

Foundational Mathematical Beliefs and Ethics in Mathematical Practice and Education

Richard Spindler
Charles A. Dana Center

Follow this and additional works at: <https://scholarship.claremont.edu/jhm>



Part of the [Applied Ethics Commons](#), [Logic and Foundations of Mathematics Commons](#), and the [Mathematics Commons](#)

Recommended Citation

Richard Spindler, "Foundational Mathematical Beliefs and Ethics in Mathematical Practice and Education," *Journal of Humanistic Mathematics*, Volume 12 Issue 2 (July 2022), pages 49-71. DOI: 10.5642/jhumath.GOSN2205. Available at: <https://scholarship.claremont.edu/jhm/vol12/iss2/6>

©2022 by the authors. This work is licensed under a Creative Commons License.

JHM is an open access bi-annual journal sponsored by the Claremont Center for the Mathematical Sciences and published by the Claremont Colleges Library | ISSN 2159-8118 | <http://scholarship.claremont.edu/jhm/>

The editorial staff of JHM works hard to make sure the scholarship disseminated in JHM is accurate and upholds professional ethical guidelines. However the views and opinions expressed in each published manuscript belong exclusively to the individual contributor(s). The publisher and the editors do not endorse or accept responsibility for them. See <https://scholarship.claremont.edu/jhm/policies.html> for more information.

Foundational Mathematical Beliefs and Ethics in Mathematical Practice and Education

Richard Spindler

Wisconsin Department of Health Services
1 West Wilson Street, Madison, Wisconsin, USA
rspindlermath@gmail.com

Abstract

Foundational philosophical beliefs about mathematics in the mathematical community may have an unappreciated yet profound impact on ethics in mathematical practice and mathematics education, which also affects practice. A philosophical and historical basis of the dominant platonic and formalist views of mathematics are described and evaluated, after which an alternative evidence-based foundation for mathematical thought is outlined. The dualistic nature of the platonic view based on intuition is then compared to parallel historical developments of universalizing ethics in Western thought. These background ideas set the stage for a discussion of the impact of traditional mathematical beliefs on ethics in the practice and education of mathematics in the mathematical community. This is compared to the potential of a belief in evidence-based mathematical foundations on mathematical practice and education.

Keywords: ethics, foundations, mathematics

1. Introduction

In an interview on a *Hidden Brain* podcast [48], organizational psychologist Adam Grant said our belief (what we think is true) can drive what we value (what we think is important). In other words, what we believe about the world can influence our ethical and moral beliefs. This article articulates how foundational beliefs about mathematics may influence ethical beliefs in mathematical practice and high school/college education, with an emphasis on how education affects practice.

I will first describe several philosophical traditions of mathematical ontology and epistemology, namely platonism and formalism, along with logicism. Although I believe the responsibility is on proponents of those traditions to give evidence for their validity, I will give a summary of the arguments against these. In the process, the evidence of what mathematics actually is will be provided: that mathematics is a historically contingent, psychological, and social enterprise constrained by empirical knowledge and mental metaphors. Next, I will discuss how platonism and formalism may be negatively affecting ethics in mathematical practice and education. Finally, I will describe how an acceptance of the human and social nature of mathematics can improve moral actions in mathematical practice.

2. Mathematical Platonism

In the West, there has been a continuous conversation about what is “real” (philosophical ontological inquiry) for over two thousand years. In mathematics, the Pythagoreans may have been the first to conclude “numbers are things” [8, page 31], i.e. that numbers are real. Incidentally, they also said “things are numbers” [8, page 31] because of their observation of how mathematics could explain the world around them. The latter is a tradition that Tegmark [45] has built on to describe a philosophy where the universe IS mathematics. As described by Brumbaugh [8], the Pythagoreans believed that numbers are independent of observers, have precise identities, and are the same for every observer. Unlike material objects, numbers have no history or location. Thus, according to the Pythagoreans, they exist in a different world from ours which humans can explore. Note, however, that the Pythagoreans still ascribed shape, gender, and other characteristics to numbers.

The Pythagoreans laid the foundations for the idea of other worldly mathematical objects, and on that foundation, Plato systematically developed a broader theory of Forms. In Plato’s Forms we see the creation of a theoretical system ordering knowledge and reality using the Pythagorean idea of an other-worldly reality for things we “know that”, such as mathematics. With these Forms representing different degrees of reality (what is), Plato develops an epistemology (what we know) based on a hypothetical connection between

a Form and the knowledge of a material object, with a progression of becoming and being [8, page 158]. Plato's efforts provided order to his system of thought and reality, ultimately stemming from the most "real" Form, the Good, at the top of a hierarchy of Forms which was also asserted by Parmenides [8, page 158]. However, Plato may have had doubts about his theory of Forms. MacIntyre [37, page 45] describes the difficulties Plato encounters when trying to combine forms having a common characteristic in Plato's dialogue *Parmenides*, leading to an infinite regress. It should be noted that early Christian philosophers essentially adopted this Platonic framework by replacing the form Good with God at the top of the system of Forms [8, page 31]. The Forms and the ultimate Good Form set up an ongoing ontological dualism in Western thought between physical and non-physical entities through medieval times with Christianity's spiritual entities, and into the Enlightenment with Descartes's mind-body dualism.

