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Kyle Singh
Simon Fraser University

Veselin Jungic
Simon Fraser University

Jun Bo Mei
Simon Fraser University

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Life of a Working Ramsey Theorist: A Conversation with Thomas C. Brown

Kyle Singh

Department of Mathematics, Simon Fraser University, British Columbia, CANADA
kvsingh@sfu.ca

Veselin Jungic

Department of Mathematics, Simon Fraser University, British Columbia, CANADA
vjungic@sfu.ca

Jun Bo Mei

Department of Mathematics, Simon Fraser University, British Columbia, CANADA
jimmy_mei@sfu.ca

Synopsis

This interview, conducted in November 2020, explores the fascinating life and work of Ramsey theorist Thomas Craig Brown. We hope, through our conversation with Dr. Brown, to show that mathematics is not a discipline that is only concerned with numbers, but that it is, in a fundamental way, also about people and connections.

In November 2020, we, the authors, had the chance to interview Ramsey theorist Professor Thomas Craig Brown. In the following, we present this fascinating conversation. We discover that Dr. Brown's dedication to mathematics and learning has always been at the heart of his work.

Dr. Brown's work explores chaos, and as we learn, his life is just as multifaceted and rich. In particular we learn that, throughout his life, Dr. Brown has interacted with some of the most notable figures in mathematics, whether it involved walking right through a beach volleyball game in conversation with Paul Erdős, or learning to juggle from Ron Graham.

Through publishing this interview, we aim not only to provide insight into the life of one of the most notable contributors to Ramsey theory, but also to demonstrate the human nature of mathematics. Thomas Brown was shaped by the people around him. His connections and interactions led him to the problems and solutions he dedicated his life to. We believe that it is of value to underline how such an outstanding mathematician was influenced by the human dimensions of the discipline.

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Wikipedia tells us that Dr. Thomas Craig Brown (born 1938) is an American–Canadian mathematician and Professor Emeritus at Simon Fraser University, Burnaby, BC, Canada. As a mathematician, Brown's primary research focus has been in the field of Ramsey theory.[1]

Professor Brown both collaborated and established life-long friendships with some of the most notable pioneers and trailblazers in Ramsey theory, including Paul Erdős, Fan Chung, Ron Graham, and Neil Hindman.

For the convenience of the reader, in the following we italicize our questions.

The Interview

What made you interested in mathematics?

> Probably it was the difficulty I had with math in grade 3. Currently, I think there are three main reasons I like math. One, it's beautiful. Two, it "exists" independent of the physical world and independent of humanity — at least, that's what many or most mathematicians like to believe. Three, it's thrilling to discover something "new" in this invisible world.

Do you have interest in other fields?

> I do read lots of books on evolution, but nothing very technical. Same, to a lesser degree, in astronomy. What I read most of all, though, are novels, mostly modern, and particularly American and Canadian.

[1] Imre Leader explains that "[t]he fundamental kind of question Ramsey theory asks is: can one always find order in chaos? If so, how much? Just how large a slice of chaos do we need to be sure to find a particular amount of order in it?" [11]. For a friendly introduction to Ramsey theory see [10].

In grade school I read every book in the (pretty small) school library. In high school, I read all of the ancient Greek plays by Aeschylus, Sophocles, Euripides, Aristophanes, and quite a few 19$^{\text{th}}$ century Russian novels, which is why I studied Russian and (ancient) Greek in college.

I translated the book *Foundations of Linear Algebra* by A. I. Mal'cev (1956) from Russian to English during the summer of 1961. It was published by W. H. Freeman in 1963, but I think it sold very few copies.

What about Ramsey Theory interests you?

van der Waerden's theorem on arithmetic progressions is a good example. I think it's very surprising that it's impossible to colour the positive integers with two colours (or any finite number of at least two colours) in such a way that every arithmetic progression of 50 billion terms gets at least two colours.

This idea, that inside every sufficiently large disorderly looking structure is an orderly structure of arbitrary size, is very appealing - why it's appealing, I really don't know.

As an undergraduate I acquired the book *Three Pearls of Number Theory* by A. Y. Khinchin (which contains a proof of van der Waerden's theorem) and was very impressed by the introduction. (It's still available at Amazon.ca for 9.25 CAD!)

The fact that many results in Ramsey theory are not only easy to understand, but in addition have elementary proofs, is very attractive: one doesn't need to master a vast field of knowledge in order to try to solve a new problem in Ramsey theory. There may be an elementary solution just around the corner.

How would you explain the concept of Ramsey Theory to someone who has never heard of it before?

It's definitely not easy. Usually I try to describe either van der Waerden's theorem or the finite Ramsey theorem for graphs. (I'm inclined to think that just explaining the meaning of $R(3,3) = 6$ is a good thing to try.)

