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In How Many Days Will He Meet His Wife?

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Abstract

In how many days will he meet his wife? This is a question asked at the end of each of two problems embedded in the verses of the last chapter of the *Vyavahāra-gaṇita* ("Mathematics of Transaction") of Rājāditya of the twelfth century. He infuses elegance in those two problems by choosing the charming idea of a husband's meeting with his wife after their quarrel. This paper not only presents the algorithms offered by Rājāditya to solve them on their own terms as well as on modern terms and discusses the historicity of the categories of those two problems but also provides an insight into why he posed them using those three terms, namely, the wife, the husband, and the quarrel between them.

1. Introduction

In how many days will he meet his wife? This is a question asked at the end of each of two problems embedded in the verses of the last chapter of the *Vyavahāra-gaṇita* ("Mathematics of Transaction" or "Determinations regarding Mathematics" or "Business Mathematics") of Rājāditya of the twelfth century. This paper aims at understanding the algorithms offered by Rājāditya to solve these two problems on their own terms as well as on modern terms, discussing historicity of the categories of the two problems, and providing an insight into why he may have posed them using the three terms he did, namely, the wife, the husband, and the quarrel between them.

Rājāditya was a poet and mathematician of Jaina faith [10, verses 1.1-1.3, pages 1-2]. He was born at Pūvinabāge. The place was located in what is now the southern Indian state of Karnataka. It was resplendent with gardens, rivers, agricultural fields, hills and mountains [10, verses 1.5-1.9, pages 3-4].

He belonged to the exclusive class of the Jaina school of Indian mathematics [3, pages 319-324]. He was philanthropic and pleasant [10, verse 1.11, page 5]. He wrote the *Vyavahāra-gaṇita* to help laymen better deal with their day-to-day transactions [10, verse 1.14, page 6].

He is credited with six works. One of them is the one that we have mentioned above; the other five ones are the *Kṣetra-gaṇita* ("Mensuration"), the *Vyavahāra-ratna* ("Gems ⟨for Mathematics⟩ of Transaction"), the *Līlāvati* ("Gorgeous Amusement ⟨of Mathematics⟩"), the *Chitrahasuge*, and the *Jaina-gaṇita-sūtrodāharaṇa* ("Rules of Mathematics ⟨developed or referred to⟩ by the Jaina(s) with Examples") alias *Jaina-gaṇita-sūtra-ṭīkā-udāharaṇa* ("Rules of Mathematics ⟨developed or referred to⟩ by the Jainas with Commentary and Examples") [10, page *xxi*].

Many historians hold that Rājāditya, on the basis that his preceptor Śubhacandra passed away in the year 1123, flourished around 1120 in the royal court of the king Viśṇuvardhana, who reigned from 1111 to 1141, of the Hoyasal dynasty [10, pages *xxiii*–*xxiv*]. On the other hand, A. Venkatasubbiah asserts, on the basis of many inscriptions and the fact that quite a few other religious teachers bearing the name Śubhacandra flourished during different times, that Rājāditya flourished certainly around 1190 in the court of the king Varaballāḷa II, who ruled from 1173 to 1220, whom he referred to as Viśṇunṛpāla [10, pages *xxiv*–*xxv*]. It is, therefore, safe to assign him to the twelfth century, whether he flourished in its early part or in its later part [2, page 42].

The *Vyavahāra-gaṇita* is believed to be the first available mathematical text in Kannaḍa. It is composed in old Kannaḍa. It is mainly concerned with arithmetic of business deals prevalent in Rājāditya's time. It has five chapters. Chapter one is introductory in nature. Terminology is discussed in chapter two. The third chapter is on Rule of Three. Rule of Five, Odd and Mixed Quantities are discussed in chapter four. The fifth chapter is on Rules of Five, Seven and Nine. The last three chapters contain rules for ration, for winnowing of husk, for buying a certain quantity on one price and to sell the same for another price, for calculating interest, for discount, for cloth, for purifying gold, for conversion of gold, for design of floor, for log of sandalwood, for receiving equal money, for wages including those of elephants, for making box, ring and ship, for motion and so on.