An important epistemological question is: how do we come to know those other-worldly entities, specifically mathematical entities? Immanuel Kant attempted to answer this question through his concept of intuition [33, page 23, pages 50–53]. He argued that foundational mathematical ideas are necessary and *a priori* truths (which Kant seemed to consider equivalent), essentially meaning they are true in all possible worlds regardless of the physical experiences for any individual that has sufficient background knowledge. In addition, by Kant's definition, they are *synthetic*, essentially meaning they aren't true just by conceptual analysis (by simply the meaning of the terms). Thus, if we don't know these ideas by conceptual analysis nor by physical information from the real world, Kant argues we know them by intuition. One of his often-cited examples is the "truth" of the concept of a triangle. Kant argues that our conceptions of space explain this intuition of foundational necessary geometric truths [31, pages 28–29]. It is asserted that this Kantian intuition provides mathematicians with a mechanism for claiming to know mathematical non-physical objects (Forms). An example of this influence can be traced forward to Gödel [22]: "But despite their remoteness from sense experience, we do have a perception also of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see why we should have less confidence in this kind of perception, i.e. *in mathematical intuition* [emphasis added],

than in sense perception.” In another example, Frege [20] argued for support of the existence of mathematical objects through semantics and truth.¹

There have been several critiques of mathematical Platonism. Bernays points out that the paradoxes discovered in set theory imply the impossibility of two simultaneous ideas: (1) The totality of all mathematical objects and (2) “the general concepts of set and function” [5, page 261]. Thus, real mathematical objects as construed by traditional Platonism is impossible. Benacerraf describes an argument, which includes different set representations of numbers, concluding that “there is no more reason to identify any individual number with any one particular object than with any other” [2, pages 290-291]. But then what can Platonism mean if we can’t identify a unique object with a number, the most basic mathematical entity? Kitcher’s solution is instead to re-orient our understanding of mathematical ideas such as sets and functions away from mathematical platonism and to ultimately base those ideas on our experiences, concluding “just as acceptance of the approximate truth of geometry entails no acceptance of the actual existence of ideal geometrical objects, so too, our acceptance of the approximate truth of set theory and arithmetic does not fill our ontology with Platonic entities” [32, page 134].

Let’s now return to the epistemological question: how do we gain knowledge of platonic mathematical objects? Knowledge of such objects relies on a prior knowledge of at least some foundational mathematical objects (for example through Kant’s intuition). Kitcher [33] has painstakingly and persuasively described how the possibility of such *a priori* mathematical knowledge is unlikely, except in the case of universal empirical knowledge. Universal empirical knowledge is *a priori* knowledge obtained from empirical perceptions, which we would obtain in any world where we had sufficiently rich experiences [33, page 31]. By defining a rigorous definition of mathematical apriorism, Kitcher argues strongly against the existence of three types of *a priori* mathematical knowledge: conceptualist, constructivist, and realist (platonist) [33, page 47]. In other words, believing that mathematical knowledge arises from “intuition” is unsupported by a rigorous analysis.

¹ Frege’s position referred to here is briefly outlined in <https://plato.stanford.edu/entries/platonism-mathematics/#FreArgForExi>, last accessed on July 21, 2022.

To summarize, although we cannot prove such non-physical entities don't exist, there are strong arguments for why they do not exist and if they did exist, then we cannot access them.

3. Formalism and Logicism

Formalism emerged with an emphasis on the rule-based approach to mathematics, where we begin with axioms and symbolic rules which then generate true propositions. Thus, formalism consists of syntactics and symbol manipulation, not semantics. There have been a variety of flavors of formalism, but probably the most influential is Hilbert's program, where Hilbert "asserted that there was a certain type of evident reasoning which was presupposed in all scientific thinking, and finitist operations were typical of these" [34, page 208]. Thus, Hilbert's hope was that of "a final solution of the problem of foundations by a reduction of all mathematical reasoning to finitist reasoning" [34, page 209]. Since Hilbert, other formulations of formalism are given in Curry [13] and Cohen [12]. Logicism differs from formalism in that the rules used in logic are the basis for fundamental mathematical truths. But, like formalism, there are no referents in the mathematical or logical terms; the terms say nothing about anything in particular but only state logical truths.

Gödel's incompleteness theorems put Hilbert's formalist program in doubt. More importantly, Hersh points out that mathematicians' proof is not the axiomatic proof as described by the formalist or logicism adherents; mathematicians' "reasoning is semantic, based on the properties of mathematical entities, rather than syntactic, based on properties of formal sentences" [30, page 103]. "Following a proof is to engage in a particular kind of psychological process", and more specifically a proof is a "sequence of statements such that every member of the sequence is either a basic a priori statement or a statement which follows from previous members of the sequence in accordance with some apriority-preserving rule of inference" [31, page 37-38]. In effect, the "truth" of a statement starts with basic *a priori* statements and that truth is preserved by allowable inferences. First, as discussed above, there are no *a priori* mathematical statements not derived from experience. Second, there are a variety of inference rules (logical systems) for which we

could generate mathematical “truths”. Which do we use? We are now back to mathematical practice being concerned with semantics (meaning) to make such fundamental decisions, and where logic is then a subfield of mathematics, not the other way around.

4. An Evidence-based Alternative to Platonism. Formalism, and Logicism

Searle [41, pages 563–564] discusses the difference between epistemic objectivity and subjectivity. Epistemic objectivity is the hallmark of science where “scientists seek truths that are equally accessible to any competent observer and that are independent of the feelings and attitudes of the experimenters in question” [41, page 563]. It is reasonable to argue that this can only be approximated, since all experimenters will bring their biases into their research. In addition, the word “truth” here must also be approximated since science consists of models of reality, not exact truths. With those caveats, the contrast to epistemic objectivity, epistemic subjectivity, then means the question at hand cannot be settled with such (approximate) independence. However, *ontological* objectivity and subjectivity are different. Ontological objectivity is the existence of the objects in question independent of consciousness, while ontological subjectivity implies the existence of the objects in consciousness.

