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Happiness in Mathematics Education: The Experiences of Preservice Elementary Teachers

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Abstract

In this paper we discuss the happiness of preservice elementary teachers (PSTs). Several times throughout a mathematics content capstone course, PSTs responded to prompts in which they described times from their past schooling experiences or during the course in which they experienced happiness or unhappiness in learning mathematics. Through thematic analysis, we examined their common experiences related to happiness and their mathematics learning. We found that PSTs' happiness is related to expectations of themselves, their teachers, their peers, and mathematics itself. The study illuminates PST beliefs about mathematics teaching, collaborative group work, and the nature of mathematical understanding.

Keywords: preservice teachers, happiness, affect, mathematics content courses.

1. Introduction

In this excerpt from Burton [1], a female mathematician describes the positive emotions associated with mathematical knowing:

> When I think I know, I feel quite euphoric. So I go out and enjoy the happiness without going back and thinking about whether

> it was right or not but enjoy the happiness. There are lots of different ways in which one understands something. The most gratifying is that sudden wave of insight in which suddenly something all becomes clear. (page 29)

Many mathematics teachers strive to create an environment in which their students have opportunities for happiness in mathematics similar to those experienced by mathematicians. However, there is a current stereotype that education (especially mathematics education) and happiness do not coincide [8]. Watson [12], in a survey of 122 preservice elementary teachers (PSTs), found that 34% of PSTs characterized mathematics as an antagonist—using words such as "monster" or "bully" to describe the subject. Moreover, because PSTs are known to have high levels of mathematics anxiety [5, 9], we infer that for many PSTs, their experiences with mathematics have been unhappy. Why have so many students experienced unhappiness in their mathematics courses? And what could teachers do to create happier mathematics learning environments?

In this paper, we report on a study in which we examined the happiness of PSTs within a capstone mathematics content course. Several times throughout the course, PSTs responded to prompts in which they described times from their past schooling experiences or during the course in which they experienced happiness or unhappiness in learning mathematics. Through thematic analysis [11], we sought to uncover the common experiences of PSTs related to happiness and their mathematics education.

2. Happiness in Mathematics Education Research

Although happiness as a topic has been discussed and studied by mathematics education scholars, to our best knowledge, there has not yet been direct research conducted on the happiness of PSTs in learning mathematics. We conjecture this is due to the subjective nature of happiness and the corresponding difficulty in operationalizing the construct. Nevertheless, happiness is an essential aspect of human experience and deserves scholarly attention [8].

Happiness came up in the work of Leung [6] as he compared and contrasted East Asian values with Western values in mathematics education. Leung found that students in the East believe that hard work and perseverance will ultimately lead to satisfaction. He wrote that East Asians seek "a contentedness derived from having put in hard work and arriving at a deep knowledge of mathematics" (page 41). Leung contrasted this valuation of struggle and perseverance to how mathematical happiness is viewed in Western schools. He found that students in the West enjoy activities "while learning mathematics" (page 41). That is, for Western students, happiness typically comes from external factors such as the teachers making learning fun through games. In the West, unhappiness associated with struggle is avoided.

More generally within mathematics education research, studies which report on students' happiness are situated within the literature on affect. This research examines the attitudes, beliefs, and emotions associated with mathematics teaching and learning [7, 10]. For insight into happiness, we look to the study of emotions. There is a consensus amongst researchers that emotions are connected to goal-directed activity. Hannula [4] explained,

> While a student is engaged in a mathematical activity, there is a continuous unconscious evaluation of the situation with respect to personal goals. This evaluation is represented as an emotion; proceeding towards goals induces positive emotions while obstacles that block the progress may induce anger, fear, sadness or other unpleasant emotions. (page 29)

We can see evidence of the interplay of goals and emotions in the research of Cobb, Yackel, and Wood [2] who reported on the emotional acts of children in a problem solving focused second-grade classroom:

> Children frequently jumped up and down, hugged each other, and rushed off to tell the teacher when they solved a particularly challenging problem. Significantly, the positive emotional acts occurred when the children completed personally challenging tasks or constructed mathematical relationships. (pages 137–138)

These children achieved their goals of mathematical problem solving and displayed positive emotional acts as a result. Cobb and colleagues compared the children's positive emotional acts to the joyous experiences described by professional mathematicians.

