

## Calculus III: Under the Influence of Peer Instruction

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## Calculus III: Under the Influence of Peer Instruction

### Cover Page Footnote

This work was supported by a Teaching Support Award from the University of Tennessee Teaching and Learning Innovation. Thanks to Dr. Lou Gross, Dr. Suzanne Lenhart, Dr. Patrick Shipman, Jonathan Clark, and Amanda Lake Heath for reading preliminary drafts and providing thoughtful suggestions for this story. BIOGRAPHICAL SKETCHES Alan Von Herrmann received his Ph.D. in Mathematics from Colorado State University. Outside mathematics, he is interested reading great nonfiction and hanging out with his dogs. Jeneva Clark holds a Master's in Mathematics and a Ph.D. in Teacher Education, with an emphasis in Mathematics Education. She enjoys writing, teaching, and writing about teaching.

# Calculus III: Under the Influence of Peer Instruction

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## Synopsis

In peer instruction [12], students engage with core course concepts and then explain those concepts to one another in small groups. Unlike in lecture format, peer instruction involves every student in the class. In Spring 2019, the first author began using a modified version of peer instruction in Calculus III classes. He started each class by discussing important Calculus III concepts from three standpoints (the formula, the geometry behind the formula, and the physics behind the formula). During the last 20 minutes of each 50-minute class session, he polled the students using questions in the “Goldilocks Zone” — not too hard and not too easy, but just right for Calculus III students. These questions ignited student-to-student discussions. Students’ attendance and achievement have improved. The paper also describes how peer instruction has influenced the first author’s own instructional practices.

**Keywords:** calculus, peer instruction, polling, active learning

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## 1. Introduction

### *1.1. Note about Authors' Positionality*

This paper is written in first person from the perspective of the first author, Dr. Alan Von Herrmann, who describes his positionality as, “I’m probably the only known deaf, nearly blind, Mexican-American mathematician with a Ph.D. in Mathematics in the world. While I occasionally face discrimination, I also value my unique opportunity to inspire students to overcome adversity.” The second author, Dr. Jeneva Clark, informally interviewed Dr. Von Herrmann over countless cups of morning brew, while eliciting the lessons learned and penned in this paper. Dr. Clark assisted with all pedagogical innovations, with data analysis described in this paper, and with manuscript composition.

### *1.2. On the Road to Recovery*

I’m Alan Von Herrmann. I am a recovering “sage on the stage,” and this is my story. After umpteen years teaching lecture-based, teacher-centered Calculus III, I noticed a wane in student engagement. My award-winning lectures which had once inspired students to dive deeper into mathematics, seemed no match for students’ absences and apathy. I wondered, did the new generation of college students now hold different expectations and needs for instruction? In December 2018, I decided to seek help and turned to the Mathematical Association of America (MAA) Instructional Practices Guide [19]. A teaching practice called peer instruction, created by Eric Mazur [4, 12, 13], caught my eye. Involving student polling and student-to-student discussions, the technique resonates with the ideas that students (a) can sharpen their understandings by interacting with others during class, (b) often listen to other students as well as they do to the instructor, and (c) can learn concepts more deeply if they individually struggle with really good questions. In this paper, I will describe my road to recovery, from a lecture-aholic to a believer in active learning. I will describe the implementation of peer instruction, how it influenced students’ success, and how it transformed me as a teacher.

## 2. Borrowing from Peer Instruction

Before implementing any changes, I submitted a detailed proposal to my department leadership to pilot a modified version of peer instruction, explaining how no rigor or content would fall through the cracks along the way. Upon approval, beginning Spring 2019, I made my first step toward active learning.

### 2.1. Class Organization

Still giving brief lectures and still giving exams, quizzes, and homework, I bravely added a peer instruction component. During the last 20 minutes of each 50-minute class session, I used Canvas to poll the students using questions I considered in the “Goldilocks Zone”—not too hard and not too easy, but just right for Calculus III students.

Based on what proportion of students got the polling question correct, I would either revisit the concept, move on to new topics, or prompt peer discussion and re-poll. If a polling question returned less than 30% correct responses, then I knew I needed to re-visit the concept. If a question yielded greater than 70% correct responses, then I knew that most students mastered the concept and were ready to move on to more material. However, if a question yielded 30-70% correct responses, then the plot thickened; these problems presented an opportunity for students to disagree, to discuss, and to determine, together, what misconceptions they may have and why. In those cases, I resisted giving them guidance, but instead, directed them to talk to their peers. Then after a few minutes, I would re-poll the students. Every time, peer discussion would move the needle in the right direction! The polling would uncover misconceptions that they might not have even known they had. Listening to their peers gave them an opportunity to hear something explained from a student’s perspective. Lastly, the debriefing and explanation were often student-led. Figure 1 shows a diagram for this class structure.