Mathematics is like science in that we have a competent mathematical community to decide if there is “truth” to tentative results made public to the community. Just as in science, the “truth” of the result is determined by the mutually agreed upon standards of reasoning of the mathematical community. This is how the mathematical community develops knowledge, and thus there is epistemic objectivity (approximately). On the other hand, mathematicians’ “objects”, mathematical concepts and ideas, are ontologically subjective; they exist in our consciousness as qualitative, subjective, and unified.² They do not exist independently, as argued in previous sections.

² Searle [41] describes three aspects of consciousness: Qualitative, Subjective, and Unity. Our conscious states have a “qualitative feel” to them and that quality could not exist without a particular subject experiencing it. We experience conscious states as a unified whole, not in pieces.

The evidence and theory of the source of our mathematical knowledge is described extensively by Lakoff and Nuñez [36]. Starting as infants, there is strong evidence that humans (and a variety of animals) have an innate understanding of numbers and very basic arithmetic. There is also evidence of specific locations in the brain where arithmetic calculations and algebraic manipulations occur. See [36, Chapter 1] for a summary of the early research on this. Thus, it does appear that we may begin life as humans with some universal *empirical* mathematical knowledge. Note, however, that there is no evidence that this knowledge derives from independent non-physical objects.

Lakoff and Nuñez then develop a theory of mathematical ideas based on evidence in the cognitive sciences on how humans conceptualize ideas, and they describe embodied cognition by “mind as it arises through interaction with the world” [36, page 350]. They argue that cognitive mechanisms lead to our mathematical ideas, which include common mechanisms used for “basic spatial relations, groupings, small quantities, motion, distributions of things in space, changes, bodily orientations, basic manipulations of objects, iterated actions” [36, page 28], and more. Note that these mechanisms arise directly from our interaction with our physical surroundings. For example, our basic spatial relations lead to various mental “schemas” such as *above*, *in contact with*, *support*, and *container*. The container schema gives us our starting point for the mathematical ideas of sets.

Another important concept is that of conceptual metaphor where we develop and understand our abstract mathematical concepts through more basic concepts, ultimately based on our interactions with the world. For example, the container schema is the basic embodied concept that are metaphors for first order logic. “An enormous range of empirical evidence has been collected that supports this view of conceptual metaphor” [36, page 47]. It is through these metaphors that we produce the huge richness of mathematics. For instance, the beginning of this process of creating mathematical ideas is addition as the adding of objects to containers or collections. We are not discovering mathematics but creating it through conceptual metaphors, ultimately based on our physical experience in the world, and validated by our mathematical community of agreed upon standards of proof and our view of mathematical “truth”.

In addition, Kitcher [33] developed a mathematical philosophy that aligns with the evidence of Lakoff and Nuñez. Kitcher describes an ideal agent theory whose basic components implicitly foreshadow the evidence-based theory of Lakoff and Nuñez and other cognitive researchers. Kitcher also illustrates several examples of how mathematics changes, such as mathematical practices, language, accepted reasoning, and what is believed to be true (and thus what is considered knowledge). Thus, the direction of mathematical change, the very “truth” of mathematical statements, and what mathematical objects are considered real or legitimate are historically contingent. For example, from approximately 1600 onward, there was disagreement over the validity and interpretation of $\sqrt{-1}$, which also caused a re-interpretation of other mathematical objects. Since there is no number whose square is -1 , then how could $\sqrt{-1}$ be a number? This required a re-interpretation of what numbers are, for example as objects that particular operations could be applied to. Note that this mathematical change was driven by the usefulness of the object $\sqrt{-1}$ in mathematical work and in scientific inquiry. In addition, this mathematical change required a change of mental representation, of what the referent for a number is. Thus, again, mathematical reality is ontologically subjective, i.e., a subjective experience of construction, not a perception of external objects.

Lakoff and Nuñez [36, pages 360–362] also give an example of historical dependency in the mathematics of floating point arithmetic, which depended on the development of computers. Similarly, Kitcher [33] provides examples of mathematics developed to solve problems within the mathematical and scientific community, not to “discover” other-worldly objects. An example is calculus, where Newton’s creation of his flavor of calculus was to describe motion while Leibniz’ creation of his version of calculus was to solve the tangent problem. Finally, there are multiple historical instances when new mathematical developments were rejected by the mathematical community but were eventually accepted. An interesting example is the construction of multiple infinities by George Cantor in the late 1800s, which were lambasted as abominations by mathematicians as prominent as Leopold Kronecker.³

³ Many good references to this history may be found; see for example Wikipedia: https://en.wikipedia.org/wiki/Georg_Cantor, last accessed on July 21, 2022.

Rather than viewing these historical examples platonically, as first not “seeing” their existence and yet somehow later “seeing” their existence, it seems a better explanation is that mathematics development is based on mathematical history, culture, and accepted practice.

Following up on this last example, it is important to briefly discuss the interaction of culture and mathematics. Culture is a set of beliefs, customs, traditions, and other aspects of a particular human society. In the last example, though more investigation on Cantor’s case would shed light, a reasonable hypothesis is that the strong reaction to Cantor’s inventions and results was that it violated what some in the mathematical community considered accepted standards of proof and what are traditional mathematical objects. This is an illustration of how cultural conditions (in this example within the mathematical community) could influence mathematics development. Regarding how pure mathematics may influence culture, there is some evidence that the Pythagoreans created a cosmological theory out of their beliefs about integers and arithmetic [6]. A more recent example is the results in number theory and their wide impact on cryptography and electronic communication. As Wilder [49] describes, multiple cultural processes have impacted the development of mathematics, including the printing press, language, and scientific developments within a particular society, and vice versa. More recently, the impact of sociopolitical ideas (such as power structures) on mathematics education has been studied, as described in [26] for example.