Research reveals that beliefs, attitude, and emotions are connected. Hannula [4], based on results of a four-year study, reported on the mathematical attitude of Rita, a lower secondary (grades 7 to 9) mathematics student. At the beginning of the study, her attitude was negative, likely due to past failures in mathematics. However, she eventually developed a positive attitude after experiencing some success and understanding in mathematics. Rita said, "mathematics has been a bit more fun, because I've been understanding it a bit more" (page 39). Hannula interpreted her positive attitude as being related to her positive emotions: "She more often than before achieved her cognitive goals and therefore her emotional experiences in the class were more pleasurable" (page 41).

This literature reveals that there may be differences in how happiness related to mathematics is experienced in different cultures [6] and that achieving mathematical understanding will result in student happiness [4]. Furthermore, meeting one's goals, whether directly or indirectly related to the mathematical activity at hand, is also expected to result in student happiness [2, 4]. Just as mathematicians experience happiness in constructing mathematical relationships [1], we know that children can experience similar happiness within the right learning environment [2].

3. Theoretical Framework

Cobb and colleagues [2] conceptualized the emotional acts of children in a problem-centered second grade classroom as being situated in a social context—emotional acts result from cognitive appraisals of local situations—the appraisals being dependent on student beliefs about the obligations and expectations of themselves as well as others in the local situation.

> For example, because the children felt obliged to figure things out and to be able to justify their answers, they did exhibit frustration, disappointment, and sometimes anger with their peers when they were denied the opportunity to do just that [engage in problem solving]. In these cases, the negative emotional acts were directed at other children and not at mathematics or the teacher. (pages 135–136)

In their work, Cobb and colleagues, after observing children's emotional acts, made inferences about children's beliefs regarding the obligations and expectations in their particular mathematics classroom. The authors distinguished between observable *emotional acts* from the internal feelings, or subjective, *emotional states*. In contrast to the work by Cobb and colleagues, we draw inferences from the self-reports of PSTs' *emotional states* rather than observed *emotional acts*. But similar to their work, we adopt the perspective that PSTs' emotions are the result of cognitive appraisals of local situations and depend upon their beliefs about the obligations of themselves, their peers, and their teacher in the mathematics classroom. We also adopt the perspective that students will experience positive emotions as they make progress towards their goals [4]. Using these perspectives, we make inferences as to the PSTs' beliefs about the obligations and expectations within the social context of their mathematics classrooms.

4. Methodology

4.1. Context

Course Info and Student Demographics. The research took place at a Hispanic serving institution in the United States within a content course for preservice elementary and middle school teachers (PSTs). The course was considered a capstone as PSTs enrolled in the course had already completed three content courses, in number and operations, geometry, and statistics, respectively. The capstone course served to not only revisit mathematical concepts learned in the previous three courses, but also make connections across the content with an emphasis on reasoning and problem solving. Class sessions were twice a week for an hour and fifty minutes each. Out of 28 PSTs who completed the course, 20 agreed to participate in this study. Of those who participated in the study, there were three males and 17 females. A diversity of races were represented in the sample. Thirteen of the participants stated explicitly during the study that they were planning to be elementary school teachers.

Course Structure. The instructor (first author) designed the course so that for the first half of each class session, PSTs worked together to solve review or "warm-up" problems typical of the content they would have seen in their

previous mathematical content courses. In an effort to mimick the problem-centered instruction of Cobb and colleagues [2], PSTs were encouraged to work collaboratively on these problems in groups of 3–4, and not move on to a new problem until everyone in the group understood the current problem and was ready to move on. There was a short time for whole-class discussion of the solutions, in which the instructor called on PSTs to verbally explain their solutions to the problems to the class. The second half of the class was devoted to student presentations of problem set exercises that had been assigned for homework. The exercises were designed to elicit reasoning and proving on concepts fundamental to elementary and middle school mathematics. PSTs completed these exercises in their personal mathematician's notebooks. PSTs generally would share their individual findings for these exercises in small groups, then volunteer to present their solutions to the class. Sometimes one PST would present a problem, and other times multiple PSTs would present the same problem. PSTs were encouraged to share different ways of solving the problems and validate the reasoning of their classmates. Most importantly, PSTs were expected to be able to clearly explain mathematical ideas. The instructor emphasized mathematical explanation both as an important role of proof [3] and as an essential act of an elementary mathematics teacher.