What awards have you won?

100 USD from Erdős (shared with Allen Freedman) for (jointly) solving one of Erdős's problems.

Here's what we showed: Let $f(n, k)$ be the minimum size of any subset B of $[1, n]$ such that B meets all of the k–term arithmetic progressions contained in $[1, n]$. Then $f(n, n^e) < Cn^{1-e}$ (where $e > 0$ and C depends on e) and $f(n, \log n) = o(n)$. For details, see [7].

What was it like working with Paul Erdős? How did working with him influence you?

Talking with Erdős, or just overhearing him talking with others, was always exciting and even exhilarating. At any big math meeting, if he was talking to one or two people, there would be six or eight people trying to listen in, or trying to ask him a question.

His memory was phenomenal. Here are three examples.

a. Once, talking about a result of mine which I thought was new, he simply remarked that he had written a related paper which had appeared in 1938 in such and such a journal. (When I found this paper later, it turned out he had done everything I had done, and more, in a better way.)

b. I asked him once in the 80s whether he had ever talked at Reed College. He immediately said something like, "Yes, in April 1955, and I talked about such and such, and had an interesting conversation with so and so."

c. He first met my wife at a math meeting in Israel. (He eventually stayed with us on two occasions for a few days each, during visits to Vancouver, BC.) She had injured her ankle badly in Cairo the week before. When she next met him, two or three years later, the first thing he said was "How is your ankle now?"

He would often talk while walking. Once at Kitsilano beach he was talking to me and walking, not looking and not noticing where he was going, and he led me right through the middle of a volleyball game. (The players graciously just waited for us to pass by.)

Allen Freedman and I once spent a lot of time with him at a meeting in Chicago. Walking down the street with him was sometimes alarming—he seemed not to notice the traffic dangers around him.

It was hard to keep up with him when trying to solve a problem. He would propose an approach, and quickly develop it, and quickly decide it wouldn't work, drop it, and start on a completely different approach. I would still be trying to grasp his first approach, and he would be half-way to discarding his second approach.

From him I learned that it was dangerous to become attached to a certain approach merely because you had invested a lot of time and energy in it. His ability to develop new ideas and then just drop them to try other new ideas was simply dazzling.

(The best book on Erdős, in my opinion, is *The Man Who Loved Only Numbers* by Paul Hoffman. Ron Graham and Fan Chung also appear in this book.)

Conversations with Ron Graham were very different. Ron was very patient, and was always concerned that you understood what he was saying. If you didn't, he would explain it in a different way.

Like Erdős, he was unbelievably quick. I had a conversation with him once in Victoria, and told him about a result I had found after weeks or maybe months of work. The next day he flew to California; in two hours on the plane he had gone far far beyond me.

Ron's lectures to a general audience were always extraordinarily clear, and at the same time full of delight.

What piece of research have you done that you feel is the most impactful or influential?

About 1963, while a graduate student, I showed that if the positive integers are finitely coloured, then some colour class is piece-wise syndetic. (I promptly forgot about this result, and didn't remember it until three years later when I needed it.)

This turned out later to be a useful fact, and is often called "Brown's Lemma," especially in the semigroup literature. See [2, 3].

Neil Hindman discovered this "partition regularity of piece–wise syndetic sets" independently in 1973. In his book (with Dona Strauss) *Algebra in the Stone–Čech Compactification* he has a wonderful one–line proof.

Another often cited paper was [6]. This paper was described in the *Added in Proof* section of Erdős and Graham's classic book *Old* *and New problems and Results in Combinatorial Number Theory* [9] as giving "a bit more evidence for the truth of the density Hales-Jewett theorem."

The paper [4] publicized a question finally solved in 1992 by V. Keränen.

Another often cited paper (about 100 times) was [5].

A problem raised in the paper [8] was stated as a problem in 1994, 2000, 2008, 2012, 2012 by five different (sets of) authors without mentioning the Brown–Freedman 1987 paper. It's odd that two of those five sets of authors included myself!

The problem, still unsolved, is the following: Does there exist an infinite sequence on a finite set of positive integers such that there do not exist two adjacent blocks having equal lengths and equal sums? (The best partial result is that there need not exist three adjacent blocks with equal lengths and equal sums.)

Is there a result in Ramsey theory that in your opinion ought to be more widely known, or at least more widely mentioned?

Yes! In the paper by Felix Behrend [1] is the following result: If Szemerédi's theorem is false for a given k then there exist pairs (n, A) such that A is a subset of $\{1, 2, 3, \ldots, n\}$ which contains no k–term arithmetic progression, n is arbitrarily large, and the density of A in $\{1, 2, 3, \ldots, n\}$ (that is, the quantity $\frac{|A|}{n}$) is arbitrarily close to 1.