The principle underlying every rule is first stated and then illustrated by a problem. At the end of every problem a brief analysis with the answer to that problem is given.

2. Two Problems

In how many days will he meet his wife? It is the question that is asked at the end of each of two problems contained in the last chapter of the *Vyavahāra-gaṇita*. The first of those problems is as follows:

> 1º "A couple quarrel «and they separated». Wife has travelled for seven days at the rate of one *gāvuda* per day. After that if the husband travels at the rate of one and half *gāvuda* per day, then in how many days will he meet his wife [10, verse 6.98, p. 264]"?

The algorithm offered by Rājāditya to solve this problem is the following. When the "value" (*dhana*, literal meaning: wealth) is divided by the remainder obtained after subtracting "constant speed" (*dhruvagati*) from "impelling speed" (*ākṣepagati*), we get the time elapsed in terms of days when those two met [10, verse 6.97, page 263].

Let us suppose that the first traveller travels with the speed of s_1. Then the second traveller starts to travel with the speed of s_2 when the first traveller has already travelled the distance of d. If t is the time required for their meeting, we have

$$ts_2 - ts_1 = d,$$

or

$$t = \frac{d}{s_2 - s_1}.$$

This is the rationale, if formulated, on which Rājāditya based his algorithm, where d, s_1, and s_2 are said to be "value" (*dhana*), "constant speed" (*dhruvagati*), and "impelling speed" (*ākṣepagati*) respectively. We shall know very soon why d was called "value" (*dhana*).

Using this formula or algorithm for $d = 7$ *gāvuda*, $s_1 = 1$ *gāvuda* per day, and $s_2 = 1 + 1/2$ *gāvuda* per day, we are able to calculate that he will take 14 days to meet his wife.

The second of the two problems is as follows:

> 2º "After becoming angry, wife travels five *gāvuda* per day. If the husband travels at the rate of one initial (*ādi*) ⟨speed⟩ and one incremental speed (*uttaragati*), then in how many days will he meet his darling [10, verse 6.103, page 268]?"[1]

The algorithm offered by Rājāditya to solve this problem is the following. Subtract "initial speed" (*ādigati*, s_2) of the following traveller from "constant speed" (*dhruvagati*, s_1) of the leading traveller. Divide the remainder by "incremental speed" (*uttaragati*, s_3) of the following traveller. If there is no remainder, then double $(s_1 - s_2)$ to obtain $2(s_1 - s_2)$, and divide it by s_3. This gives the time t elapsed in terms of days if one is added to the result [10, verse 6.100, page 266]. Thus, we have

$$\frac{2(s_1 - s_2)}{s_3} + 1 = t,$$

or

$$s_1 t = \frac{t}{2}\left[2s_2 + (t-1)s_3\right].$$

This is the formula for finding the sum (i.e., "total distance travelled" or "constant speed multiplied by meeting days" or $s_1 t$) of an arithmetic progression of which "first term" (*ādi*) is s_2, "common difference" (*uttara*) is s_3, and the "number of terms" (days) is t.

This shows that Rājāditya solved 2º by applying the approach of an arithmetic progression. Since the sum of an arithmetic progression was called "value" (*dhana*) in ancient Indian mathematics [17, verse 2.19 and under it, pages 105-106]; [1, Rule 39, page 28]; [11, verse 126, page 106] and he equated it with the distance as we noticed above, d has been called "value" (*dhana*) by him in his algorithm offered to solve 1º.

[1] The present author has inserted the terms "initial" and "incremental speed" in the statement of this problem to help the sentence flow better in English. He has also put the terms *ādi* and *uttaragati* in parentheses for the same reason though these terms are present in the sourced translation [10].

Using the algorithm for $s_1 = 5$ *gāvuda* per day, $s_2 = 1$ *gāvuda* per day, and $s_3 = 1$ *gāvuda* per day, we are able to calculate that he will take 9 days to meet his wife.

