The Human Face of Mathematics: Reuben Hersh (1927-2020) In Memoriam

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Cover Page Footnote

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The Human Face of Mathematics:
Reuben Hersh (1927-2020) In Memoriam:[1]

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Synopsis

Reuben Hersh (1927-2020) celebrated mathematics as a human endeavor, historically evolved and intelligible only in a social context. It is therefore appropriate to remember him in the *Journal of Humanistic Mathematics*. There have been many tributes to Reuben, which commemorate his life and impressive mathematical and philosophical achievements. This memoriam to Reuben instead focuses on showing how his humanistic philosophy was so indicative of the person he was.

With the *Journal of Humanistic Mathematics*, editors Mark Huber and Gizem Karaali, standing on the shoulders of Alvin White (cf. [9]), have been tireless in providing a forum to promote the human face of mathematics. Because Reuben Hersh celebrated mathematics as a human endeavor, historically evolved and intelligible only in a social context, it is appropriate to remember him in this journal. There have been many tributes to Reuben, which commemorate his life and impressive mathematical and philosophical achievements. This rather personal tribute instead focuses on showing how Reuben's humanistic philosophy was so indicative of the person he was.

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[1] This memoriam to Reuben Hersh includes parts excerpted from a chapter entitled "A Case Study in Reuben Hersh's Philosophy: Bézout's Theorem" [18] which had been written for a 2017 Festschrift [21] honoring Reuben on his 90th birthday.

Generosity of Spirit – The Social Context for Mathematics and Beyond

I met Reuben in the 1980s. I wanted to design a general education mathematics course around *The Mathematical Experience* [2], a trade book Reuben wrote with Philip J. Davis.[2] Their intention was to promote an understanding of mathematics from historical, philosophical, and psychological perspectives. At an annual AMS/MAA meeting in Phoenix, the publisher's representative told me I could not immediately purchase the book because it was the last copy she had at the booth. At that very moment, a booming voice proclaimed, "Give her the book!" It was Reuben. I could barely find my voice, when this big bear of a man then invited me to discuss the book over coffee. It would be the first of many experiences and conversations we shared.

Reuben and I corresponded frequently in the decades that followed. His letters would always bring a smile to my face. He would apologize for his atrocious handwriting, explaining, for example, it was because it was 1:30 in the morning. He would scrunch sentences together, positioning them around the borders of a sheet of ruled loose-leaf paper, saying "Sorry, running out of paper." Reuben often asked and answered his own questions. When trying to encourage me to come for a few months to work with him, he said, "Since you've never been to New Mexico, I would say you owe it to yourself to visit Santa Fe. Don't you agree? Of course you do."

Reuben embraced me as a friend and collaborator. I am among many who benefitted from his generosity of spirit – always there to listen, argue, and inspire. He truly embraced the power of making mathematics intelligible through social interaction. I often sent him papers asking for his comments. It was always my honor when he frequently sent me his papers to review.

Our conversations extended beyond mathematics. He was adept at deepening my understanding of whatever topic we were discussing. One of my favorite letters from Reuben is one in which he shared his thoughts about a short story by Anton Chekhov that I had been urging him to read. Reuben wrote,

> I was frustrated in Phoenix because I never got to tell you that

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[2] This 1981 book won a National Book Award in Science in 1983.

> I have finally read "The lady with the dog". I really wanted to
> talk with you about it. Both the art of Chekhov's fiction and his
> vision of human life. Joy as inseparable from tragedy. Reality
> indistinguishable from the imaginary and the ideal. Maybe even
> good and evil inseparable, impossible without each other, then.

Reuben's largesse expressed itself in his attitude toward his students. He
once told me, "What makes teaching so enjoyable for me nowadays" is that
students want to talk about "all sorts of things besides the material I'm
teaching. Sometimes they have big problems, and I may be able to help a
little."

