, and thank them for their time.

This attitude arises naturally from a belief in personal variations of internal experiences, and a facility with the history of monotheism and its practices. More often than not, the more intense types of Bible studies — where the text is interpreted in a specific manner to promote a certain belief — are completely upended by the Hebrew I will offer. For example, at my university campus, there was a group whose members invited people to study the Bible with them on the spot for half an hour at a time. The word salt came up in the text, as in

You are the salt of the earth, but if the salt has lost its
flavor, with what will it be salted? It is then good for nothing,
but to be cast out and trodden under the feet of men.
(Matthew 5:13, *World English Bible*)

I was able to describe how the Hebrew word for salt has a sound that is very similar in sound to the Hebrew word for king, and that the author of those words was probably saying something clever about whether or not a political leader or king has an innate feature of kingliness that is immutable. It is an interesting play on words, and useful from a historical perspective.

Notwithstanding, the people in this group stopped inviting me to share in Bible studies with them; the invitations ceased all at the same time, so I imagine that there was some private discussion about whether study with me would be worthwhile.

At home, missionaries coming to my door included Mormons and Jehovah's Witnesses, plus a few other Christian denominations. I had Mormon neighbors who invited me to come across the street to their house to study their holy book. I agreed, but only participated a few times.

Once the book started to describe scenes of violence, I took that as a sign that I ought to discontinue my attendance.

I consider it an oddity in a sacred text if the characters presented must resort to violence to meet their aims. I told my neighbor so, and politely stopped coming. I felt comfortable doing so, having taken the time to explore the text and to learn first-hand what was being promoted.

Taken as a violation of one's body, violence is promoted in stories as if it had a positive social use [12]. Disney movies, for example, are marketed to children, and often present violence as a solution to some problem that the characters encounter [15].

Sometimes I will be blunt about this type of flaw that is seen in a sacred text that is meant by the owner to be taken literally. A few times I have asked the descendants of slaves in the USA why they are promoting a text which promotes slavery, as Jesus did not condemn the practice, and other letters within the book encourage obedience of slaves to their masters (Ephesians 6:5; Colossians 3:22).

My aim in this regard is both learning and teaching, and I come with a compassionate intention. Engagement is a pro-social sentiment. Indeed, the practice of sharing one's personal faith in this manner is a social endeavor [16]. The main object I have seen is a sincere desire to share some specified kind of teaching, and to do so in a manner that concludes on good terms socially. Participating in this type of interaction has necessitated some skill on my part in negotiating the interaction's boundaries, i.e. I try to promote whatever perspective is being offered in some manner.

Since this time of my graduate studies, from about 2007 onwards, I have considered that proselytizing mathematics might be worth exploring. I have mentioned the idea to a few people, and also have considered what form a fruitful exploration might take. I have not myself engaged in the practice. This article is meant as a preliminary treatment of some ideas that might be personally useful in doing missionary work in mathematics, in the same manner that religious people do missionary work with religious texts, but is grounded within a specific context.

It is a context wherein the sharing of a personal truth of mathematical sentiment isn't simply a parody of religious sentiment, and doesn't compete with religion out of disrespect. In a very real sense, people who suffer from addic-

tion, and in particular alcoholism, are often faced with religious pressure to find God or a suitable higher power in order to participate in a Twelve-Step recovery program.

In Alcoholics Anonymous (AA), the first of the Twelve-Step programs, the basis for recovery from alcoholism is seen as a combination of internal reflection that includes reliance on that which is greater than we are (i.e., a higher power) as well as service work to maintain the institutions of recovery and to help others along the way. Recovery rests on what could be termed *role psychology*. In recovery, one takes on a role in a group that may be of use to others, and in so doing, assumes certain responsibility and acquires skills in assuming that role. The skills acquired, such as the ability to communicate in a manner that is compassionate with others who are suffering, also find utility in helping the psyche of the person taking on that role. To sponsor others in their recovery, for example, requires that a person have their own stable recovery practice so that they have something to share. By the same token, teachers learn their subjects very deeply in the process of delivering the materials to students [20]. The role helps to determine the effect, since assuming a role comes with a social expectation.