To summarize, what we have seen in our analysis of mathematical “foundations” is that the essence of mathematics is mathematical practice based on human cognition interacting with the external physical reality through metaphors and with our cultural environment. Mathematics is constrained by (1) the basic innate mathematics of universal empirical mathematical knowledge and (2) the cognitive metaphors that we use for reasoning in mathematics. Furthermore, its development is historically contingent and depends on what is considered acceptable reasoning by the mathematical community at the time, based on the cultural context during its development. Thus, mathematical practice is a creative, human, and cognitive-based endeavor restricted by accepted reasoning and conceptual metaphors, ultimately based on real world experiences. “Because of its cultural bases, there is no such thing as the absolute in mathematics; there is only the relative” [49, page 196].

To put it in Searle's terms, mathematics is ontologically subjective, but may be also considered external in that it subsists in a historical and cultural context. Note that having historical and cultural aspects does not imply that mathematics is arbitrary; this is because of the above constraints imposed on it. These constraints are substantial and explain the extreme stability of mathematics over time.

5. A Brief History of Western Ethics as related to Mathematics

The myths of platonism and formalism ignore the historical and social context and contingencies of mathematics, as described in the last section, dehumanizing mathematical practice. Similarly, Western ethical theories tended to be produced as if untethered to the historical point or cultural milieu they were created in. Once created, they were presented as fixed and permanent. Indeed, “[p]hilosophers and other intellectuals still betray, even if they less frequently parade, a cluster of habits of conceiving reason (moral or otherwise) as: separable from any cultural conditioning, transcending time and not historically conditioned, at its best when detached from a community of co-investigators, substantially separated from the limitations of physical life, and able to be cleaved from feelings and bodily inclinations. Reason is in a word pure, structured, and operating independently of the practical pressures of living” [19, page 29].

That idea of individualistic ethical reasoning untethered from our humanized state or our daily lives is unfounded just as platonism is. Let's briefly review this theme in Western ethical thought. In ancient Greece, an action by a person was judged to be good or moral if it the action was due to virtues that were developed because of the role that person had in social life and in the community. In other words, an action was ethical if a person expressed certain virtues as well as completed his (and then it always was about males) duty as circumscribed by their role in the community [37, Chapter 2 and pages 166-167].⁴ Aristotle was a prototypical proponent of this virtue theory

⁴ Hume argued that morality cannot arise from factual truth, described succinctly that “ought” cannot be determined by “is”. However, Hume admits that our wants and needs influences our norms [37, page 175]. Macintyre's argument countering Hume is as follows:

of ethics, which in his view was developed through habit. The ancient Greek model is that one's role implies one's duty: *is* affects what *ought*. Both our psychological nature and social context (what *is*) contribute to and influence our moral actions (*ought*).

Over the centuries, the Greek idea of roles changed and were made more malleable, coinciding with the rise of individualism. For example, the rise of the protestant reformation stressed the individual's personal contact with God. Thus, the original moral meaning of duty, and hence social norms, was lost. Kant reaches an important historical inflection of universality and yet individualistic trend by (1) arguing that moral rules are a priori, independent of experience and the material world [37, page 192] (see also [46, pages 84–86]), and (2) the moral rules satisfy a categorical imperative. This means that a moral actor would want the rule to universally apply to all moral beings and in all appropriate situations. We as *individuals* judge if a rule meets this categorical imperative and should be an ethical norm; individualism triumphs.⁵ An important aspect of this is that consequences are not considered. Kant defines duty to be following the ethical norms, satisfying the categorical imperative, shown to us by intuition. Thus, Kantian ethics falls back to intuition as justifying universal ethical norms, independent of history, culture, or practical human action, just as mathematical platonism relies on intuition.

This use of intuition in ethics was presaged by St. Thomas Aquinas' claim that we "rationally intuit natural law" [7, page 69], and of course Aquinas specified particular principles and natural inclinations that he felt persons should intuit as "natural". Early in the 20th century, Moore [38] continues this tradition of intuitionism in determining what is considered the "good". And this individualistic tradition continued in various subsequent theories.

Hume asserts that reasoning alone cannot cause action and only moral evaluations can, though reasoning can inform those actions. Clearly this cannot be true for all actions. For example, if I know that a robber is planning on robbing my house tonight, then reason tells me I should take some preventative action.

⁵ The other part of Kant's categorical imperative is that persons should be treated as ends, not as a means. Brannigan [7, page 133] argues that Kant is saying this means respecting their moral right to self-determination and autonomy. Again, individualism is paramount.

For example, MacIntyre [37, page 262], while discussing Hare's analysis [29], argues that "Hare's prescriptivism is, in the end, a reissue of the view that behind moral evaluations there is not and cannot be any greater authority than that of my own choice. This is a repetition of Kant's view of the moral subject as lawgiver", as given through intuition.