Making a Difference – Endeavoring to Contribute

The story of Reuben's journey to mathematics reveals this commitment to
help — to make a difference in our world. He was the first in his family to
get a university degree. After graduating from Harvard at age 19, earning
a B.A. with honors in English literature in 1946, Reuben sought a career in
journalism. He revealed to me that he made this choice because he hoped to
influence the world in a positive way. That not happening to his satisfaction,
he then took a job as a lathe operator, believing, in his words, that at least he
could contribute something "concrete" to the world. An industrial accident
cut that career short. He next became a graduate student at the Courant
Institute of Mathematical Sciences at New York University. Reuben earned
his PhD in 1962, writing his thesis on hyperbolic partial differential equations,
under the direction of Peter Lax. Apparently, Peter had invited Reuben to
be his thesis student when he was enrolled in Peter's real variables course.
Reuben dryly mused (cf. [20, page 4]), "I assume my homework must have
been above average."

Reuben may not have achieved all he hoped for as a journalist or machinist.
However, in his long career in the Mathematics Department at the University
of New Mexico, through his research, publications, talks, teaching, and social
interactions, Reuben indeed was able to make a difference.

In 2017, Bharath Sriraman invited a group of scholars, whose work Reuben
had identified as having influenced him, to contribute essays for a *Festschrift*
[21] – entitled *Humanizing Mathematics and Its Philosophy* – on the occasion
of Reuben's 90th birthday. Sriraman reported that Reuben had agreed to

the publication of the book provided he would be able to "shape this in order to break convention and to be different!" [19, page *vi*]

To accomplish this objective, Reuben had posed to the potential contributors a series of questions about the future of mathematics research, mathematics education, and philosophy of mathematics emanating from this provocative statement that Paul Cohen had made: "At some unspecified future time, mathematicians would be replaced by computers."[3] Reuben asked, in the context of Cohen's premise, "Will the computer change mathematical research, mathematical philosophy, mathematical teaching?"

The mathematicians, philosophers, logicians, and linguists who contributed to the Festschrift offered widely diverse responses to that question. In what have been described as "colorful and eclectic" essays [1, page 1425], the contributors expressed "heartfelt appreciation" [19, page *vi*] of Reuben and his Humanist philosophy.

Challenging the Status Quo: Exposing the "Back" of Mathematics

Reuben once noted that his intellectual life centered on categories and evaluative modes learned as a graduate student, about which he was only dimly conscious. "They were part of the way I saw the world, not part of the world I was looking at." [2, page 1]

He described his early career as that of a "conventional" mathematician who "proved things," largely in his field. Indeed, reflecting on his conventional "mathematical self," he had made this observation: "I find mathematics an infinitely complex and mysterious world; exploring it is an addiction from which I hope never to be cured. In this I am a mathematician like all others." [2, page 2]

By the 1970s, however, Reuben began to explore the "unconventional" side of himself. In the decades that followed, he distinguished between the "front" and the "back" of mathematics — the former consisting of polished results and the latter consisting of what mathematicians must do to obtain them. The "front" is the traditional face of mathematics. The "back" is its human face. Reuben developed his humanistic philosophy, focusing on the "back" of

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[3] Reuben did not specify when, but noted that Cohen's prediction "enraged him" [19, page *vi*].

mathematics. He endeavored, in a myriad of ways, to share his wonderment and fascination about the meaning and purpose, the why and how, of doing mathematics.

Reuben and I spoke often over the years about these two aspects of mathematics, largely in relation to teaching. Students exposed to the "back" of mathematics come to appreciate that mathematics is not infallible nor unique, not physical nor mental. Instead, it is like all those very real things that are real only as part of collective human consciousness.

In deciding what to write for Reuben's *Festschrift*, I chose to explore the "back" of a famous theorem concerning the precise number of points of intersection of two plane curves. Despite the multitudes of mathematicians who contributed to the evolution of thought over centuries about this theorem, it bears the name of the eighteenth century French algebraist, Étienne Bézout who proved and generalized it. I had only experienced Bézout's theorem as part of the "front" of mathematics, "attached to a number of theorems in algebraic geometry concerning intersections of arbitrary cycles" (formal sums of irreducible varieties) on projective space "and often to more general situations whenever an intersection ring of a variety is explicitly computed."[4, page 152] When I began researching the history of the theorem, I truly understood its "human face." It was my joy to share with Reuben, in one of our last conversations, my debt to him for encouraging me to pursue its "back." It turned out to be a fascinating story, rich in human activity replete with twists and turns, truths and untruths, failure and successes, derivations and innovations, and multiple examples of social interactions among mathematicians conversing over decades and centuries. See [18].