What does "if there is no remainder" mean? There may be two cases. One is the remainder obtained when s_2 is subtracted from s_1. The other is the remainder obtained when $s_1 - s_2$ is divided by s_3. In the former case when $s_1 - s_2 = 0$ or $s_1 = s_2$, nothing remains to be done further. Hence, the latter case seems to be plausible. However, neither Śrīdhara (c. 799) [16, verse 96, pages 139 and 78] nor Mahāvīra (c. 850) [12, verse 6.319, pages 106 and 177] refers to any condition of "if there is no remainder" sort.

3. Discussion

3.1. The meaning of gāvuda

Both 1º and 2º contain the term *gāvuda*. It is intelligible that it is a measure of distance. It is not contained in the list of various measures of length, using which *yojana* is defined in the *Vyavahāra-gaṇita* [10, verses 1.28, 1.33, 1.35-1.36, and 1.38, pages 11 and 13-15; also see page *xxxi*]. Either it is missing from the manuscripts of the *Vyavahāra-gaṇita* or Rājāditya himself missed it to be referred to. It seems that the term changed to suit the tongue of the Kannaḍa people from the Sanskrit term *gavyūta*.

The term *gavyūta* applies to the distance up to which the bellowing of a cow can be heard. 2000 *daṇḍas* (staffs) makes one *gavyūta*. This is stated in each of the *Jambūdvīpa-prajñapti Sūtra* ("Aphorisms for Communications of *Jambū* Island") (earlier than 362 BCE) [15, 2.25, page 29], the *Bhagavatī Sūtra* ("Aphorisms from the Venerable") (some date between 362 BCE and 466) [7, 6.7.134, pages 432-433], the *Anuyoga-dvāra Sūtra* ("Aphorisms for Entrance of Discipline") (some date between 75 and 300) [8, 345, page 257], the *Tattvārtha-vārtika* ("Explanatory Commentary on the Meaning of the Fundamental Principles") of Akalaṅka (seventh century) [5, 3.38.6, page 208], the commentary by Mādhavacandra Traividya (c. 982) on the *Trilokasāra* ("An Essence of the Three ⟨Regions of the⟩ Universe") of Nemicandra (c. 981) [9, page 22], and *the Jambūdīva-paṇṇatti-saṃgaho* ("Compendium of the Communications of the Jambū Island") of Padmanandi (c. 1000) [18, 13.32-34, page 238].

3.2. Historicity of the two problem categories

1º comes under the category in which the time of meeting of two travellers is to be found out when the traveller with fast speed begins travelling in the same direction on the same path after the traveller with slow speed has already covered a specified distance. Long before Rājāditya, the rule for solving the problems of this category is found in the *Bakhshali Manuscript* ("Treatise ⟨of Mathematics⟩ named after ⟨Village⟩ Bakhshali") (c. 400 or seventh century), the *Pāṭīgaṇita* ("Mathematics by ⟨means of⟩ Algorithms") of Śrīdhara (c. 799), and the *Gaṇita-sāra-saṅgraha* ("Compendium of Essence of Mathematics) of Mahāvīra (c. 850). Each of them contains one problem. The characters used in the problem given in the *Bakhshali Manuscript* are impersonal [13, page 89]. In the *Pāṭīgaṇita* of Śrīdhara they are no specific by name [16, verse 65, examples 81-82, pages 84-85 and 52]. In the *Gaṇita-sāra-saṅgraha* of Mahāvīra, the slow traveller is man and the fast traveller is messenger (*dūta*) [12, verses $326\frac{1}{2}$-$327\frac{1}{2}$, pages 107 and 178-179].

2º comes under the category in which the time of meeting of two travellers is to be found out when both the traveller with a constant speed and the traveller increasing his initial speed by a given quantity per day begin travelling from the same place at the same time and move in the same direction on the same path. Again long before Rājāditya, the rule for solving the problems of this category is found in the *Pāṭīgaṇita* of Śrīdhara and the *Gaṇita-sāra-saṅgraha* of Mahāvīra. They each contain one problem. Each of the two characters is man (*nara*) in the *Pāṭīgaṇita* of Śrīdhara [16, example 111, pages 139 and 78]. Similar is the case in the *Gaṇita-sāra-saṅgraha* of Mahāvīra [12, verse 320, pages 106 and 177].