Another feature that mathematical platonism and Western ethics share is the inherent dualisms in these theories. Mathematicians have created incredibly beautiful ideas but then, as described in an earlier section, some ascribe to those ideas an otherworldly ontological status separate from our own human existence, the platonic myth. In its more extreme form of Kantianism, ethical theory is determined by individualistic intuition also divorced from the social context in which conduct (which defines ethical action) is performed. All that matters is what the moral actor intuits as one's duty consistent with the categorical imperative. As Fesmire points out, "So, for adaptive purposes, we speak of thought and thing, mental and physical, internal and external, idea and sensation. We split experience from nature, signifier from signified, representation from represented, word from object. Then we mistakenly conclude that these binaries correspond to independently existing entities or states of affairs, like Descartes's *res cogitantes* and *res extensa*" [19, page 47]. However, as described in earlier sections, our mathematical ideas are not independent of our brains, our natural world, our social context, and our historical location. Similarly, our individual conduct and direction in mathematical practice are not independent of our culture and our placement in society, mathematical or otherwise.

To summarize, there appear to be important similarities between the currently dominant threads of mathematical philosophy and Western ethical philosophy. Namely, that the objects (in mathematics) or the proper ethical actions to follow (in ethics) are independent of human experience and context, operating separately from social and historical trends, and that we have a mysterious intuition of these objects or right actions. It seems, then, it should not be surprising that mainstream voices in the mathematical community would deny that mathematical practice and development (at least in "pure" mathematics) has any meaningful relationship with current human experience or with ethics beyond one's individual actions.

6. Ethics and Mathematical Foundations

Now let's consider some specific situations for how our view about mathematical foundations may impact ethics in mathematical practice and instruction. First, we examine who is considered worthy to do mathematics. Grant and Kahneman [24] describe two model systems of the mind: System 1 and System 2. System 1 is that system that operates automatically with little voluntary control (sometimes also called intuition). System 2 is our mind focusing attention and effort on mental activities that we experience as conscious control and choice. According to [24], the mind's System 1 forms a narrative for us that fits our pre-conceived unconscious network of knowledge and beliefs. Now recall Grant's statement that "our belief can drive what we value" [48]. Thus, our mathematical ontological and epistemological beliefs will impact our values and hence our conduct through our mathematical practice. Specifically, a danger of the Platonic view of mathematics is that it provides a convenient reason for a practicing mathematician to hold pre-conceived beliefs, where the mathematician may not be aware of why they really hold such beliefs, due to the workings of System 1.

Suppose now that we have been subsumed in the Platonic view of mathematics, where mathematical objects are discovered by a mathematical intuition. The often-unstated argument is: If you don't possess that special intuition, then you are not capable of doing real mathematics. That mathematicians might see only some as capable mathematical researchers is actually driven by our current social, historical, and ethical context informing our unconscious network of beliefs, a context where sexism and racism in our society are starting to be recognized more fully. Thus, our ethical decisions as practicing mathematicians are determined by the social context we are in, which in turn determines who practices what mathematics, and thus guides mathematical development.

In effect, the platonic myth of intuition provides an outward justification for our pre-conceived ideas of who is capable. The idea that there are people who can do scholarly mathematics and there are those who should only do practical mathematics has an ancient history in mathematics. D'Ambrosia describes this history, from Egyptians to Greeks to Romans in [14].

Once platonism, and its enabler, the supposed intuition of mathematical objects, are established as the paradigms within an already systemically racially biased society, then bias and racism may establish themselves as the lens in viewing who is able or worthy to learn mathematics. For example, Chestnut *et al.* [10] and Storage *et al.* [44] describe how groups that are not considered to possess brilliance and talent in a field, such as mathematics, are under-represented in that field. This “myth of brilliance” appears to impede the mathematical progress of practitioners and students alike. And the racism can be much more direct. For example, several early pioneers of statistics, such as Francis Galton, used statistics to justify their eugenics beliefs. This racial and ethnic bias in Western mathematics in general (beyond foundational beliefs) is described in [40]. “We cannot escape the fact that statistics is a human enterprise subject to human desire, prejudice, consensus, and interpretation” [11]. As described earlier, mathematics is no different; it is a human enterprise.

Similarly in high school and college instruction, it is plausible that belief in mathematical platonism has the potential effect of perpetuating, through teachers, the belief that only some people are worthy enough to engage in mathematical practice. If you are not considered capable of doing real mathematics, then why even try? This leads directly to the fixed mindset of many students [16]. Lakoff and Nuñez call the belief in mathematical platonism the “Romance of Mathematics” [36, page 338] and discuss the consequences of this Romance: “It intimidates people. It makes mathematics seem beyond the reach. Of even excellent students with other primary interests and skills. It leads many students to give up on mathematics” [36, page 341]. Any high school or university teacher can attest to the common statement of students that “I am bad at math” or “I cannot do mathematics”. “The Romance serves the purposes of the mathematical community. It helps maintain an elite and then justify it” [36, page 341]. In other words, it leads to a vicious circle where people who have been elevated as capable become practicing mathematicians and then carry that philosophy forward in their instruction.

Next let’s examine WHAT areas of mathematics are considered worthy of study. Consider the divide between pure and applied mathematics as exemplified by Hardy’s statement that “[w]e have concluded that the trivial mathematics is, on the whole, useful, and that the real mathematics,

on the whole, is not" [28, page 43]. Though originally stated in 1940, discussions of this biased binary view of mathematics can regularly be found subsequently, such as expressed by panelists in the 1962 symposium on applied mathematics [9], up to today. The idea that the greatness of mathematics derives from its abstractness and independence from physical reality, as described by Stone [43] implies a hierarchy of worthy subject areas, creating division that damages the profession and mathematical development. Besides being incorrect about mathematics being independent of physical reality (as described in Section 2), the interplay between abstraction and applied mathematical pursuits supports each other and creates progress for both.⁶ Again, Newton's and Leibniz' invention of calculus are excellent examples of this. Finally, note that which scholars are given leadership positions (in mathematical and research associations) and which ones receive attention are also affected by what mathematics is considered worthy.