4.2. Data Collection

PSTs gave consent to participate in the research study at the beginning of the semester and agreed to have all of their course work analyzed for the research purpose of understanding their happiness and unhappiness in the mathematics classroom. The primary sources of data included beginning-, middle-, and end-of-the-semester student reflections, as well as bi-weekly exit tickets, in which the PSTs were asked to reflect upon the circumstances in which they were happy or unhappy learning mathematics. More specifically, on the first day of class PSTs were given the following prompt: "Think back over all the years you have been learning mathematics. Do you recall times when you were happy learning mathematics? Do you recall times when you were unhappy learning mathematics?" At the midway point of the semester, PSTs submitted a midterm questionnaire in which they responded to the prompt: "Please describe in detail any aspects of the course that have made

you happy or unhappy." Finally at the end of the semester, PSTs responded to the following prompt:

> Think back over *this semester* as you have been learning mathematics. Do you recall times when you were happy learning mathematics? Do you recall times when you were unhappy learning mathematics? What was your happiest moment? What was your unhappiest moment?

Throughout the semester, PSTs also wrote bi-weekly exit tickets in which they wrote about whether they were happy or unhappy learning mathematics during class. All of these PST reflections were collected and analyzed as data for this research study.

4.3. Analysis

Two research questions informed this study—the first descriptive and the second interpretive: 1) How do PSTs in a mathematics content course describe their happiness/unhappiness as it relates to mathematics? and 2) How is PSTs' happiness influenced by the obligations and expectations of the mathematics classroom? The purpose of analysis was to uncover and summarize the general trends of PSTs' experiences of happiness in relation to the teaching and learning of mathematics and understand those experiences in terms of their goals and the obligations and expectations within the mathematics classroom.

First stage of analysis. The first author (JP) collected all of the data and then transcribed all of the reflections that were related to a PST's happiness. After transcribing all relevant quotations, he then wrote a summary paragraph for each PST, detailing how the PST described their happiness and unhappiness as it related to mathematics. Throughout this process, JP kept analytic memos, noting when he noticed a trend in the data, or perhaps taking time to write about possible implications. He also began to create a list of the things that PSTs cited as related to their happiness and unhappiness. Next, he read through the summaries for all of the PSTs, adding to the list any other ideas that were related to PSTs' happiness. He then sorted this list into themes [11]. Next he went back to the transcribed quotes and sought to identify all of the PST quotes relevant to each theme.

In summary, during the first stage of analysis, JP transcribed relevant data, created a summary of each PSTs' happy / unhappy moments / tendencies, looked across the summaries for trends, and organized PST quotations according to themes [11].

Second stage of analysis. JP then conveyed his preliminary findings to a second researcher (second author KD). Then KD went back to the transcribed quotations and summaries, wrote a summary paragraph for each PST based on the quotations, and noted their own takeaways from the data. We then discussed preliminary findings and began to review related literature in mathematics education. After this discussion we decided to reanalyze the data together through the lens of expectations and obligations [2, 4]. We reviewed and discussed the summaries and reflection quotations for each PST, paying explicit attention to the ways in which PSTs' happiness was related to their expectations of themselves, their peers, and the teacher within the mathematics classroom. During this phase, we also re-sorted the PST data quotations according to themes. Finally, using our analysis documents as a guide, we wrote up the following results section, with the goal of conveying the major themes expressed in the data.

5. Results

Before continuing, we ask that the reader consider the question: What makes the students you teach happy or unhappy when learning mathematics? After considering the question yourself, consider whether our findings align with your expectations.

After searching for general themes in PSTs' responses, we interpreted PSTs' descriptions of their happiness through a lens of expectations and obligations within the mathematics classroom. We found that PSTs' happiness is related to expectations of themselves, their teachers, their peers, and mathematics itself. In the following, PSTs' names have been replaced by pseudonyms.