To implement this new teaching practice, I followed recommendations for effective task design in team-based learning [17], and I screen-projected clear instructions for each task, communicated time limits for each task, and began peer instruction activities on day one of the course. Using the very first day of class as an opportunity to share my vision with the students and model

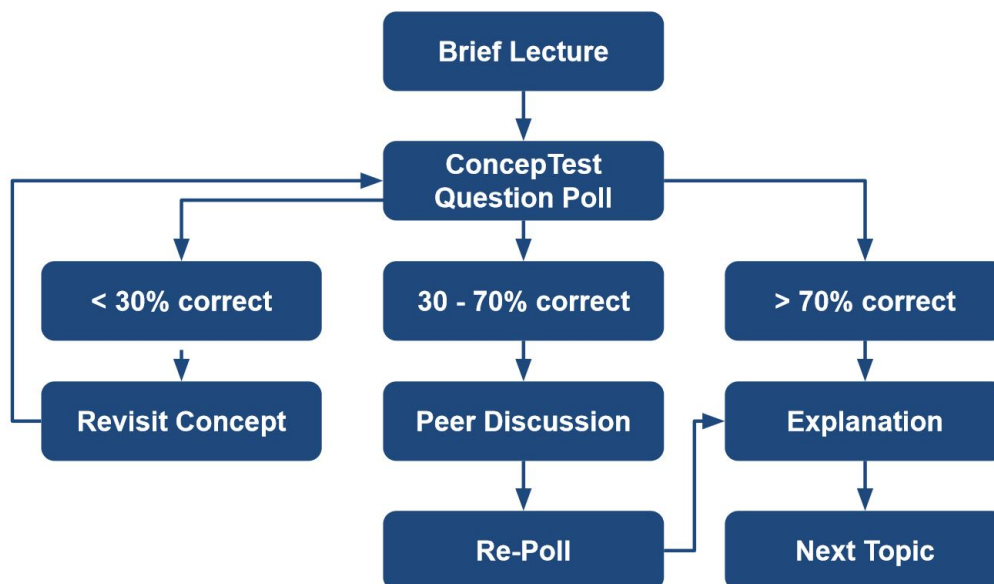


Figure 1: Class organization.

actual active learning has proven extremely helpful for classroom management. Showing the diagram in Figure 1, I explained to students how peer instruction would work. With conviction, I explained to students why I believed peer instruction would be beneficial to their learning. Then, on day one, we practiced the peer instruction process with the question, “Humpty Dumpty was pushed. True or false?” During the timed peer discussion, students playfully turned to one another and discussed whether or not the King’s men really put forth any effort into Humpty’s recovery. Was he a sad casualty of a royal conspiracy? With this silly starter question, students became acquainted with the process without the complicating factors that sometimes interfere with math, such as math anxiety and stereotype threat. When this silly starter question was replaced with math questions, I proactively emphasized that students should not be afraid to make mistakes and learn from them. My slides read, “The incorrect answers are an important part of learning the concepts, and you will NOT be penalized for incorrect responses!”

## 2.2. The Lecture Part

Lectures had always been a part of my courses, but delivering these brief pre-peer-instruction lectures began to elicit more student engagement. Students listened more intently because they anticipated the polling questions. Also, the students seemed to listen differently. Instead of simply being note-taking scribes who postpone higher-order thinking, my students began to think in class! I could detect this because they would ask more questions about deeper concepts and applications than before.

Each class began with a brief lecture about important Calculus III concepts from three standpoints – the formula, the geometry behind the formula, and the physics behind the formula. After I realized that some students were now doing more critical thinking than transcribing in class, I decided to encourage this more broadly. In Fall 2019, I designed slideshows to enhance the lectures. Most students accessed these on Canvas both before and after in-class lectures, often taking notes outside of class, which enabled them to have better discussions during class.

In each lecture I aim to show three viewpoints of each concept, an algebraic or symbolic viewpoint, a geometric viewpoint, and a physics-based viewpoint.