The danger of formalism is different but also problematic. In a formalistic view, mathematics becomes meaningless, consisting of moving symbols according to pre-defined rules.⁷ Though I believe it would be difficult to find a practicing mathematician agreeing that their work is completely meaningless, that undercurrent of meaninglessness appears to imply that there are no ethical quandaries to consider in mathematics itself. However, surely the potential and actual use of the variety of mathematical creations throughout society requires ethical choices to be made, as in any scientific discipline. A recent example is the mathematics used in artificial intelligence, which faces fundamental ethical questions [25].

In education, every mathematics high school and college instructor has encountered students that view mathematics in this purely formal manner (possibly most of their students!). Where does this come from? If an instructor begins to absorb the formalist view as their own foundation, then it would become natural to teach this formalism AS mathematics, regardless of what actual mathematical practice really is. Some mathematics students (and even instructors) find following established rules and determining the "correct" answers comforting. Unfortunately, as emphasized above, that is alien

⁶ Courant makes an eloquent statement on this in [9].

⁷ In cultural terms, this is called symbolic reflex behavior [49, page 193].

to how mathematical practice actually works and, in fact, can be detrimental to meaningful student learning and problem-solving. “It is part of a culture that rewards incomprehensibility . . . Socially, the inaccessibility of mathematics has contributed to the lack of adequate mathematical training in the populace in general” [36, page 341]. Finally, this educator belief in formalism impacts mathematics itself, not just education, as Wilder states “It moves one to wonder how many potentially great mathematicians are being constantly lost to mathematics because of ‘symbolic reflect’ types of teaching” [49, page 194].

So why is mathematics taught in this formalist framework? Certainly, educating formally is easier than demanding a fuller student understanding and expecting a student to apply the mathematics to new situations. Also, as described by Handal [27], traditional instructional beliefs are difficult to dislodge, with teacher’s beliefs guiding their instruction. But there may be a deeper reason which also illustrates the impact of a formalist view. Kollosche stated that “it gives us the opportunity to judge your ability and willingness to be a logico-bureaucratic subject” [35, page 307]. His argument is that mathematical education based on formalism is training in following rules, needed for many jobs and roles in our society. Closely related, the formalist reasoning in math education leads to a de-humanization of our lives in society. “By focusing on technical means and not on the ends of their actions, persons, governments and corporations risk complicity in the treatment of human beings as objects to be manipulated, in actions that threaten social well-being, the environment and nature” [18, page 8].

7. Moving Forward

What is ethics? Ultimately, ethics is about making judgements to guide conduct, and, as expressed by John Dewey, the purpose of ethics is to live a rich and meaningful life. John Dewey believed that conduct has two aspects. The first aspect is having a life of purpose, where we have “thoughts and feelings, ideals and motives, valuation and choice” [15, page 2]. The second aspect is an individual’s relationship with nature and human society. Thus, ethics cannot be separated from our personal goals, practical situation, historical context, and our social life. Note the similarity of this approach to mathe-

mathematical practice that occurs within a social and historical context, based on the goals and predilections of the mathematical community of the period. Thus, Dewey's ethical approach is more aligned with mathematical practice than the traditional Western ethical trend as described in an earlier section.

It is time we re-consider our roles as mathematicians and how our foundational and ethical beliefs impact our mathematical practice, which then impact our ethical conduct. In contrast to the traditional Western ethical philosophy, John Dewey argued that we should stop searching for a single ultimate principle that will answer our ethical problems for us because there are none.⁸ This is because in our practical social lives, there will always be inherent conflicts in which we must balance incompatible aims and values. As a pragmatist, he identified three types of ethics that we will balance: virtue ethics, deontological ethics (that is, ethics determined by duty, which could include a sense of justice), and consequential ethics (of which John Stuart Mill's utilitarianism is the most well-known example) [19, page 56]).

Sometimes a mathematician might say they just research "math" and just teach "math", as if mathematics is a body of knowledge independent of everything else, which also provides a convenient excuse not to reflect on one's role in mathematical practice. Instead, we should consider what it means to practice and impart mathematics from a humanistic standpoint, balancing all three types of ethics in that practice. From the point of view of virtue, what character traits do we want to encourage in ourselves as mathematicians and students as future mathematicians? From the deontological standpoint of justice, we should actively resist the fables of brilliance and so practice and educate mathematics in a manner that counters those fables, promoting equity and justice. In addition, we should consider the potential consequences of our mathematical work and instruction on society. For example, what are the consequences of formalism? Not only does it remove the understanding

⁸ For example, to apply Kant's categorical imperative to any practical situation, we must restrict its applicability to those actors in that particular situation. The problem with this is that with sufficient thought and restriction, nearly any proposed rule can be universalized, and this thus makes the categorical imperative without content [37, pages 197–198]. This also provides an argument that Kant's rule does not provide an ironclad universal ethical principle but instead one of several possible guides in deciding one's conduct, much like "do unto others as you would have done to you".

of mathematics, but its basic formulation is strict rule-following. Is that what we think is best for a democratic society? Finally, a re-orientation from traditional philosophic beliefs to a humanistic, evidence-based orientation may move us toward cooperation, instead of division and elevation of favored areas of mathematics.