Implementing the Humanistic Philosophy in the Classroom: Teaching Mathematics Humanistically and Teaching Humanistic Mathematics

In 1987, at my invitation, Reuben gave a series of talks at California State University Northridge. In one of them, he explained his humanistic philosophy in the context of teaching. I was teaching a general education mathematics class at that time using *The Mathematical Experience* [2]. I gave the students in my class an assignment to attend Reuben's presentation and write their evaluation of it.

During his lecture, Reuben discussed different teaching methodologies, and asked the students to vote on which they preferred, and to what degree. One student indicated that she voted for the total humanistic teaching method because she felt that it created an open and creative classroom atmosphere, and "who knows?, you could possibly discover new ways of doing old problems and brand new concepts." She described Reuben as a "very laid-back guy who doesn't want to push his views on us — just open new doors for us to look into."

Another student indicated he wished Reuben could have stayed longer because he was so interesting. Another wrote,

> All the math classes I've had up to this point have been non-humanistic. And if I missed something (which happened) the rest of the mathematics was lost to me). If I had had a humanist Math teacher before now, I wouldn't have this math phobia and would understand math. I really appreciate this type of teaching as I am understanding math for the first time in about 10 years.

In the 1990s Reuben sent a two-page document to me, entitled "My Opinions on Teaching College Math".[4] He wrote,

> My opinions and methods have changed a lot since I started teaching in 1957. Then I thought of my job as teaching the subject, or teaching the material. The main thing was how much material I covered. Now, I think of my job as teaching the students. The main thing is how much the students accomplish.

The change in focus from subject to students is at the heart of Reuben's humanistic philosophy of teaching. In this context, he defined the term "humanistic" in two different, but complementary ways — as "humane-ness" in teaching — treating students as if they were human beings, and as linking mathematics education with the humanities. Emphasizing mathematics as fallible, evolving, and real, as part of collective human consciousness, brings the subject down to earth for students, and makes it accessible to them psychologically. It increases the likelihood that someone can learn mathematics, because "it's just one of the things that people do."[6, pages 1, 4]

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[4] Reuben retired in 1995, but continued to teach until 2009, when he was 81 years old.

Reuben and Phil had written *The Mathematical Experience* [2] in 1981 for general audiences. I found it wonderfully applicable for use as a textbook in general education classes as well as in capstone courses for students majoring in mathematics, science, and philosophy of science, and for prospective teachers of those subjects. Reuben, Phil, and I conversed often about assignments for these courses. Then, in the mid-1990s, they invited me to collaborate with them on a textbook version of the book.[5] Reuben submitted our proposal to the publisher, writing,

> The benefit to students will be in improving their understanding of what mathematics is and what it's good for, in repairing their damaged self-esteem caused by disasters in earlier mathematics courses, and in providing a substantial addition to their knowledge of human culture, not only of mathematics and science but also of history and philosophy.

These words reflect Reuben's sense of responsibility for students' psyches as well as their learning. Responsibility was a term that he frequently used. I once asked Reuben why he and Phil decided to write *The Mathematical Experience* using "one voice." He replied, "We agreed to be jointly responsible for each other."

Sharing his Humanism – Opening a Window to his Life

Reuben was born in the Bronx, New York, to Phillip and Mildred Hersh. He had one sister Deena. He and his wife, Phyllis, had two children, Daniel and Eva, and three grandchildren, David, Jessica, and Ze'ev. Reuben often opened a window to his personal life by circulating letters (advising the recipients of his intention to do so). Knowing Reuben, I do believe he would not disapprove of my revealing some of the correspondence with his mother that he shared with me.

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[5] This eventually became [3]. I was fortunate to be in close contact with Phil Davis until he died on March 13, 2018. In addition to our collaboration on *The Mathematical Experience Study Edition*, our love of Pappus's theorem, Nathan's mustard, and humanistic mathematics were among the many things that bound us to one another. Just four months before he died, Phil had sent me "a few ideas ... that may interest you as educational possibilities" which he described as "A humanistic approach to the mathematics in a newspaper, often on the front page."