- I. First, I introduce the concept itself and important notation and possibly an algebraic viewpoint, such as for the Divergence Theorem: “Let  $S$  be a closed surface (e.g., sphere, box, paraboloid bounded by a plane) that encloses a solid  $E$  in 3-D space. Assume  $S$  is piecewise smooth and is oriented by normal vectors pointing toward the outside of  $E$ . Let  $\vec{F} = \vec{F}(x, y, z)$  be a vector field whose domain contains  $E$ . Then,  $\int \int_S \vec{F} \cdot d\vec{S} = \int \int \int_E \nabla \cdot \vec{F} dV$ .” I also remind them of Green’s Theorem, and frame it as a 2-D analogue, and I discuss how this can measure outward flow per unit volume.
- II. Another viewpoint I intentionally incorporate is geometric. For example, to motivate the need for the Divergence Theorem, I ask students how they would calculate the flux of a vector field out of a box, shown in Figure 2. Do they want to calculate a sum of six surface integrals, one for each cube face? Instead, the students can call on their new friend, the Divergence Theorem.

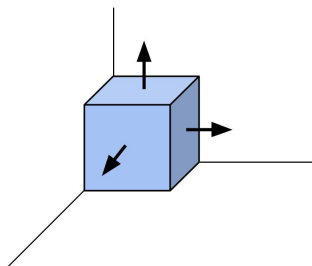


Figure 2: Flux of a vector field out of a box.

III. I also include an application from a physical point of view. For example, “Let’s say I have a rigid container filled with some gas. If the gas starts to expand, but the container does not, what must happen?” Students answer that the gas must leak or the container must burst. “You go to a gas station and pump air into one of your tires. What must happen to the air inside the tire?” The students answer that the air must compress. “These scenarios illustrate the Divergence Theorem. A vector field represents a flow of a fluid, and its divergence represents its compression or expansion.”

To provide an overview of the course itself, below is a list of topics included:

- vectors and vector functions
- functions of two and three variables
- partial and directional derivatives
- optimization
- multiple integrals
- calculus of vector field
- line and surface integrals
- the theorems of Green, Gauss, and Stokes



### 2.3. The Peer Instruction Part

For each class session, I had prepared 2 to 4 polling questions. I almost always wrote original questions, but I used the GoodQuestions Project [3], ConcepTest Questions [12], and MathQUEST/MathVote [11] as inspiration. A sample polling question from my class is shown in Figure 3. With each question, my goal was to land in the “Goldilocks Zone,” reminiscent of Vygotsky’s zone of proximal development [20]. I had heard of Vygotskian constructivism before, but it was no longer an educational philosophy that felt fluffy in my mathematician mind. I now had aligned measurable and observable goals, such as eliciting 30-70% correct on the initial poll, setting up the perfect atmosphere for productive peer discussion.

Which of these iterated integrals is equal to

$$\int \int_D \frac{\sin x}{x} dA$$

where  $D$  is the triangle in the  $xy$ -plane bounded by the  $x$ -axis, the line  $y = x$ , and the line  $x = 1$ ?

A.  $\int_0^1 \int_x^1 \frac{\sin x}{x} dy dx$

B.  $\int_0^1 \int_0^x \frac{\sin x}{x} dy dx$

C.  $\int_0^1 \int_1^y \frac{\sin x}{x} dx dy$

D.  $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$

E. More than one of the above is correct.

*Figure 3: A polling question.*

I traded in one addiction for another. Instead of a dependency on lecturing, I now began to crave the creative process of composing productive questions.

My water-cooler talk, which had once been commiserating about student performance, now regularly began with, “Look at this great question.”

Miller [15] identified some characteristics of really good questions. For example, they should stimulate students’ mathematical interest and curiosity, help students monitor their understanding, offer students frequent opportunities to make conjectures and argue their validity, draw on students’ prior knowledge and misconceptions, provide formative assessment, and foster an active learning environment. From my new experience, I would add a few more characteristics of good questions.