How we educate students is also an ethical choice that impacts future mathematical practice, our society, and future generations in countless ways. Ernest [17] argues that the first philosophy of mathematics education should be ethics. Peck [39] claims that five aspects of mathematics education are affected by the foundational philosophic choice of educators and the educational system. These are (1) mode of learning (authenticity and how active the pedagogy is), (2) classroom cultural environment, (3) historical viewpoint, (4) critical examination of mathematics in society, and (5) social interaction in mathematical practice. A humanistic belief in mathematics leads to authentic and active learning with a classroom culture that empowers students to invent and create. An evidence-based approach to foundations, based on our shared humanity, leads to a rich historical account of mathematics and thus a way for students to see themselves as active “actors” [39, page 8] in mathematics. Seeing the direction of mathematics as historically contingent can lead mathematicians and students to critically examine choices made in mathematics past and present. Finally, through authentic activities and the humanistic approach to mathematics, students and researchers alike can appreciate mathematics as the social enterprise it is, where the community of practitioners decide what results are acceptable and worthwhile in the human context they currently live in.

The evidence is that we first experience basic mathematics in our interactions with the material world, and mathematicians build on that experience through a psychological and socially created enterprise within certain constraints. Thus, contrary to mainstream Western philosophical thought, mathematics is inseparable from physical, psychological, social, and historical context, as is ethics. Because of that, mathematics does not exist in some separate realm; on the contrary it is fundamentally tied to mathematical practice, a human enterprise. Mathematical practice can be performed ethically or not, and because mathematics and practice are inseparable, mathematics itself have ethical aspects, along with mathematics education.

Mathematics is an intensely human and social endeavor. In that view, mathematical practitioners and instructors would see *each* person as having the potential to live a complete and fulfilling life which includes experiencing mathematics for what it is: as a human and social enterprise. This is what it means for mathematics, its practice, and mathematics education to be ethical. It is time our mathematical community seriously reflects on and transforms its foundational beliefs so it can grow and contribute to our society in an ethical manner.

References

- [1] Keturah Anderson, Jo Boaler, and Jack Diekmann, “Challenging Myths about Learning: A Blended Approach,” *Education Sciences*, Volume 8 Issue 3 (2018), page 98. doi:[10.3390/educsci8030098](https://doi.org/10.3390/educsci8030098)
- [2] Paul Benacerraf, “What numbers could not be,” *Philosophical Review*, Volume 74 (1965), pages 47–73. Reprinted on pages 272–294 in [4].
- [3] Paul Benacerraf, “Mathematical Truth,” *Journal of Philosophy*, Volume 70 Issue 19 (1973), pages 661–679. Reprinted on pages 403–420 in [4].
- [4] Paul Benacerraf and Hilary Putnam (editors), (*Philosophy of Mathematics: Selected Readings*, Cambridge University Press, New York, 1983.
- [5] Paul Bernays, “On Platonism in Mathematics.” *L’enseignement mathématique*, Volume 34 (1935), pages 52–69. Reprinted on pages 258–271 in [4].
- [6] Carl Boyer, *A History of Mathematics*, 2nd edition, John Wiley & Sons, New York, 1968.
- [7] Michael Brannigan, *Ethics Across Cultures*, McGraw Hill, Boston, 2005.
- [8] Robert Brumbaugh, *The Philosophers of Greece*, State University of New York, Albany, 1981.
- [9] George F. Carrier, R. Courant, Paul Rosenbloom, C. N. Yang, and H. J. Greenberg, “Applied mathematics what is needed in research and education,” *SIAM Review*, Volume 4 Issue 4 (1962), pages 297–324. doi:[10.1137/1004087](https://doi.org/10.1137/1004087)

- [10] Eleanor K. Chestnut, Ryan F. Lei, Sarah-Jane Leslie, and Andrei Cimpian, “The Myth That Only Brilliant People Are Good at Math and Its Implications for Diversity,” *Education Sciences*, Volume 8 Issue 2 (2018), page 65. doi:[10.3390/educsci8020065](https://doi.org/10.3390/educsci8020065)
- [11] Aubrey Clayton, “How Eugenics Shaped Statistics: Exposing the damned lies of three science pioneers,” *Nautilus*, Issue 092, October 28, 2020. Web article at <https://nautil.us/issue/92/frontiers/how-eugenics-shaped-statistics>, last accessed on July 22, 2022.
- [12] Paul Cohen, “Comments on the Foundations of Set Theory,” pages 9–15 in *Axiomatic Set Theory*, Proceedings of Symposia in Pure Mathematics 13, Part 1, edited by Dana S. Scott (American Mathematical Society, Providence, 1971).
- [13] Haskell B. Curry, *Outlines of a Formalist Philosophy of Mathematics*, North-Holland, Amsterdam, 1958.
- [14] Ubiratan D’Ambrosia, “Ethnomathematics and its Place in the History and Pedagogy of Mathematics,” pages 13–24 in [40].
- [15] John Dewey and James Hayden Tufts, *Ethics*, Henry Hold and Co., London, 1909.
- [16] Carol Dweck, *Mindset: The New Psychology of Success*, Ballantine Books, New York, 2016.
- [17] Paul Ernest, “What is Our First Philosophy in Mathematics Education?” *For the Learning of Mathematics*, Volume 32 Issue 3 (2012), pages 8–14.
- [18] Paul Ernest, “The Ethics of Mathematics: Is Mathematics Harmful?” pages 187–216 in *The Philosophy of Mathematics Education Today* edited by Paul Ernest (ICME-13 Monographs, Springer, Cham). doi:[10.1007/978-3-319-77760-3_12](https://doi.org/10.1007/978-3-319-77760-3_12)
- [19] Steven Fesmire, *John Dewey and Moral Imagination: Pragmatism in Ethics*, Indiana University Press, Bloomington, 2003.
- [20] Gottlob Frege, *Foundations of Arithmetic*, 1884. English translation by J.L. Austin, Blackwell, Oxford, 1980.