One of my favorite letters was one Reuben wrote to his mom on June 7, 1992. He had been invited by Anatoli Swishchuk, currently a professor at the University of Calgary, to give a talk at a conference sponsored by the Mathematics Institute of the Ukranian Academy of Science in Katsively, a village in the Crimea. Reuben wrote, "People there kept referring to me as a 'famous mathematician,' and using the term "random evolution," which was first thought of by me and Richard Griego way back in 1967."[6] After the conference concluded, Reuben took the opportunity to visit his mother's birthplace; a small village in the Ukraine now called "Butzni." In the remaining five pages of the letter, Reuben described for his mother, in great detail, the state of the village and his interactions with its inhabitants. Reuben revealed that he was told no Jewish people resided in the village since World War II.

Reuben described himself as a "New York Jew" [10, page *xv*] who grew up in the Depression decades. He wrote an article [8] about transformations in American society, and in particular in its mathematical community, with respect to Jews, as a consequence of World War II. Reuben wrote,

> When I studied at the Courant Institute of NYU from 1957 to 1962, its Jewish (specifically, Ashkenazi) flavor was impossible to miss. Of course, it was in large part the creation of Richard Courant, who came to New York in 1934 as a Jewish refugee expelled by Adolf Hitler from his post as the leader of the great and famous mathematical school at Góttingen in Germany. ... Of the younger members of the brilliant faculty, Peter Lax (my mentor and adviser) and his good friend Louis Nirenberg, world leaders in their specialty of partial differential equations, were themselves graduates of NYU (and Jewish). [8, page 2]

In 2015, Reuben wrote a wonderfully detailed and beautifully illustrated account of the life and career of his mentor and advisor – *Peter Lax, Mathematician: An Illustrated Memoir* [10]. James (Jim) D. Lax, M.D., the son of Peter, and his wife Anneli (my thesis advisor! see [17]) was an important consultant for Reuben during the process of its publication. Jim recently revealed to me that during one of their conversations, Reuben had explained what motivated him to write about mathematicians and mathematics for the layperson.

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[6] Professor Griego was Reuben's colleague at the University of New Mexico.

It was to "show that they were no different from other people."

Within the pages of Peter's biography [10], there is a reference to Dr. Ferenc Polgár (pages 12, 87), from whom we can trace a path to Reuben. Dr. Polgár was chief of radiology at the Jewish Hospital in Budapest, Hungary, where Jim's grandfather (Peter's father), the brilliant diagnostician Dr. Henry Lax served as chief of medicine. The Lax family narrowly escaped the devastation of the Holocaust (see §1). They emigrated in 1941. However, the Polgárs delayed. Thankfully, Dr. Henry Lax was able to sponsor their emigration in 1946 (page 12).

Jim recently discovered a cache of post-war letters that his grandmother Klara had saved. Among them were those written to his grandfather Henry from Dr. Polgár. Jim forwarded the letters to Reuben before he died. What brings Reuben into the picture is that Dr. Polgár's daughter, the linguist Vera John-Steiner (née Polgár) was Reuben's partner and collaborator. Reuben forwarded the letters Jim had sent to Vera's family, as by that time, sadly, Vera had passed away.

An Enduring Focus of his Humanist View – The Social, Historical-Cultural Realm of Mathematical Entities

Reuben contributed four essays to the 2017 *Festschrift*, the first three of which are re-publications (cf. [11, 12, 13]). The fourth [14] is a three-page summary of his position on the nature of mathematical entities. It is not surprising that Reuben would choose this topic for his only original essay of the collection. It was a topic that had engaged his attention for decades—see for example [2, 6, 7].

In Chapter 8 of *The Mathematical Experience* [2], entitled "Mathematical Reality", Reuben endeavored to examine examples of mathematical work to see "what philosophical lessons we can draw from them" (page 404).[7] In an epilogue, entitled "Philosophical Afterword," added to the 2012 revised Study Edition of the book as pages 492–496, Reuben reflected once again on the "major philosophical dilemma ("fic-tionalism" vs. "realism") with respect to the nature of mathematical entities. He noted that in the

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[7] That chapter also appeared in the Study Editions (1995) and (2012) [3].

intervening years since the first publication of the book in 1981, scholars such as Paolo Mancosu (cf. [16]) had recognized "actual mathematical practice as a legitimate philosophical topic" (page 492). Citing the "famous anthropologist Leslie White," Reuben claimed a solution to the dilemma, proposing a third major philosophical realm, the "cultural" (or public or intersubjective), to take its place alongside the two traditional ones on subjective (private or interior) and physical (material or exterior).