This implies that while van der Waerden's theorem can be stated: "For all k, there exists n such that if $\{1, 2, 3, \ldots, n\} = A \cup B$, then A or B contains a k–term arithmetic progression," Szemerédi's theorem can be stated: "For all k, there exists n such that if $\{1, 2, 3, \ldots, n\} = A \cup B$, and $|B| \geq |A|$, then B contains a k–term arithmetic progression."

Or: Using Behrend's theorem, Szemeré's theorem is equivalent to the statement: Let k be given. Then there exists n such that if A is a subset of $\{1, 2, 3, \ldots, n\}$ with $\frac{|A|}{n} > 0.9999$, then A must contain a k–term arithmetic progression. (One might expect that this would be easier to prove than the corresponding statement with 0.9999 replaced by an arbitrarily small positive constant. Perhaps it's surprising that it isn't!)

> I find it odd that I've never seen Behrend's result mentioned in standard works on Ramsey theory.[2]

Is there a theory or idea that has not been proved that you hope to prove in your life? Which ones and why?

> No, but 35 or 40 years ago, I spent about three years trying hard to prove the density version of the Hales–Jewett theorem. When I quit trying, it was such a relief!! It was like finally getting rid of a piece of furniture — an unused grand piano, maybe — which had filled up the whole living room!

Would you mind sharing a little bit about yourself growing up?

> My father (whose mother was German and immigrated to the United States with her family in the 1890s) was born in 1905 in a logging camp on the Columbia River in Oregon, not far from Portland. My mother (whose father was Dutch and immigrated to the United States at age 15 about 1900), was born in 1909 in a small town in southern Wyoming.
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> I was born in 1938 in Portland. Shortly after the end of World War II, my family moved to Beaverton, a small town (then) in the country about an hours' drive from Portland, where I entered a two–room school in third grade. (The principal was also the janitor and the school–bus driver!) My new class had already mastered subtraction (of one digit numbers), which I found completely baffling.
>
> My third grade teacher (who also taught grades 1, 2, and 4 in the same room!) patiently spent hours with me helping me to catch up — without her, I would never have become a mathematician.
>
> In high school, the math teacher was the coolest teacher in the school. In the 12th grade, I realized I really liked math and was good at it. However, in high school I spent most of my time playing the trumpet, and considered a career (as a performer) in music. I graduated in 1956.

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[2] While reviewing the proofs of our interview before publication, Professor Brown wanted us to note that "this result now has been stated in a standard work, namely, on page 61 of the really excellent book *Fundamentals of Ramsey Theory*, by Aaron Robertson, published in 2021 by CRC Press." Robertson writes: "We start with a result due to Behrend [14] that seems to have been overlooked. This was communicated to the author by Tom Brown." After another sentence, Behrend's result is stated as Theorem 2.67, on page 62." Robertson's reference [14] is our [1].

I was given a four-year "General Motors scholarship" which could be used at any college or university in the United States. I chose the California Institute of Technology, because of an article I read in Time magazine. After one year I transferred (because of limited course offerings at Caltech) to Reed College in Portland, where I took Russian, ancient Greek, philosophy, music, psychology, ...

Because the USSR had launched Sputnik in 1957, the US poured millions into higher education, and I had an NDEA (National Defence Education Act) Fellowship good for three years of graduate study. I chose Washington University in St. Louis, Missouri, because a math student one year ahead of me told me how much he liked his classes there.

With my new Ph.D., in 1964 I took a one-year appointment as an instructor at Reed College. At Reed, I learned of a US–USSR exchange program (it involved only about 25 US students and 25 USSR students), and I spent the next academic year at Kiev State University.

While in Kiev, I learned (from a Canadian student) of the existence of Simon Fraser (which had opened in 1965) and applied for a job there, and spent 1966–2003 at SFU. (Except for three nonconsecutive full years when I took leaves of absence from SFU and taught at the Bosphorus University in Istanbul. It was there in 1978 that I met my German wife, Astrid — we have been married for 42 years now. There were other research semesters or sabbaticals which I spent in Cairo, Nairobi, various places in Europe, and Turkey.)

In the early 70s, I saw Ron Graham juggling at a math meeting, and from then on I received occasional lessons from him. Learning to juggle three balls is about as hard as learning to ride a bicycle, and you never lose this ability. Five balls is a different story. Once with Ron we stood side-by-side, facing the same direction, and with my right hand and Ron's left hand, acted like a single person juggling five balls. Eventually I could juggle five balls for ten seconds or so (about 40 throws). Currently I'm trying to revive this ability.

Is there any extra information you want to give that you feel is important or noteworthy about yourself or Ramsey Theory?

Ron Graham died on July 6, 2020, sad news to everyone who knew him or knew of him.

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