- [21] Gottlob Frege, “The Concept of Number.” 1884. Reprinted on pages 130–159 in [4].
- [22] Kurt Gödel, “What is Cantor’s Continuum Hypothesis?” *The American Mathematical Monthly*, Volume **54** Number 9 (November 1947), pages 515–525. Revised and expanded version published on pages 470–485 in [4].
- [23] Nicholas D. Goodman, “Mathematics as an Objective Science,” *American Mathematical Monthly*, Volume **86** Issue 7 (August-September 1979), pages 540–551. Reprinted on pages 79–94 in [47].
- [24] Adam Grant and Daniel Kahneman, *Thinking, Fast and Slow*, Farrar, Straus and Giroux, New York, 2011.
- [25] Brian Patrick Green, “Artificial Intelligence and Ethics: Ten areas of interest,” Markkula Center for Applied Ethics website, Nov 21, 2017. An updated version (“Artificial Intelligence and Ethics: Sixteen Challenges and Opportunities”) is available at <https://www.scu.edu/ethics/all-about-ethics/artificial-intelligence-and-ethics-sixteen-challenges-and-opportunities/>, last accessed on July 22, 2022.
- [26] Rochelle Gutiérrez, “The Sociopolitical Turn in Mathematics Education,” *Journal for Research in Mathematics Education*, Volume **44** Issue 1 (2013), pages 37–68. doi:10.5951/jresmetheduc.44.1.0037
- [27] Boris Handal, “Teachers’ Mathematical Beliefs: A Review,” *The Mathematics Educator*, Volume **13** Issue 2 (2003), pages 47–57.
- [28] G.H. Hardy, *A Mathematician’s Apology*, Cambridge University Press, Cambridge, 1940 / 1992.
- [29] R.M. Hare, *Freedom and Reason*, Oxford University Press, London, 1963.
- [30] Reuben Hersh, *Experiencing Mathematics: What do we do when we do mathematics?*, American Mathematical Society, Providence, 2010.
- [31] Philip Kitcher, “Kant and the Foundations of Mathematics,” *The Philosophical Review*, Volume **84** Issue 1 (1975), pages 23–50.

- [32] Philip Kitcher, “The Plight of the Platonist,” *Noûs*, Volume **12** Issue 2 (May 1978), pages 119–136.
- [33] Philip Kitcher, *The Nature of Mathematical Knowledge* Oxford University Press, New York, 1984.
- [34] Georg Kreisel, “Hilbert’s programme,” *Dialectica*, Volume **12** Issue 3/4 (1958), pages 346–372. Revised version on pages 207–238 in [4].
- [35] David Kollosche, “The true purpose of mathematics education: A provocation,” *The Mathematics Enthusiast*, Volume **15** Issue 1/2 (2018), pages 303–319.
- [36] George Lakoff and Rafael Nuñez. *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*, Basic Books, New York, 2000.
- [37] Alasdair MacIntyre, *A Short History of Ethics: A History of Moral Philosophy from the Homeric Age to the Twentieth Century*, Macmillan Co., New York, 1966.
- [38] G.E. Moore, *Principia Ethica*, Dover Publications, Garden City, NY, 1903.
- [39] Frederick A. Peck, “Rejecting Platonism: Recovering Humanity in Mathematics Education,” *Education Sciences*, Volume **8** Issue 2 (2018), page 43. doi:[10.3390/educsci8020043](https://doi.org/10.3390/educsci8020043)
- [40] Arthur Powell and Marilyn Frankenstein (editors), *Ethnomathematics: Challenging Eurocentrism in Mathematics Education*, State University of New York Press, Albany, 1997.
- [41] John R. Searle, “Consciousness,” *Annual Review of Neuroscience*, Volume **23** (2000), pages 557–578. doi:[10.1146/annurev.neuro.23.1.557](https://doi.org/10.1146/annurev.neuro.23.1.557)
- [42] Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology*, Oxford University Press, Oxford, 1997.
- [43] Marshall Stone, “The Revolution in Mathematics,” *The American Mathematical Monthly*, Volume **68** Issue 8 (1961), pages 715–734.

- [44] Daniel Storage, Zachary Horne, Andrei Cimpian, and Sarah-Jane Leslie, “The Frequency of ‘Brilliant’ and ‘Genius’ in Teaching Evaluations Predicts the Representation of Women and African Americans across Fields,” *PLoS ONE*, Volume **11** Issue 3 (2016), e0150194. doi:[10.1371/journal.pone.0150194](https://doi.org/10.1371/journal.pone.0150194)
- [45] Max Tegmark, *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*, Knopf, New York, 2014.
- [46] Mel Thompson, *Teach Yourself Ethics*, NTC/Contemporary Publishing, Chicago, 2000.
- [47] Thomas Tymoczko (editor), *New Directions in the Philosophy of Mathematics: An Anthology* (Revised and Expanded Edition), Princeton University Press, Princeton, 1998.
- [48] Shankar Vedantam, “The Easiest Person to Fool,” *Hidden Brain*, episode published in 2021. Available at <https://hiddenbrain.org/podcast/the-easiest-person-to-fool/>, last accessed on July 22, 2022.
- [49] Raymond Wilder, “The Cultural Basis of Mathematics,” pages 258–271 in *Proceedings of The International Congress of Mathematicians* Volume 1 (1950). Reprinted on pages 185–200 in [47].