5.1. PSTs' Expectations of their Mathematics Teachers

On the first day of class, PSTs recalled their past experiences of happiness and unhappiness when they were learning mathematics. In those reflections,

many PSTs remembered their mathematics teachers. PSTs' descriptions of those teachers alluded to beliefs about *good* and *bad* mathematics teaching—good mathematics teachers should explain clearly and thoroughly, should care about students, make learning fun with manipulatives, should not move to a new topic until all students understand the current topic, etc.... Above all, good teachers make sure students understand mathematics. These expectations are reflected in many of the students' first day reflections.

- When I was small I had a great math teacher who did everything possible so her students could understand math. She used objects to teach us how to add and multiply. Her class was great so I brought in beans and rice, that's how I started counting and adding. I will always remember this as a happy math moment. (Luisa)

- In high school, I had horrible teachers who either didn't teach, or assumed we knew what s/he was talking about and went way too fast. Then when questions were asked they treated students like they were dumb for not knowing the answer. In college my stats professor was horrible, constantly making mistakes, didn't seem to know the material, definitely couldn't teach the material – and yet still expected you to know everything for the exams. (Melissa)

PSTs' happiness in the mathematics content course under study was also related to their expectations of the teacher and how mathematics should be taught. PSTs were unhappy when the instructor did not go over problems they were confused about or when the class took too long to work on a mathematics problem. PSTs were happy when the teacher had a caring attitude and took extra steps to meet the needs of the PSTs; such as creating challenge problems for those who completed their work early, or having a caring attitude in the classroom.

> [I was happy] when you gave us the bonus problems to work on. (I really appreciated you taking the time to make this for those who wanted an extra grade boost or challenge). The tests were always what you told us to expect, yet still challenging us to think outside the box. I enjoyed your enthusiasm for math with every problem. The way you would approach telling people they were

> wrong was very cool. You wanted people to understand where we were coming from before correction. (Ariana)

5.2. Expectations of Mathematical Problem Solving

Unhappiness related to new mathematical expectations. For most PSTs, the course was different from other mathematics courses they had taken in the past. They were used to solving mathematics problems quickly and were not accustomed to spending a lot of time on a problem or going in-depth. For instance, Ariana wrote that she was "Unhappy when we talked for 45 minutes on one problem." This sentiment was shared by several other PSTs. We infer that the mathematical norms in their previous classes conflicted with the norms the teacher was attempting to negotiate in the content class. Students were required to solve challenging problems that required searching for patterns and finding novel solutions. They also were expected to justify and discuss their results. This was not normal for most students who were used to finishing problems quickly without discussion.

Early in the semester Mckenzie wrote "The uncertainty of solving problems makes me very unhappy." This PST was a mathematics minor and enjoyed completing exercises for which she understood the mathematical steps. But to work on challenging problems for more than several minutes was a challenge for Mckenzie and other PSTs. This unhappiness was not necessarily caused by boredom, but because PSTs found it confusing to explore a problem deeply. Beth wrote, "I would become unhappy when we spent too much time on one problem because I went from understanding it to confused back to understanding back to confused." The deeper the discussion and the more perspectives considered on a problem, the more confused Beth became. She wanted the comfort of knowing she understood the mathematics, but this comfort was compromised when she was forced to look at problems on a deeper level. Bernice, at the end of the semester, provides a succinct summary of how she felt in the new mathematical environment:

> I have always liked math class but I never thought why I liked math. After this semester I have a much clearer idea as to why I enjoyed mathematics as a child, I liked getting the right answer. I never questioned how or why a formula worked, I only cared

> about being sure how to apply it to get an answer. When I began the semester and we started problem set #1, I was very unhappy. It forced me to rethink what I knew about mathematics and look for patterns I never bothered to look for before. It was an extremely difficult process at first. Once we started doing the in class presentations and group work, I began to embrace this new way of mathematical thinking. (Bernice)

For PSTs like Mckenzie, Beth, and Bernice, they had come to interact with mathematics problems in a certain way in their previous classes. Their new experiences with problem solving did not meet their expectations for mathematics, and resulted in unhappiness. The problems in the course were challenging and required more creative thinking than those they were used to solving. Eventually they adjusted to the new expectations, and found happiness in the course. At the end of the semester, Beth wrote,