Many people newly coming to Twelve-Step groups have difficulty with the idea of a higher power. The assumption is that one can look to any phenomenon that might have some power to lead to health, and in adopting a loving attitude towards that phenomenon, one builds a connection leading towards health. Although most people in a Twelve-Step program will share that their higher power “whom they choose to call God” is active in their lives, a large number of people leave recovery programs because of their difficulty in understanding the nature of the connection and commitment to a higher power. I have seen people call on a Spirit of Nature as a higher power, though this is a rarity in the groups I attended. I have never seen someone call on mathematics as a higher power in a Twelve-Step meeting.

Notwithstanding, mathematics would make a useful and approachable higher power. The practice of working mathematics problems can be seen as having a calming effect [11]. The social nature of sharing in the gospel of mathematics can highlight how appealing mathematics is to some people and allow for bonds of friendship to arise out of a common interest [4]. Mathematics can be seen as a source of mystery, since much of mathematics is yet unknown, and all practitioners will relate that their knowledge in mathematics is not all-encompassing. Finally, mathematics can be seen as a source of healing.

The neural pathways in those accustomed to a drug or a fix are, in practice, very fragile as the drug or fix is removed and withdrawal occurs [17]. The supposed neurological rewiring that occurs during recovery is enhanced by fine motor tasks, such as those promoted by occupational therapists [18]. Outcomes are better when clients are engaged in fine-motor activities. The same is likely true of mathematics [10]. The stable procedures involved in working problems allows for the healthy generation of new neural pathways [3], and at the same time the reflective insights that occur during the practice of mathematics can be taken as a source of gratitude for being alive and well [8].

It might seem offputting to think of mathematics as a higher power, and yet the necessary features of mystery and transformation are present. Indeed, some people might reasonably consider adopting mathematics as their personal higher power in their Twelve-Step recovery. Hence it makes sense to look more deeply at how the Twelve Steps are used in recovery, and how mathematics might be implemented as a higher power within this context. For reference, I have included a list of the Twelve Steps of Alcoholics Anonymous as Table 1; see for example <https://www.aa.org/the-twelve-steps>.

Table 1: The Twelve Steps of Alcoholics Anonymous.

Step	Action
1	We admitted we were powerless over alcohol — that our lives had become unmanageable.
2	Came to believe that a Power greater than ourselves could restore us to sanity.
3	Made a decision to turn our will and our lives over to the care of God as we understood Him.
4	Made a searching and fearless moral inventory of ourselves.
5	Admitted to God, to ourselves, and to another human being the exact nature of our wrongs.
6	Were entirely ready to have God remove all these defects of character.
7	Humbly asked Him to remove our shortcomings.
8	Made a list of all persons we had harmed, and became willing to make amends to them all.
9	Made direct amends to such people wherever possible, except when to do so would injure them or others.
10	Continued to take personal inventory and when we were wrong promptly admitted it.
11	Sought through prayer and meditation to improve our conscious contact with God, as we understood Him, praying only for knowledge of His will for us and the power to carry that out.
12	Having had a spiritual awakening as the result of these Steps, we tried to carry this message to alcoholics, and to practice these principles in all our affairs.

Typically, Step 1 is worked with a Sponsor in the AA group, who helps the newcomer catalog their experience with alcohol, and in particular focuses on finding examples within their own life of how decision-making is influenced by the substance's powerful draw. The aim is to see how an invisible line has already been crossed, namely that one's rational faculties are no longer trustworthy when it comes to questions of drink. There may be crisis-based decision making, where relationships are torn apart, and the associated anger ends up as a trigger to drink. What happens typically is the crisis precedes the drink as well as following it. The effect becomes the cause.

Instead of, for example, needing a drink to cope with something difficult, the alcoholic creates a crisis, and that then acts as a reason to drink. It's a subtle difference, but is important in understanding how a person's agency has been warped by the substance abuse. Once this pattern becomes clear for a person, they are well on their way to recovery. In fact, it forms the basis for the active part of the Twelve Steps, that of writing inventories and making amends, all with the hope that these will undo some of the damage caused by this inversion of agency.