- (a) **Good questions often require students to transfer a concept to a new context.** Calculus students sometimes fail to recognize when a mathematical concept can be applied to a new context. Jo Boaler found that teaching in ways that integrate process with content can assist in knowledge transfer [1]. It also helps use polling questions to challenge students to transfer knowledge early. Whether a polling question is doing this can only be assessed by considering what preceded the polling. For example, if an instructor over-prepares the students for a polling question, then that becomes what Roberson and Franchini call a “naive task” [17]. In my lectures, I purposely withhold hints that might cheapen the students’ peer instruction experience or cheat them out of productive struggle.
- (b) **Good questions often challenge students to see mathematical concepts from multiple perspectives.** Calculus students often overlook connections between important algebraic, geometric, and physical contexts. Thus, I am very intentional about provoking student thinking about these connections, both in lecture and in polling questions. These three emphasized perspectives are informed by my course and institutional context, but more generally questions can be guided by frameworks such as the Harvard Calculus “Rule of Three” (graphical, numerical, symbolic) [7] or Lesh, Post, and Behr’s framework (physical, pictorial, linguistic, symbolic) [8]. Students should be challenged to see the same problem through various mathematical lenses, whatever those lenses are, in order to develop what Greer calls “representational flexibility” [6].

- (c) **Good questions often lead to more questions, in particular students' questions about underlying reasoning.** Sometimes mathematicians avoid asking students “Why?” about math because the range of possible student responses is unbounded. However, Roberson and Franchini explained that “*Why?* is the doorway to course content and disciplinary thinking—and to meaningful inter-team conversations” [17, p. 290]. During my implementation of peer instruction, after the re-poll results are disclosed, when students are excited to share out loud their explanations, I often follow their explanations with *Why?* questions. In addition, sometimes the tables are turned, and the students pose *Why?* questions to me, often questions encountered during their peer discussions.

Although good Goldilocks questions are essential to effective peer discussion, Miller’s data imply that using good questions is not enough; students benefit from the peer discussion about the questions, rather than the questions themselves [15]. The discussion part impacts students for a few reasons. They have to articulate their reasoning to others. This obligates them to think about the question at hand, but to also collect their thoughts well enough to communicate them. The polling also plays an important role. When peer discussion is book-ended by student polling, students are more engaged in the discussion. Because each student was forced to make a decision on the first poll, each student has “skin in the game.” According to Roberson and Franchini, “The most clarifying action a student can take is to make a decision” [17, page 278]. When I call on a student to make an independent judgement and to then commit to that verdict by electronically selecting an answer, I am providing an opportunity for clarity.

### 3. Student Learning, Under the Influence

Studies have indicated that peer instruction improves student participation and achievement [4, 10], and my experience integrating peer instruction into a Calculus III course encountered a similar trend.

### 3.1. Student Attendance

The improvement in student attendance rates can be seen in the red plot in Figure 4. To represent the before and after pictures, I chose the three most recent semesters I taught Calculus III, excluding summers, which may have anomalous attendance. In the fall of 2017, when I taught one class of 33 students, on an a typical class day, 70% of the 33 enrollees would attend. In the Spring 2019 semester, I taught three sections of Calculus III, for a total enrollment of 87. This was was my first time to try peer instruction, and I included peer instruction in each class session except for exam days. The average attendance rate rose to 83%. The following semester, Fall 2019, teaching two sections for a total enrollment of 77, I continued peer instruction and I added slideshows, and the average attendance rate rose to 90%. Attendance rates since then, as I currently continue these teaching methods, have remained 90% on average, with expected variation during flu season.

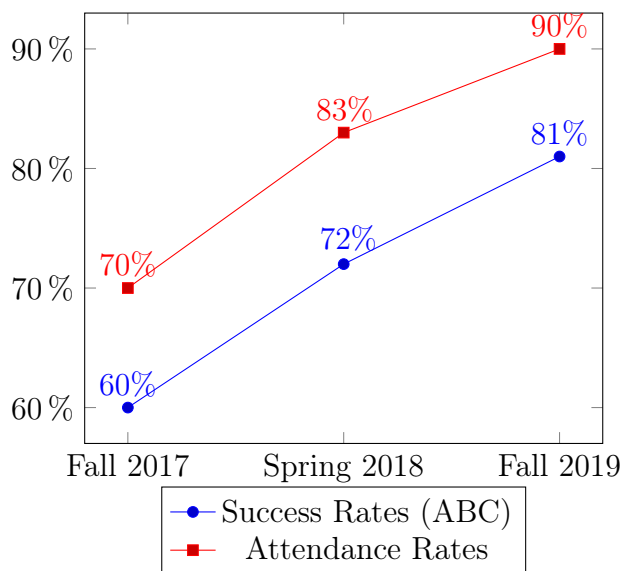


Figure 4: Sample sizes were  $n = 33$ ,  $n = 87$ , and  $n = 77$ , respectively.