> [I was happy] when I finally understood a concept we were working on after I had struggled with it. I don't have any specific examples because I feel like it happened many times throughout the semester. There were some problems that I kept working on and trying different things just to see what would happen. (Beth)

Happiness from deeper understanding of mathematics. Many PSTs came to embrace this new way of mathematical understanding, and found it a welcome reprise from what they expected of mathematics. The classroom activities that led to deep understanding resulted in happiness. One PST, Melissa struggled with the course originally, finding it difficult to meet expectations. But during the middle of the semester she adjusted and became happier:

> I actually like what we're doing now – seemingly simple concepts that on the surface seem easily doable, but once you get into it, there's so much more to answering the question than originally thought. I think it's interesting to look at the same problem multiple ways – initially confusing – but interesting. (Melissa)

Many students enjoyed exploring concepts deeply, and the course was a refreshing change of pace from their prior mathematics courses. These students

often contrasted the course to their prior courses, emphasizing the course activities that resulted in a deeper understanding. For instance, Jillian wrote,

> I enjoy learning about math in these types of classroom environments where there's open discussion about concepts and big ideas rather than strictly lecture, note-taking and drill exercises. It helps me understand the why, not just how. (Jillian)

Jillian was happier understanding mathematics deeply, and the structure of the course, with its emphasis on discussion of concepts and big ideas rather than lecture, helped her achieve that goal. Many students enjoyed "understanding the why" and contrasted this experience with prior mathematical experiences.

> I was proud of the way I was able to understand why distribution works, as opposed to just doing it because I was taught that it works. I appreciate that this class teaches the reasoning behind why mathematics works, as opposed to just telling us that it works and leaving it at that. (Peggy)

Another PST, Susie, never had much success in mathematics classes before, and confided that she always needed tutors to succeed in her previous mathematics courses. It was difficult for her to achieve mathematical understanding in lecture classes where she had to rely on memorization. But in the content course, she had new opportunities for mathematical creativity.

> I like this new style of learning math and I enjoy trying to figure out proofs and getting to be creative. I also am happy watching others present because they have different methods that I never thought of. This semester's class has been the time I was happiest learning mathematics because it was different from all my other math classes. I enjoyed trying to solve problems by myself (and usually with my groups). I also enjoyed the times when our class was able to have a discussion about the challenging problems. (Susie)

PSTs like Susie found it easier to accomplish the mathematical goals in the content course than in previous mathematics courses. As the emphasis in

the course was understanding why mathematical claims were true, and explaining those claims to others, deep understanding, and thus happiness was achieved. PSTs were happier when they saw multiple solution paths and representations of mathematical problems. Seeing multiple solutions, and hearing their classmates' explain them, helped the PSTs to meet their goal of mathematical understanding. In an exit ticket, Enrique explained,

> My favorite part of today's lesson was how different minds can see things in a much different way. People's thought processes are much different from others—we all had different charts and representations of the patterns of triangular numbers. I was happy today. We tackled challenging warm up exercises that had us a bit confused initially but as we worked we came up with the answer. (Enrique)

6. Social Aspects of Students' Happiness

For the most part, increased collaboration led to opportunities for deeper understanding and thus to happiness. PSTs were happy when they achieved new understanding through group discussion, and happy explaining ideas and helping classmates with their work. For instance, Priscila wrote "I was happy when we were in groups because when I was stuck they would help me and explain the problem and solution in a way that I could understand. I was also happy when I was able to help them solve a problem."

While there was significant happiness that came from working in small groups, there was also unhappiness. Student small groups were randomized at the beginning of every class meeting, so each day students worked with different classmates. Each day, the instructor (JP) told the class that small group members should work on problems collaboratively, and not move on to a new problem before all group members were ready. This expectation was even explicitly stated at the top of the warm-up handouts. We believe that students have expectations of their peers as well as themselves in the classroom and breaches of these expectations resulted in unhappiness. The following quote from Estrella demonstrates how her unhappiness stemmed from peers' actions which we interpreted as violating her expectations:

> I get unhappy when I am the slowest of my group members and when group members don't really care for what I or others have to say. I dislike when "future teachers" see this as a competition and rush even when their members are falling behind. I dislike it when team members are not open to hearing new/other approaches. I get happy when I understand the problems, when I learn, when I get grouped with kind and helpful group members, when I help others and when I present. I get happy when I receive positive feedback and constructive criticism. (Estrella)

Estrella was one of several students who described unhappiness resulting from working in small groups with group members who left them behind and/or disregarded what they had to say. Students described these experiences in terms of feeling "dumb" as they were not working "at the same pace as others." Other students described unhappiness from being grouped with unfriendly people who were not willing to slow down or help. This unhappiness, we believe, is related not only to expectations of others, but also to the expectations a student puts upon themselves and what they believe are expectations that others hold them to.