Once a person gains a little insight into their illness, they are in a tight spot. That is where Step 2 comes in. The work of Step 2 is personal. Whether someone can genuinely look at mathematics as embodying the kind of personal, even magical power that will guide their recovery is a real challenge. But that is normal. Any person working Step 2 is faced with the same challenge, whether mathematics or God or Jesus or Mohammed or Krishna . . . or any number of personal deities are invoked. At this point, a person needs the assurance from the recovery group that their own personal idea will suffice.

2. A Three-Categorized Epistemology and Its Use in Twelve-Step Recovery

For the person looking to use mathematics as a higher power in Twelve-Step recovery, it makes sense to explore now what is meant by mathematics. The most common definition of mathematics is practical rather than theoretical. It states that mathematics is the study of mathematical objects, and then proceeds to list a few examples of certain classes of objects that are considered mathematical, such as numbers, operations, structures, etc. [14]. The lack of precision seems to be no worry to mathematicians — in the same fashion that the lack of a consensus for a definition of life does not impair the professional

lives of biologists [13]. In both, there are historical reasons for the matter studied being included in the subject, and each is coherent and fruitful. Yet in mathematics, unlike biology, there is an alternative to this common definition that may be sufficient to describe the field and also lend insight into what is studied. I favor the Davis and Hersh definition [6], highlighted by David Mumford and Alexandre Borovik: Mathematics is the study of mental objects with reproducible (or communicable) properties.

If I suggest that the reader imagines three points, and now imagines three lines connecting these points, I have demonstrated a practical example of a mental object with communicable properties. From this I can suggest that the definition is useful. The reader will have imagined either a triangle, or three colinear points connected by a line, based on my use of mathematical language.

If mathematics is taken as the study of all mental objects with communicable properties, there will be included mental objects that perhaps previously have not been included within mathematics, but whose membership is achieved by the communicability of their properties. I am not able to conceive of an object that falls in this class and that is not mathematical, in the sense that all such objects can be described mathematically. Indeed, so long as a property can be specified, mathematics can and does treat such objects despite any potential difficulty in visualizing them. Note that this definition of mathematics is not commonly used among professionals. It is from mathematics communication and pedagogy.

This qualified-mental-objects definition of mathematics is useful. To see how this works, an analogy can be made between mathematics and science. Consider the parallelism. If mathematics is the study of mental objects with communicable properties, then science can be taken as the study of physical phenomena that have sensible properties, and upon which reproducible experiments can be undertaken. The scientist cannot study that which cannot be measured. (That which is physical but lacks sensible properties and on which reproducible experiments cannot be undertaken — might be taken as another kind of object, the supernatural.)

More commonly we attribute things that are insensible to the mental realm, and consider that mental objects lacking reproducible properties are in the area of personal, psychological, philosophical, or religious knowledge. See Table 2.

Table 2: A Three-Categorized Epistemology: Mathematics, Science and the Personal.

	Communicable Reproducible	Not Communicable Not Reproducible
Physical	Science	(Supernatural)
Mental	Mathematics	Personal

In the category of personal knowledge, I am not trying to assert any one definition for such things, but rather suggesting that this category — of mental objects lacking reproducible properties — is not mathematical nor scientific, and yet valid within its own parameters. If these parameters are personal, then respect for a person ought to produce an inclination to accept that people may have personal experiences whose properties are not necessarily communicable. Likewise with psychological, philosophical, or religious experiences.

The challenge of deciding what one may communicate of a mental experience forms the basis for asserting that there exist mental objects that are an amalgam of both types, i.e. with some properties that may be communicated, and other properties that may not. If one takes the example of romantic attraction as a case, one can see that my romantic attraction to another person has meaning in particular to myself, and although some parts of that attraction might be communicated in a way that will share some information about my experience, fundamentally the experience is mental and lacks reproducible or communicable properties. My love life is of great importance to no one in particular but myself. It stands as something which exists in my mental field, and yet lacks a communicable basis that would produce an objective outcome that is predictable. In attempting to communicate my inner world, I may develop some skill at identifying what is communicable, and in this fashion may make some progress in translating my personal experience to a communicable one.