### 3.2. Student Achievement

Figure 4 also shows, in the blue plot, student success rates (C or better) from Fall 2017, Spring 2018, and Fall 2019. The first semester included lecture only, the second included lecture and peer instruction, and the third

included all of the above plus slideshows. With only lecture, 60% of the 33 enrolled students passed with a C or better. During the first semester I implemented peer instruction, 72% of the 87 earned a C or better. Finally, when I implemented peer instruction and utilized slideshows during the mini-lectures, 81% of the 77 scored a C or better. This change in success rates causes one to wonder whether the material has changed in rigor or difficulty. I promise the exams did not become easier. In fact, I often include novel exam questions, which are different from any the students have seen in class prior to the exam. The students seemed to get better at applying concepts to new scenarios.

Success rates only tell part of the story of student achievement. For example, did more students score A's, B's, or C's? The changes in grade distributions can be seen in Figure 5.

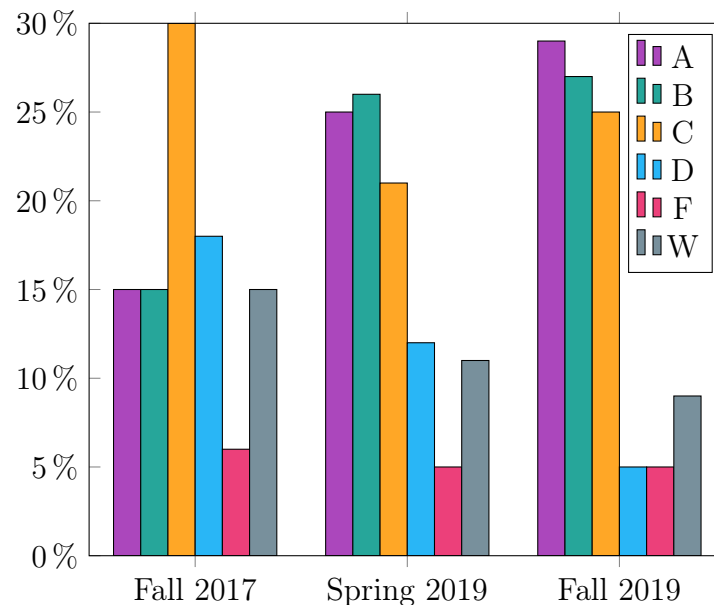


Figure 5: Grade distributions.

In Fall 2017, with lecture only, the most frequent course grade was C, earned by 30% of the students. In Spring 2019, with peer instruction, the most frequent course grade was B, earned by 26%. In Fall 2019, with peer instruction

and slideshows, the most frequent course grade was A, earned by 29%. The most stark difference in distributions is between the lecture-only semester and the first peer instruction semester. In particular, the sub-distribution of A's, B's, and C's changed from left-skewed to more uniform. This indicates that peer instruction may help "C students" the most. Between Spring 2019 and Fall 2019, when slideshows were added, it seemed to help all students achieve higher course grades, except for those who failed. This makes sense because it made the lecture material more accessible to all students who chose to attend.

### 3.3. Student Perceptions

On anonymous student course evaluations that are administered online, the students are presented Likert-type statements, with which they strongly agree (5), agree (4), are neutral (3), disagree (2), or strongly disagree (1). The following survey items showed notable increase in mean scores when I changed teaching formats.

- "The instructor contributed to your understanding of course content."
- "The class sessions were well organized."
- "The course materials (readings, homework, laboratories, etc.) enhanced your learning in this course."

Figure 6 on the next page shows how each of the mean ratings for these statements improved from Fall 2017 (lecture only), to Spring 2019 (lecture + peer instruction), to Fall 2019 (lecture + peer instruction + slideshows). In particular, the mean ratings for the instructor's contribution to student understanding rose from 3.21, to 3.38, and then to 3.96. The mean for the organization of class sessions rose from 3.11, to 3.53, to 4.27. The mean for the course materials' enhancement of learning rose from 4, to 4.11, to 4.43. In some studies, it might be essential to compare these means to campus averages, but I choose not to focus on such comparisons. This manuscript is a story of one instructor who is seeking to improve by reflecting on his own teaching. Like a golfer or bowler who just wants to beat their own score, I only want to compete with my former lecture-holic self. Forming a trajectory of my own development helps me most, and I endorse this outlook for others who venture to try active learning in their classes.