Rājāditya gives two problems under the first category. One of them is 1º itself. And in the other problem, the character with slow speed is a girl while the character with fast speed is an elephant [10, verse 6.99, page 265]. There are four problems given under the second category. 2º is the third one of them. Both the character with a constant speed and the character increasing his initial speed are the same boatman in the first of those four problems [10, verse 6.101, page 266]. The second problem seems to the present author to contain the character with a constant speed to be an animal and the character increasing his initial speed to be a huntsman [10, verse 6.102, page 267].

The following is the fourth problem.

> "One pig travels a distance of fifteen *gāvuda* per day. A dog chases that pig at the rate of one initial (*ādi*) ⟨speed⟩ and one incremental (*uttara*) ⟨speed⟩ (*gāvuda*). O mathematician, tell me in how many days will that dog meet the pig [10, verse 6.105, page 269]." [2] Answer 29 days [10, page 269].

Rājāditya offers a separate algorithm to solve this problem. The algorithm, if formulated, reads $t = (2s_1 - 1)/s_1$ [10, verse 6.104, pages 268-269]. But the algorithm, when applied, does not give 29 days. The expression $t = (2s_1 - 1)/s_1$ seems to be the damaged form of an adapted formula. On the other hand, the algorithm offered to solve 2º, when applied to the fourth problem, gives 29 days. The expression $t = (2(s_1 - s_2)/s_3) + 1$ reduces to $t = 2s_1 - 1$ when adapted for $s_2 = s_3 = 1$. This adapted formula gives 29 days when applied to solve the fourth problem.

On the basis of the above historical account we can say that even if problems involving two objects or persons in motion might have been posed by his predecessors, too, Rājāditya seems to be the only mathematician who posed two problems on the subject of speed in the context of a married couple's quarrel, in the manner in which the character with a slow or constant speed is the wife and the character with a fast speed or with an increase in his initial speed is the husband.

3.3. Posing problems that connect with students

Recently, Professor Vignesh Muthuvijayan of Indian Institute of Technology, Madras, posed a question in an end-of-semester examination [6]. Students were asked to assist Chennai Super Kings captain Mahendra Singh Dhoni, a former Indian international cricketer who captained the Indian national team. More specifically they were to help Dhoni to choose whether to bat or field if he won the toss in his Indian Premier League match on the basis of psychrometric charts and dew point calculation. See Figure 1.

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[2] Once again, the present author has inserted the terms "initial" and "incremental" in the statement of this problem to help the sentence flow better in English. He has also put the terms *ādi* and *uttara* in parentheses for the same reason though these terms are present in the sourced translation [10].

BT1020 Material and Energy Balances,
Date: May 6, 2019; Time: 9:00 AM onwards
End Semester Exam (40 points)

1. (5 points) Answer the following questions:
   a. Dew can play a major role in day-night cricket matches. Excessive dew in the outfield makes the ball wet. For spinners, the challenge is to grip the wet ball and to impart spin while delivering. For fast bowlers, it is difficult to hit the desired length. Hence, this can put the fielding team at a disadvantage. During Indian Premier League 2019, Chennai Super Kings are likely to play a qualifier in Chepauk Stadium on May 7. As per the weather forecast for May 7, the relative humidity in Chennai is expected be 70%. The temperature at the start of the game is predicted to be 39˚C. At the beginning the second innings, the temperature is expected to drop to 27˚C. Based on this info, if Mahendra Singh Dhoni wins the toss, would you recommend batting or fielding first? Justify your answer. Note: Points will not be awarded without the correct justification.
   b. …[6].

Figure 1: Professor Vignesh's problem.

Professor Vignesh says that he had set the problem as a way to test students' understanding of psychrometric charts and dew point calculation. He further says that he tries to use any day-to-day reference that can catch the attention of students. Ultimately, the goal is to convey the concept that needs to be learnt [6]. The same pedagogical spirit must have inspired Rājāditya to pose 1° and 2°. "In some places," remark Padmavathamma *et al.*, "we find the poet ⟨i.e., Rājāditya⟩ to be humourous. We find this on problems related to travelling distances. He says that the husband and wife separate after becoming angry [10, page *xxxiii*]." Compare this with Professor Vignesh who says, "I designed the question [. . .] including Indian Premier League because it just makes the learning experience fun for the students [6]."