> Some students know more than others. So they tend to finish the work faster and at times they don't wait for the rest of the group. That adds a lot of pressure to some of us. There was a day when I sat down with a group of students in class. You passed the warm up problems and I realized I needed help with some of the problems. To my surprise, the rest of the group understood the problems and finished quickly, however, when I asked for help they looked at me differently and I felt unhappy and lost. From that day on I requested warm ups to be posted online so I could review them before coming to class. (Luisa)

Luisa's expectation was that her group members should help her and complete the problems together as the teacher instructed. When she is not helped and feels that her group members look down on her, she is unhappy. While Estrella's unhappiness (see above) was directed at her group members, Luisa's seems directed at herself, perhaps in response to perceived social expectations. She felt pressure to keep up with others and worried about other

slowing group members down. In response, she made a plan to self-study and complete assignments before she works on them in small groups.

7. Personal Accomplishments and Expectations of Self

We found that PSTs described many happy moments as times when they were meeting self-expectations: fulfilling their goals, meeting classroom expectations, and achieving personal accomplishments. Happiness was experienced when PSTs presented a problem at the board for the first time, did well on tests, found a solution to a challenging problem after extended effort, and so on.

- I was happy when I was able to explain base 5 to my table. (Celeste)
- I was happy when I passed the second test we had this semester. Also I was happy to understand the multiplication problem sets and learned how to explain them. It was difficult for me to understand angles when I was in middle school and I felt more confident about angles in problem set 3. (Anita)
- Happy coming up with unorthodox proofs to supplement what the class finds. (Mark)

Analogously, unhappy moments came from not being able to meet expectations or facing obstacles in the path towards achieving their goals: for example: failing to come up with a satisfactory explanation, making a mistake when presenting, being unable to find a problem solution, etc...

- Times when I was unhappy were when I felt like I was not understanding. It was definitely difficult coming up with general proofs. Unhappiest moment was when I got my first test back and received an 80%. (Gabriella)
- Not very happy when I present, I feel like I forget things when I go up to the board. (Luisa)

There are many more examples of these types of descriptions within the data. Students are happy when they meet their own expectations. These are

usually expectations within the class—producing satisfactory explanations, solving a mathematics problem, etc... We see our students are happy when they meet the expectation of understanding mathematics, and unhappy when they do not.

8. Happiness When the Class Prepares PSTs to Become Teachers

PSTs desire to become teachers, and they expect that their university courses will prepare them to be effective teachers. Although the course under study was a content rather than a methods course, many students hoped that the course would support them in learning how to be an effective mathematics teacher. When the class met these expectations, PSTs were happy. These expectations were met through reading practitioner articles from *Teaching Children Mathematics* or chapters from *Mathematical Mindsets* by Jo Boaler (2015). Peggy described the happiness she got from these readings when she wrote, "What I have enjoyed the most is the articles that we read every week. They give an insight into real teaching situations that we would otherwise have to experience for the first time on our own."

Other students were happy that they had the chance to explain mathematics to their classmates, recognizing that, even though it was difficult, it was an important skill to learn as a future teacher. Bernice wrote, "As a future teacher, it is only natural that my happiest moments doing mathematics during the semester were when I was able to teach someone else." A few others reflected that the class helped them to see a different perspective of mathematics, and would influence their teaching in the future. For instance, Maritza wrote,

> Times that I recall when I was happy was when I was able to understand a problem and was able to explain it to other people. It also made me happy that this classroom wasn't held as a right or wrong class. The professor heard everyone's input and even when they were off track he tried to understand as to why it happened and what we can do differently next time; he even sometimes took that information and made us think about if a student did that how we would explain it to them. This class also helped and made me happy because it gave me a different