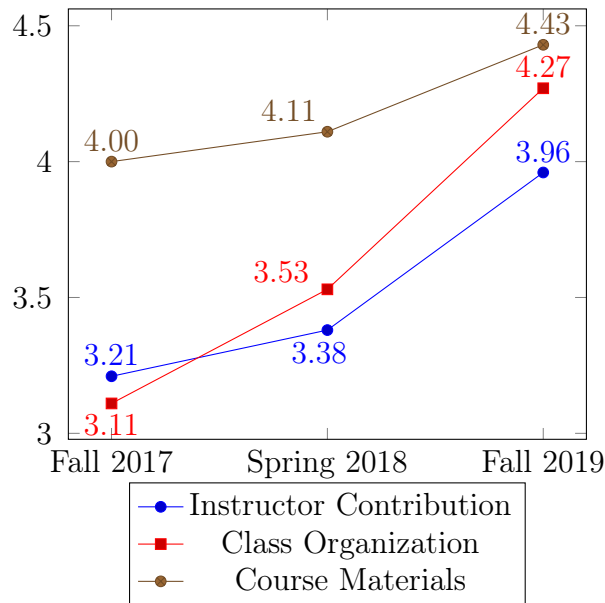


Figure 6: Means from course evaluations.  
 (5 = strongly agree, 4 = agree, 3 = neutral,  
 2 = disagree, 1 = strongly disagree)

Similar to the experiences of other faculty [16] [18] [21], the written responses from student evaluations were more positive than they had been in years, especially in Fall 2019. Below are some comments that specifically mention my new teaching practices. Yes, they are cherry-picked, but still helpful for revealing peer-instruction-specific impacts.

- “I really liked how he challenged us to understand the concepts and not memorize formulas.”
- “I really enjoyed the TEAM aspect of learning. It encouraged me to always stay on top of my understanding of each topic rather than being passive about it.”
- “He provides many aspects for one problem, and makes real life connections to the material.”
- “The class is very well structured and the Professor’s teaching style was easy to follow. The material is difficult but the professor provided many resources for students to utilize and he had group activities that helped with conceptual learning before leaving class.”

- “Fantastic course. I appreciated the ability to learn with others through the team activities.”

#### 4. My Teaching, Under the Influence

The MAA Instructional Practices Guide identifies three pillars of teaching: design practices, classroom practices, and assessment practices [19]. My personal design practices (e.g., how I prepare and plan for my classes), have been strongly influenced by peer instruction implementation. In my early years as faculty, I might have taken pride in being able to walk into a classroom with nothing but a piece of chalk and let my expertise take center stage with an impromptu lecture. My younger self would have seen preparation for a Calculus III class as a sign of weakness in a world where instructor content knowledge was king. Now, however, I spend many hours preparing for my classes, honoring the part of class that matters most - student learning.

My classroom and assessment practices have changed as well. I now have more opportunities to ask my students *Why?* questions, promoting deeper conceptual understanding and abilities to transfer knowledge to new contexts. I also have more opportunities to listen to my students. Like others have reported [2], this helps me identify student misconceptions much earlier than exam-grading season.

#### 5. Conclusion

I acknowledge that there is already a mountain of evidence that interactive teaching and active learning approaches benefit students, including a meta-analysis of 225 studies [5] and a cross-sectional study that shows peer instruction can reduce gender gaps [9]. However, instructor-guided lecture is still the predominant mode of instruction in undergraduate calculus courses [14]. Then again, why would instructors favor evidence-based interactive teaching approaches if their department leaders still expect them to rely on lecture? This prevalent expectation was revealed by the MAA National Study of College Calculus, which analyzed institutional documents for 12 different mathematics departments, and lecture was identified as the primary mode of instruction at 9 of the 12 institutions [14]. Thus, there is a misalignment



between what is, and what ought to be. This leads me to my rationale for sharing. I share my story, not to add to the pre-existing evidence of active learning's efficacy, but to add a testimony, a personal endorsement, an encouraging voice. Academic traditions can change. The change agents are the instructors reading this article, who value student learning more than status quo.

### **Acknowledgments**

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### **BIOGRAPHICAL SKETCHES**

Alan Von Herrmann received his Ph.D. in Mathematics from Colorado State University. Outside mathematics, he is interested in reading great nonfiction and hanging out with his dogs.

Jeneva Clark holds a Master's in Mathematics and a Ph.D. in Teacher Education, with an emphasis in Mathematics Education. She enjoys writing, teaching, and writing about teaching.

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