For Reuben, "mathematical entities exist and have three principle manifestations - social, mental, and neural - none of which can stand alone" [14, page 363]. He stressed the humanism of knowing something that is manifested in multiple ways, indicating that these multiplicities

> are the different ways we know things — any kind of things — including mathematical things which are manifested as cultural items, as personal experience, and/or currents in our flesh and blood. [14, page 363]

In the preface to *Humanizing Mathematics and Its Philosophy* [19, page *vi*], its editor noted, "Reuben Hersh epitomizes the phrase 'Humanist mathematician and philosopher'." A reviewer of the book, Joseph Auslander, contextualized this characterization observing that Reuben's "Humanist view is probably the most cogent expression contrasting the dominant philosophies of Platonism and Formalism." [1, page 1424]

A Tribute in the *Journal of Humanistic Mathematics* for an Ethical, Self-Reflective, and Humble Man

Fewer than two years before he died, Reuben asked me to review a talk he was planning to give at Cambridge. He was writing about ethics in the sense of it being a value system for his own conduct.[8] Reuben explained that for him, ethics means simply "doing the right thing." Yet, he observed that while "it seems so simple and straightforward," it is really "so dubious, so ambiguous, and so undecidable." Reuben lamented, "The thing is, when I look at my life in math education, and beyond that at my long history of attempts to help make this a better world, I see the same painful duality. Curative or palliative?" The call for "a humane, humanistic, student-centered pedagogy"

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[8] He had already published an article in 1990 entitled "Mathematics and Ethics" [5].

to counter the "evils of formalized, routinized, soulless math education", has resulted in disappointments and unintended consequences which have "left many of my much depleted generation, with no faith that we know how to carry out a cure. . . . Yet we cannot and do not give up all activity. Where we can manage it, we try for any reforms that are likely to improve the situation, lessen the pain and damage and make it all go on a few years longer. In other words, we have given up on the curative mode, and are in a palliative one."

Reuben was so focused on being responsible to the world, making it a better place. He saw mathematics education as integral to that effort. These comments made in the twilight of his life suggest that in his own mind he was unable to live up to his own expectations for his doing that.

My assessment is different. I am grateful, personally, for the windows Reuben opened in my world — with respect to mathematics and beyond. In this, I am certain I am not alone. I am especially appreciative that I was invited to write this tribute to Reuben for the *Journal of Humanistic Mathematics*.

When we were preparing for the 2012 study edition of *The Mathematical Experience*, Reuben, Phil, and I were in conversation about a possible article for the journal to talk about new epilogues we were adding. On October 26, 2011, Reuben wrote,

> I think it would be fine if you or Phil wrote something for the Humanistic Math J. about our new edition. I would rather not take it on, as I am in the middle of two other demanding projects. Also, I have had articles in the first two issues of the Hum. M. J., so perhaps I am in danger of "over-exposure" there.

Mark Huber and Gizem Karaali have described the focus of the journal in this way, "The term humanistic mathematics could include a broad range of topics; for our purposes, it means "the human face of mathematics." Thus, our emphasis is on the aesthetic, cultural, historical, literary, pedagogical, philosophical, psychological, and sociological aspects as we look at mathematics as a human endeavor."[9] I do not believe Reuben was in danger of being "over-exposed" here — neither as a frequent contributor nor as an editorial advisor — nor in this tribute to him.[10]

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[9] https://scholarship.claremont.edu/jhm/about.html; also see [15].
[10] EDITORS' NOTE: Editors agree with the author.

On March 23, 2018, Reuben emailed me from Brookdale, a senior residence in Santa Fe, where he was living. He described several of his ailments, and with benign resignation asked, "But what do you expect, 90 years old? A miracle???" This time Reuben did not answer his own question. I would certainly have said "yes." Would that he would still be still here to contribute.

Rest in Peace Reuben.

Figure 1: Reuben in James D. Lax's New York apartment, post interview with Peter Lax for [10]. Courtesy of James D. Lax.

Acknowledgement.

My heartfelt thanks to Dr. James D. Lax, for the information he shared, and most especially, for his insights and advice on how I conveyed it.

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