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### Recommended Citation

Adriano Demattè, "Violence in Mathematics Teaching. Reflections Inspired by Levinas' Totality and Infinity," *Journal of Humanistic Mathematics*, Volume 12 Issue 2 (July 2022), pages 72-97. DOI: 10.5642/jhumath.CJDW2311. Available at: <https://scholarship.claremont.edu/jhm/vol12/iss2/7>

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# Violence in Mathematics Teaching: Reflections Inspired by Levinas' *Totality and Infinity*

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## Abstract

In mathematics class, violence is carried out in some usually not recognized situations. In this article, I share some reflections on the topic inspired by some passages of *Totality and Infinity*, work of the French-Lithuanian philosopher Emmanuel Levinas (1906–1995). I find violence by the teacher in not promoting students' understanding of mathematics, in a distorted use of rhetoric, and in interrupting an ethical relation. This article analyses situations taken from class activities, focusing on the teacher's presentation of mathematical content and students' interventions. I also propose that the improvement of interventions in mathematics education is possible and suggest theoretical ideas for acting on situations of violence.

**Keywords:** Levinas, violence, ethics, mathematics teaching

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## 1. Introduction

*Totalité et infini: essai sur l'extériorité* (*Totality and Infinity: An Essay on Exteriority*, hereafter TI [11]) is considered one of the fundamental works of the French-Lithuanian philosopher Emmanuel Levinas (1906–1995). In TI, Levinas rethinks man through a phenomenological analysis. His research recovers the roots of ethics and frames ethics as a different description of human subjectivity and not an *a priori* definition of a code. Levinas illustrates the ethics of the face which is the epiphany of the Other. The concept of face concerns the authority experienced in front of the Other.

The face says that Other's life must be protected according to a mysterious relation that is established in every manifestation of the face.

Derrida [4] includes a chapter entitled "Violence and metaphysics – Essay on the thought of Emmanuel Levinas," thus creating, through the concept of violence, a sort of nucleus around which to gather the entire thought of the French-Lithuanian philosopher. For Levinas, violence is a "thematization of the Other," a reification of the Other, a way of grasping the Other. Violence is injustice.

The term "violence" appears frequently in the works of Levinas, and its meaning is progressively enriched, also including non-negative meanings. In the initial part of TI [11], Levinas says:

[V]iolence does not consist so much in injuring and annihilating persons as in interrupting their continuity, making them playing roles in which they no longer recognize themselves, making them betray not only commitments but their own substance, making them carry out actions that will destroy every possibility for action" [11, page 21].

This excerpt cannot be considered a definition of "violence" as Levinas intends it. We will see in the rest of this article some examples of class situations: to discuss them we will refer to other TI passages. However, I believe it can be extremely useful for introducing the topics related to mathematics education that I wish to address. When Levinas speaks of "playing roles in which they no longer recognize themselves," I think of students who consider mathematics as a discipline they cannot make sense of, which they do not believe can be part of their life (for a challenge with themselves, for the sake of solving a problem or to have a tool for analysing the reality). Because of this, the teacher's requirements risk turning into a way of making students betray their own substance. What the mathematics teacher requires, then, are actions that cannot leave a constructive mark in students' education, actions that certainly do not allow them other actions that can lead to mathematical understanding.

## 2. Face of the Other, ethics, discourse

In various works, Ernest deals with the theme of Ethics and Mathematics: see, in particular, [6]. In another work, he highlights that mathematics “is widely and uncritically assumed not only to be wholly beneficial but also to be beyond any ethical responsibility”, but he proposes “a critical analysis of both the good and the harm mathematics causes [7].” Radford [13] refers to “mathematics teaching and learning as an ethical event”. I agree with both authors in believing that the teaching/learning of mathematics is intrinsically connected with ethical relations.

In this article, I offer a different analysis from that of Ernest, focusing on the teacher-student relation. Furthermore, I propose examples in which Radford’s statement is referred to the concreteness of class work. I focus on how the teaching of mathematics can determine situations that have generally neglected ethical relevance. I consider first of all the students as those who suffer violence. I do not mean to refer to cases in which they might be subjected to beatings or physical violence which, until a few decades ago, were widespread in the Italian school system, and in other countries. Rather, I want to highlight that violence against the students is putting them in a situation that does not allow them to start from their background, to express themselves in an authentic way. I believe this can have very negative repercussions, hindering the possibility of giving meaning to the content they are facing and therefore of understanding.

Levinas illustrates [11, page 194] the ethics of the face which is revelation of the Other. The face is the manifestation of another human being, his exteriority. It introduces the possibility of the I and the Other to enter a relation. On the other hand, the face is only a trace of the Other and it does not exhaust its being because there is always a beyond that cannot be grasped. In this way, the Other is placed in an infinity, in a position of transcendence. According to Levinas, the ego is “captured” by the Other, who may be weak and defenceless but, paradoxically, commands. Wanting to enclose the Other in a description and therefore in a totality is an unattainable goal. Levinas says that “the relation with the Other alone introduces a dimension of transcendence and leads us to a relation totally different from experience in the sensible sense of the term” [11, page 193].

That is, we cannot exhaust the relation with the Other through a sensitive experience. If that were the case and we were able to give a description that includes it, the relation would cease. These, in a nutshell, are some ideas that I believe can help to explain the title of Levinas' work from which we are drawing inspiration.

The meaning of the face goes beyond that of "part of the human body". Let's think about when we say: "He revealed his true face". We mean that he has revealed his true nature, made up of beliefs, desires, personal reasonings etc.

Considering the mathematical text as the production of a human being, I reflected on the fact that the text has a content that the author intentionally wanted to insert into it. The author's primary purpose is to involve the reader. The mathematical text is an exteriority, a trace, a means to establish a relation between human beings.