Besides being lighthearted, both 1^{o} and 2^{o} are value-based questions. In this sort of question, the question setter tries to see the understanding level of a student before whom the question is put, and the student, keeping the value or values in mind, has to apply his or her own knowledge and understanding to solve. Many different values might come into play. For example, responsible behavior, self discipline, punctuality, team work, social concern, to help, awareness, and so forth are values. In Professor Vignesh's problem, we might see the value of helping; similarly in the two problems of Rājāditya, the values of family, help, support, and forgiveness might be evoked.

3.4. Other possible explanations for the context of the problems

If we would like to know why Rājāditya posed 1^{o} and 2^{o} using a husband and wife who had a quarrel, the above consideration of making learning fun or situating a mathematical problem in a value-evoking context so as to make the mathematics more palatable seems like a plausible explanation. However there might be other explanations too. In this section we explore two alternatives.

3.4.1. A real-life quarrel

It is likely that Rājāditya got to witness at some point in his life before writing the *Vyavahāra-gaṇita* a family situation where a married couple quarreled over something, after becoming angry the wife went away, and then her husband made efforts to meet her again. Witnessing such a real-life incident may have led him to think of it as a possible setting to pose 1^{o} and 2^{o}. This possible explanation may be supported by the following view. "Hundreds of problems in this book," write Padmavathamma *et al.* in their preface to the *Vyavahāra-gaṇita*, "are taken from real life which enable us to have an idea of the socio-economic conditions which prevailed in Karnataka during the 12th century A.D. [10, page *xxiii*]." In other words, Rājāditya is known to have incorporated instances of real life into his book, so a married couple's quarrel might just have been another such instance.

3.4.2. Possible literary considerations

Now let us see if there might be any literary considerations which might have caused Rājāditya to set 1^{o} and 2^{o} in the setting of a married couple's quarrel.

"Broken string and scattered pearls" has been a motif in Indian poetry and mathematics composed in Sanskrit. It is connected with the pearl necklace, the string of which keeps breaking at every slender excuse, scattering the pearls all around. For details regarding this motif see [14, pages 463-476].

The motif is said to have been employed for the first time by Kālidāsa, a great classical Sanskrit poet and dramatist, in his minor poem *Meghadūta* ("The Cloud Messenger") in the following manner.

> "In the blessed city of Alakā, the nocturnal wanderings of the lovely ladies can be discerned the next morning from the various items that they unknowingly scattered on the path owing to the agitation in their gait: *mandāra* ⟨i.e., Coral tree's)⟩ flowers from their hair, decorative paint on their cheeks, golden lotuses with which they had decked their ears, and above all, strings of pearls that once rested on their breasts, as they hurried in darkness towards the lover; or, more appropriately, when they returned from him [14, page 466]."

The essence of a variation offered by Kālidāsa himself in his epic poem *Kumārasambhava* ('The Birth of a Son') on this motif reads as follows.

> "On the mountain Kailāsa, *Siddhas* ⟨i.e., the blessed ones⟩ live in houses made of rock crystal. The stars reflecting in these crystal palaces look as though they are the pearls, spilled from the necklaces that broke during lovemaking [14, page 467]."

This motif was also used by Padmagupta alias Parimala (c. 1005) in his *Navasāhasāṅkacarita* ("Conduct of New Sāhasāṅka") to describe the opulence of Ujjayinī [14, page 467]. Ujjayinī was an ancient city besides the Kṣiprā River. It is now known as Ujjain located in what is now the central Indian state of Madhya Pradesh. Mammaṭa (eleventh century) describes King Bhoja's opulence in an indirect manner in which he suggests it through the description of his court poets on whom Bhoja conferred immense wealth. His description includes the elaboration on the motif. The essence of the description from Mammaṭa's *Kāvyaprakāśa* ("Light of Poetry") is as follows.