This process of translating mental objects from ideas lacking communicable features to those that have these features is useful in countering an objection that I have encountered when I have presented this three-categorized model of knowledge. Sometimes, a person will give me the example that there are people who hold some awful personal beliefs. These are beliefs that others find objectionable, based on racism, nationalism, etc. They wonder if this three-categorized model gives such people a free pass to persist in these odious

personal beliefs, and if it does, they assert that perhaps the model is not useful.

I can counter this argument by asserting that these odious personal beliefs are an amalgam of mental objects, some of which have communicable properties, and some of which do not. I assert that mental objects with communicable properties are not in a privileged personal realm, but rather belong to the realm of mathematical or scientific knowledge. It is only that which is not communicable which garners the privileged status of being personal. Thus, the only part of bigotry that stands as personally privileged is that which is without form. Once it has taken form, it no longer has a protected status, and is obviously false. This process of separating the communicable from the personal could be seen as a form of sublimation, about which there have been volumes written in the psychological literature. The process is one of healing [9].

Thus, this three-categorized model is useful in contributing to the ongoing dialogue that exists around questions of individualism and collectivism, as well as group membership and rights. It is an important contribution in that it redefines what is considered personal in a consistent fashion, and allows for policy to intervene successfully while still respecting that which is personal.

Furthermore, the process of determining that which is communicable in one's mental field has a developmental benefit. The historian of philosophy can recognize such a process in Kant's *Critiques* clearly, for example, and can see how considering the nature of mental objects can lead to a change of mental state as the process progresses, that is, as one considers the kind of internal reflection that produces beneficial results. The process is experiential. Philosophy thus belongs to this third category of knowledge, along with the religious, psychological, and personal. See Table 3 for a schematic treatment of these examples.

Table 3: Examples of Refining Mental Objects in Different Categories.

Category	Example
Personal	Romantic thoughts
	Personal experiences
Religious	Religious experiences
Psychological	Sublimation
Philosophical	Development of new terminology

Having identified that mental objects may be of at least two types, i.e. those with communicable properties and those without, then one might consider that consciousness can be trivially defined as that which provides the basis for both of two types of experiences: those that have reproducible or communicable properties and those that lack reproducible or communicable properties. See Figure 1. The definition of consciousness in this way may lead to useful results in computer science. One can see that a coupled system that includes both communicable and non-communicable phenomena might be feasible to create, and further work in this area can be pursued [7].

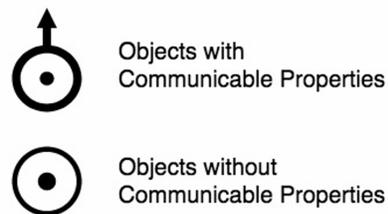


Figure 1: Two types of objects of consciousness. Artificial intelligence ought to include both types.

3. Turning Your Will and Life Over to a Higher Power

Once a person has a clear view of mathematics as the study of mental objects with reproducible properties, and is aware that there are other mental objects which are not easily communicated, it is not difficult to see that progress in working Step 2 has been made. There is a dignity present in recognizing that people have a personal side to them that will not lend itself to language, but instead unfolds through living. That mathematics, as a study, has a hand in how we think about this process is enough to create the belief that mathematics can serve as a focus for the restoration of proper agency on the road of recovery. Studying mathematics doesn't just lend itself as a roadmap, it treats roadmaps themselves in the abstract, and that is where its power lies. Here is both a pathway to understanding abstraction, and a language with which to make progress, while at the same time an affirmation of the person and their experiences. The newcomer has now come to believe that mathematics has the power to restore them to a healthier frame of mind.

Step 3 comes next, and is decidedly simpler. It calls just for making a decision. The person in recovery has, first, seen that their agency is not as it should be when it comes to questions of drink, and, next, has seen that their inner world has features to it which suggest that transformation is possible. Step 3 is simply an affirmation. It is simply a question of saying, “OK. I believe that mathematics and studying mathematics will be helpful in transforming my agency, and healing my thoughts, emotions and relations. Now, let’s do it.” A person then gets in the habit of “turning it over” to their higher power. If something is unsettling, the newcomer reminds themselves that they are in the middle of the process. There is no hurry. Their higher power is in charge.