> perspective on mathematics and how I want to teach it in the future. The professor gave us problems such as "Why is explaining place value important?" which was helpful to future teachers because it gave us a different point of view and definitely made me appreciate more as to why each concept is important. (Maritza)

9. Implications for Teacher Educators

What are the implications for instructors of PSTs in mathematics content courses who wish to create happy learning environments? As student happiness is connected to classroom expectations and pursuing goals related to those expectations, it is paramount that teacher educators make the classroom expectations clear. Norms take time to establish, but we believe clear expectations will increase the likelihood of student happiness. Consider how Cobb and colleagues [2] described the implications of their study.

> The most general implication of our work is that the teacher should negotiate the social context within which children attempt to solve mathematical problems and thus influence their beliefs about their own and the teacher's roles and the nature of mathematical activity. The objective is for both the teacher and the students to create a social context in which construals that warrant detrimental negative emotions such as frustration are simply not made while solving mathematical problems. (pages 143–144)

PSTs in the mathematics content course struggled at first to meet the expectations required—to engage in both mathematical problem solving and mathematical communication (that JP attempted to normalize). PSTs initially experienced unhappiness as they were asked to explore novel mathematical situations in which they had not yet been shown a method for finding a solution. And yet with time, the PSTs adjusted to the expectations and found happiness in searching for patterns, collaboration, explanation, and perseverance in problem solving. We believe it is important for instructors to be patient with PSTs, especially in the beginning of the course, and to encourage them in their gradual progress. Moreover, instructors can explain that mathematical roadblocks and struggle are to be expected during mathematical problem solving. As Cobb *et al.* [2] reported, "Children in the project

classroom quickly learned that not knowing what to do was routine" (page 133). Because the norm of not knowing contrasts with prior experiences of PSTs in a traditional classroom, frustration may be the immediate result of not knowing what to do. Teacher educators can play a role in reframing acceptable emotions for mathematical activity by normalizing mathematical problem solving, but it requires time and encouragement.

Every PST in the content course reported that they were happy when they understood mathematics. Several PSTs described happiness that came through the activities that resulted in deep conceptual understanding (e.g. looking for patterns, collaboration, seeing multiple representations). These PSTs contrasted these satisfying learning experiences to the unsatisfying experiences they had in the past when they were required to learn mathematics through memorization and by completing exercises they did not quite understand. Our data suggests that PSTs not only have the potential for exploring challenging mathematics deeply, but they ultimately find it rewarding. Of course, teacher educators should do everything in their power to promote deep mathematical understanding.

When facilitating small group work, teacher educators must be aware of the social dynamics that can result in unhappiness and work to promote productive small-group collaboration. Inspired by the preliminary findings of this study, JP asked PSTs in a subsequent content course to describe their experiences working in small groups on the first day of class, and answer whether or not the following statements described them:

- I am happy to help others who may be slow to understand a problem.
- I am annoyed when other group members are moving slowly.
- I am a slow mathematical thinker and worry about slowing other group members down.
- I like to finish my work quickly and would like to work with group members who move fast.

JP then organized small-groups according to these preferences, being careful not to put students together who might be incompatible group members. At the beginning of the class, he emphasized the importance of working together

and not leaving group members behind. JP observed that group members did a better job of adhering to the norm that all group members understand the problem before moving on to a new problem than in previous semesters. We believe that purposeful grouping, and conversations that bring awareness to the issues of unhappiness that can occur while working in small groups helped the students be more empathetic towards their group members and collaborate more effectively.

A final implication for teacher educators concerns every PST's goal of becoming a teacher. While usually reserved for methods courses, we believe instructors should incorporate at least some activities that will help PSTs feel that they are being prepared to be effective teachers. PSTs in the content course were happy when the course activities were relevant to their future teaching practice (e.g. reading practitioner articles, teaching their classmates, working on problems that were placed in a school context). Encouragingly, some PSTs wrote that the course provided them a new perspective of what mathematics instruction could be. These PSTs believed that the learning experiences would carry over into their teaching—which will hopefully result in positive mathematical experiences for children in the future.

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