> "In their ⟨i.e., the poets'⟩ palaces too, heaps of pearls get strewn about from necklaces broken in course of love sports. And these

too are swept with brooms into the courtyards 'in the morning after'. Not only that; coming into contact with the lac dye on the feet of women who go about there, the pearls are tinged with red. Looking at these red pearls, the pet parrots of the palace think they are pomegranate seeds and try to bite into them [14, page 468]."

Bilhaṇa was an eleventh-century Kashmiri poet. He introduces a twist in his *Vikramāṅkadevacarita* ("Conduct of Vikramāditya ⟨VI⟩") to the motif. The essence of the twist is as follows.

> "Pleasure palaces line up on both banks of the river Vitastā. In the balconies of these palaces, couples engage in uninhibited amorous sports with the usual consequence that the pearl strings break. The pearls then roll down into the river, and the river shines with these floating pearls like the Milky Way with the twinkling stars [14, page 468]."

Bhartṛhari was a Sanskrit writer of the early medieval period. While describing the importance of good deeds, Bhartṛhari says:

> "Marvellous house, vivacious women, Goddess of wealth, radiant with white parasol «reigns there, as it were». One thinks this lasts forever, when past good deeds are abundant. But as soon as these are exhausted, wealth scatters in all directions in front of your very eyes, like the rows of pearls when the string is broken in love quarrel [14, page 470]."

Before we proceed to note that the motif of the "broken string and scattered pearls" has given rise to mathematical problems as well, we briefly explain why lovemaking can be a quarrel. The *Kāmasūtra*, attributed to Vātsyāyana, is an ancient Indian Sanskrit text on pleasure-oriented faculties of human life. It avers that "the sexual union itself is a kind of quarrel (*kalaha*), because lovemaking is a kind of contest (*vivāda*) and pitiless in nature [14, page 470]." For more information see [14, pages 470-471].

Back to math! Śrīdhara (c. 799) appears to be the first mathematician who used the motif to give rise to mathematical problem. The problem posed in his *Triśatikā* ("⟨Algorithms in⟩ Three Hundred ⟨Verses⟩") is as follows.

> "In ⟨their⟩ love-quarrel (*suratakalaha*, quarrel of amorous pleasure), the pearl-necklace (*muktā-hāra*) of a certain 'loving woman' (*kāminī*) broke. On the floor (*bhūmi*) fell one-third of the pearls, on the bed (*śayanatala*) was found one-fifth. The 'woman with beautiful hair' (*sukeśyā*) grabbed one-sixth and ⟨her⟩ lover (*priya*) collected one-tenth. Six pearls were seen on the string (*sūtra*). Tell, mathematician (*gaṇaka*), how many pearls (*muktās*) were there in the necklace (*hāra*) [1, example 26, page 14]?"

Mathematically, the problem is

$$x\left(1-\left(\frac{1}{3}+\frac{1}{5}+\frac{1}{6}+\frac{1}{10}\right)\right)=6,$$

where x is the total number of pearls. The equation yields an answer of 30.

Mahāvīra (c. 850) enjoyed the patronage of the famous and benevolent *Rāṣṭrakūṭa* king Amoghavarṣa Nṛpatuṅga who ruled at Mānyakheta in south India, much of what is known as Karnataka today. In his *Gaṇita-sāra-saṅgraha*, he not only incorporated new, big and more data to use the motif to pose a new mathematical problem but also elaborated Śrīdhara's problem to a great extent. The following is his problem.

> "At night (*rātra*), in a month of the spring (*vasanta*) ⟨season⟩, a certain 'young woman' (*yuvati*) with the orbs (*bimbas*) of hips (*nitambas*) like the hoods (*phaṇas*) of a serpent (*phaṇi*), the limbs (*aṅgas*) adorned with sparkling (*kana*) pure (*amala*) jewellery (*bharaṇa*), the eyes (*nayanas*) like the belly (*jaṭhara*) of a fish (*pāṭhīna*), ⟨and⟩ the slender (*tanu*) waist (*madhya*) bent by ⟨the load of⟩ the heavy (*kaṭhina*, literal meaning: hard) ⟨multi-layered⟩princess-necklace (*stanahāra*) ⟨lying between the lower neck and the bust line⟩, was loving to her husband (*pati*) joyfully on a thick (*sāndra*), wide (*rundra*) and soft (*mṛdu*) bed (*talpa*) on the wide (*pṛthu*) terrace (*tala*) of a big mansion (*saudha*), white (*dhavala*) like the moon (*himakara*), situated in a garden with trees bent down with the load of the bunches (*gucchas*) of flowers (*prasūnas*) and fruits (*phalas*) and resonant with the melodious sounds of parrots (*śukas*), ⟨Indian⟩ cuckoos (*kokilas*) and bees (*madhupas*) which were all delighted with the juice (*rasa*)