Any difficulty or challenge can be met with this type of reflection. It is not altogether different from counting numbers until your emotions cool down. In fact, that is one common example of how people “turn it over” using math. There is empowerment in this practice.

Twelve-Step work also requires doing an inventory and making amends, that is, doing Steps 4 through 9. These are a hurdle that group members sometimes have trouble with. The process of looking deeply at what causes anger, fear and other negative emotions in our lives is too big. There is too much there to put down succinctly and see the patterns that are present. And at this point, a Sponsor from the group will help to set up a process and timeline so that these inventories can be accomplished initially, setting the stage for making amends; and then afterwards, for ongoing inventories and amends (Step 10). There is no difficulty here in having mathematics as a higher power working Steps 4 through 9, since the foundation of Steps 1, 2, and 3 have already been accomplished. The newcomer is already familiar with turning it over. Steps 4 through 10 are worked much the same way that everyone works them.

This leaves Step 11, which suggests “prayer and meditation” as a way of deepening a connection with a higher power, as an obvious place to promote the continuing study of mathematics. The most famous society for the study of mathematics in the ancient world were the Pythagoreans. They took their insight from the irrational nature of the square root of two, i.e. the diagonal of a square whose sides are unitary. The proof that this diagonal is not a rational number was taken as an invitation to consider the nature of reality as composed of numbers, and that mathematics was sufficient to include the mysteries that one might encounter in the real world [19].

Currently, in more mundane instances, people do gather together to practice mathematics in what are called math circles or math clubs. These are social spaces wherein people practice mathematics together. They do not focus on any mystical implication from mathematics generally. I believe that they are open to all comers. The main point is simply that there be an interest.

As far as I know, there are not currently people in Twelve-Step programs using mathematics as a higher power for recovery. In this article I have shown that mathematics may indeed serve as a higher power for Twelve-Step recovery, and that it is reasonable for people to attempt to do so if they are faced with the prospect of working the steps. I would like to caution that missionary work may be seen as an attempt at selling something, and care should be taken to preserve the dignity of whomever you wish to talk to about mathematics.

4. Final Thoughts

On a recent trip to Singapore, it was pointed out to me that children in the primary grades are already learning basic calculus. Student test scores in Singapore on the Programme for International Student Assessment test are the highest in the world [5], and students devote incredible amounts of time to the work, sacrificing extracurricular activities in favor of more schooling that occurs after regular school hours. The mood surrounding mathematics here is serious, and students are proud of their accomplishments, though some youth say the study, and especially the amount of time devoted to it, is overbearing.

Missionary work has a different focus. It is meant to bring people into contact who might otherwise not be in contact, and to allow for a peer-to-peer opportunity for both learning and healing. I don't think that the study of mathematics will generally usher in a new age of enlightenment, yet the promotion of disbelief in science does threaten to erode the gains made possible since the seventeenth and eighteenth centuries, and the idea of banishing ignorance in favor of reason. Postmodernism rejects this idea of reason as monolith [1], and there is no good reason not to accept that there can be multiple ways of knowing, and multiple truths. Yet a path forward, one that includes greater facility in communication between different groups and factions is not without merit.

A respect for different realms of knowledge, and for which parts of a mental object may be communicable, may lead to greater respect for people who hold different perspectives. The idea that these might be seen as complementary and not as exclusionary holds a postmodern frame to its modernist progenitor, and may inspire additional work for what seems historically to have been a religious aim, namely harmonious living. It is in describing mental objects, and discerning which part is communicable, that philosophical knowledge grows inside a person. The distinction between healthy and unhealthy emotions becomes clearer.

If this process can help to bring about recovery from addiction, then it ought to be practiced. This article has shown how recovery is possible with mathematics as a higher power, and how clarity of thought can be developed through practice. Moreover, the intent of the original writing of the Twelve Steps of Alcoholics Anonymous was never to keep people out who could not pass a religious test. Opening the doors to taking mathematics as a higher power is in line with the idea of sharing the message that recovery is possible. Mathematics helps.

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