of 'floral nectar' (*kusumāsava*). ⟨When their lovemaking⟩ raised to a love-quarrel (*praṇayakalaha*, quarrel of desire), the pearl-necklace (*muktāmayakaṇṭhikā*, necklace made of pearls) of that woman (*abalā*) broke and fell on the floor. One-third ⟨of those pearls⟩ reached the maid (*cheṭikā*). One-sixth ⟨fell⟩ on the bed (*śayyā*); a half of that, and a half of that, and so on for six times in succession, fell everywhere. ⟨And⟩ one-thousand-one-hundred-sixty-one pearls (*muktāphalas*) were seen ⟨still on the string⟩. Tell the 'total number' (*pramāṇa*) of pearls (*muktās*) in the string if you know ⟨how to solve⟩ miscellaneous (*prakīrṇaka*) ⟨problems based on fractions⟩ [12, verses 4.17-22, page 49]."

This leads to the equation

$$x\left(1-\left(\frac{1}{3}+\frac{1}{6}+\frac{1}{2\cdot 6}+\frac{1}{2^2\cdot 6}+\frac{1}{2^3\cdot 6}+\frac{1}{2^4\cdot 6}+\frac{1}{2^5\cdot 6}+\frac{1}{2^6\cdot 6}\right)\right)=1161,$$

with an answer of 3456.

Bhāskara II (born in 1114 and died after 1183) not only repeated Śrīdhara's problem but also copied it except for a few verbal changes in his *Līlāvatī* ("Gorgeous Amusement ⟨of Mathematics⟩"). Both Śrīdhara and Bhāskara II composed the problem in two verses or four hemistiches. Bhāskara II copied Śrīdhara's last three hemistiches as they were. He made some slight verbal change in Śrīdhara's first hemistich by replacing "'loving-woman' (*kāminī*)" and "love-quarrel (*suratakalaha*)" by "young-woman (*taruṇyā*)" and "quarrel of coition (*nidhuvanakalaha*)" respectively [11, verse 56, page 58]. The term "quarrel of coition" clearly indicates which quarrel was being talked about in Indian poetry and mathematics through the motif.

Śrīdhara (c. 799), Mahāvīra (c. 850), and Bhāskara II (born in 1114) each were the celebrated mathematicians of India. Śrīdhara and Mahāvīra belonged to the exclusive class of the Jaina school of Indian mathematics, to which Rājāditya himself belonged. Rājāditya seems to be a contemporary to Bhāskara II (died after 1183) if it is true that he flourished around 1190. He seems to have been inspired from the works of Śrīdhara and Mahāvīra to employ the motif to give rise to his two problems, i.e., 1º and 2º. But he employed the motif in a lesser way. He reduced the motif to such an extent that the motif itself disappeared in his two problems. He was bound to do so for he did not base his two problems on the subject of fractions as his predecessors did. He based them on the subject of speed. But the characters such as the lovers (wife and husband) and the quarrel (definitely, love-quarrel, because the same was prevalent in Indian poetry and mathematics) between them, which have been linked with the theme of the motif, are visible in his two problems. They are reminiscent of the theme. They produce the theme together in his problems but not as precisely as the motif ("broken string and scattered pearls") itself does in his predecessors' problems. The following fact revealed and views expressed by S R Sarma support our plea that Rājāditya employed the motif in a lesser way.

> "To Bhaṭṭa Tauta, the *guru* ⟨i.e., preceptor⟩ of the celebrated Abhinavagupta ⟨(c. 950 –1016)⟩, is attributed the saying that the inborn poetic genius consists in the ability to invent ever new or original modes of expression. The originality most cherished in Sanskrit *kāvya* ⟨i.e., poetry⟩, however, is not so much an originality in themes, images, comparisons or motifs, but rather the originality in inventing new variations or twists to what are already stock motifs. It should, therefore, be interesting to trace how poets at different times play upon the variations of a single motif. It is these variations that show the imagination, the creativity, and the innovation of a poet. The poet or, for that matter, any artist in Indian classical tradition, though working within strictly laid down parameters, has the freedom to be innovative in inventing original variations to the stock motif [14, page 463]."

Before we discuss the matter of language, we would like to point out that three of the *Triśatikā* of Śrīdhara, the *Gaṇita-sāra-saṅgraha* of Mahāvīra, and the *Līlāvatī* of Bhāskara II are composed in Sanskrit. However, the first two are relevant for our present discussion as we cannot say for certain that Rājāditya was contemporary to Bhāskara II. The motif had been prevalent in Indian poetry and mathematics composed in Sanskrit while Rājāditya's *Vyavahāra-gaṇita* is in *Kannaḍa*. It was a common occurrence in India that thoughts expressed in one language was transmitted in another language. Authors in ancient and medieval India used to take responsibility to introduce what was prevalent in any other language, especially in Sanskrit, the then pan-Indian medium, or Prakrit, into his regional language.

Rājāditya was also an author of that kind. Not only the motif, although in a lesser manner, but also mathematical thoughts composed in Sanskrit he brought in his *Vyavahāra-gaṇita*. For example, he brought word-numerals or, to be more exact, object-numerals prevalent in Sanskrit into the *Vyavahāra-gaṇita*, which he composed in *Kannaḍa*. He has offered a list of 242 object-numerals in its 'Chapter on Definitions' (*Paribhāṣā Prakaraṇa*). Most of them are either in Sanskrit in their true form or in the form, called *tadbhava*, in which Sanskrit terms changed to suit the tongue of the *Kannaḍa* people [4, pages 53-66].

4. Concluding Remarks

Rājāditya infuses elegance in his two problems by choosing the charming idea of a husband's chasing after and eventually meeting with his wife after their quarrel. In this paper we explored two mysteries associated to these two problems.

The first mystery is resolved. The rationale behind the algorithm Rājāditya offers to solve the second problem is based on the rule for finding the sum of an arithmetic progression and this explains why the term used for distance in his algorithm to solve the first problem is value (i.e., the sum of an arithmetic progression, *dhana*). However we still do not really know why he chose to set these two problems in the context of a married couple's quarrel, but we have explored a few possible hypotheses in this paper. We have explored the possibility that Rājāditya might be aiming to "make learning fun by posing a value-based question" or he might simply be using a real-life experience he had to spice up a mathematical exercise.

In the last section of the paper we explored the possibility that the context of a married couple's quarrel (definitely, love-quarrel) might have been inspired by the familiar literary motif of "broken string and scattered pearls". This indeed seems quite plausible. The problems set by Śrīdhara, Mahāvīra, Bhāskara II (who, in fact, only retained Śrīdhara's variation), and Rājāditya cover a range of diverse contexts and situations, making learning fun. But these masters seem to have posed these problems not just for fun but also to keep up with the already available variations of or to invent new variations to the stock motif of "broken string and scattered pearls". Of the variations invented in Indian mathematics to this motif, Mahāvīra's variation is highly structured and, we might conclude, Rājāditya's variation is just indicative.

Notation

( ) A pair of parentheses, in addition to its common usage, encloses either the original Sanskrit words(s) or the present author's explanation of the immediately preceding words(s). Sometimes it encloses both of them or more. In the latter case, they are separated by comma.

⟨ ⟩ A pair of pointing angle quotation marks, wherever used, contains a paraphrase supplied by the present author to achieve comprehensiveness together with clarity.

« » A pair of pointing double angle quotation marks contains paraphrase supplied by Padmavathamma *et al.* [10] themselves and Sarma himself [14].

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