According to what Levinas says, ethics originates from the I-Other relation. The relation between human beings that is established through a mathematical text (or any medium concerning mathematical knowledge) requires that the reader be willing to be commanded — or guided — by the Other behind the text, i.e., to pay attention to, to trust the proposal by dedicating time and exercising the patience necessary to follow the author's reasoning, to be willing to review prejudices, to receive a teaching. If this is not done, understanding cannot be developed. I think I can therefore say that there is an ethics in dealing with a mathematical document. On the part of the students, it is in the relation with the Other that the possibility of accessing new horizons of knowledge is realized, to be open to teaching, without assuming that only what they possess — which is part of their horizon — can be the source of advancement of knowledge.

Gadamer [9] speaks of "horizon" as a range of vision that influences the ability to interpret a text, to understand. Knowledge is in itself appropriation, possession. In a form that may appear provocative, Levinas states that "[k]nowledge would be the suppression of the Other by the grasp, by the hold" [11, page 302], and compares knowledge and violence. However, knowledge is an always open process due to the fact that it is achieved through discourse together with the Other. The Other conditions me and calls me to come out of myself to build a common world, for example mathematical concepts.

It is part of mathematics education to always offer students new horizons. In fact, the discourse through which the relation with the Other is built (relation generated from the manifestation of the face and from which ethics arises, but without the need to thematize it and reflect on moral principles) does not determine an I-Thou relation, closed in on itself. In the eyes of the Other there is all of humanity. The author, the teacher, and anyone from whom the student accepts a teaching are bearers of mathematical knowledge whose boundaries can be those of humanity itself. In the words of Levinas:

The relation with the Other, discourse, is not only the putting in question of my freedom, the appeal coming from the other to call me to responsibility, is not only the speech by which I divest myself of the possession that encircles me by setting forth an objective and common world, but is also sermon exhortation, the prophetic word. By essence the prophetic word responds to the epiphany of the face, doubles all discourse not as a discourse about moral themes, but as an irreducible movement of a discourse which by essence is aroused by the epiphany of the face inasmuch as it attests the presence of the third party, the whole of humanity, in the eyes that look at me. [11, page 213]

This quote summarizes many of the aspects seen above but also introduces the “third-party” — for us, a starting point to see mathematics in the context of the culture and heritage of all humanity; this opens up the possibility of reflections that go beyond the scope of this article.

Let us now try to bring ourselves to what happens in the classroom, focusing on aspects of the relation between teacher and student. We will start once again from a passage of TI, and from its generality we will move on to examine a specific situation. I want first to consider that the term *teaching* used by Levinas has a broader meaning than what we can refer to with the teacher-student relation. The I-Other relation proposed by Levinas provides for the establishment of authority. He affirms that the willingness to accept a teaching originates within this relation. “The relation with the Other, or Conversation, is [. . .] an ethical relation; but inasmuch as it is welcomed this conversation is a teaching [enseignement]” [11, page 51]. Faced with the Other, the thematization is converted into conversation, into an ethical relation.

From focusing on elements of knowledge, one moves on to waiting for novelty, remaining suspended in the face of the possible, of which the Other is the bearer. Approaching the Other in a conversation is a way to welcome what he can tell, to accept that it can go beyond what our thinking can be. This is also what the attitude of students — facing a text or, in general, a material of a new study proposal — should be: first of all, open to what the document will present. Behind this document they must sense the presence of another human being — independent of their initiative and their power — who can guide, stimulate, urge them.

In the encounter with the Other, one can be animated by the desire to know it. Let's think about the teacher in front of a student she has never met before. We think that the teacher wants to have information regarding the personal situation of the student, above all his preparation in mathematics. From these, she will start her educational intervention. Let's think about what will happen next. The teacher might take into account the initial information about that student — about his being. She could interact with that student by remembering his situation, helping him to overcome those possible difficulties he initially showed he had, suggesting ways for him to improve his way of doing mathematics.

The teacher will expect responses from the student, both during the lesson and through the verification tests. The relation between the teacher and the student has changed over time because at the beginning it was based on knowledge, on thematizing the student's characteristics, then it has turned into an exchange in which the teacher also expects a teaching — that is, what the student can reveal her about how his mathematics learning is progressing — in order to modify, if necessary, the educational intervention. In other words, the thematization changed into conversation.

The latter term is the translation of “discours” which appears in the French original. I agree with the translator's choice because “conversation” expresses better than “discourse” the idea of an interaction that is not planned, which is open to what the Other is the bearer of. This suggests, in essence, that the relation between two human beings is “ethical”. Knowing — thematizing — is a way to possess, to make one's own, in a certain way to dominate the Other; on the contrary, being in conversation means not having the relation with the Other under control, accepting the novelty, responding accordingly

and, indeed, not in a planned way. What has happened is not simply a modification of thought, but the conversion into an original relation with exterior being: see [11, page 66].

### 3. Lack of understanding?

In some cases, students perform a task correctly but reveal, for example, that they do not recognise the logical links between the different parts of what they write or say, do not know to explain why one proposition is the premise of another, cannot produce examples and counterexamples, and so on. I think that the widespread “banking model of education” [8] — in which the teacher makes deposits that the students patiently receive, memorise, and repeat — tends to give rise to unsatisfactory situations of that type. In addition, the school does not implement interventions that allow students with difficulties to remedy their shortcomings.

Another author who made a critique of education in various respects similar to, despite having lived in a different period and context than, Paulo Freire (1921–1997), is the Italian philosopher Antonio Gramsci (1891–1937). Freire worked in Brazil, but during his long period of exile, he read Gramsci. He himself stated: “I discovered that I had been greatly influenced by Gramsci long before I had read him” (quoted in [12]). I would like to remind the reader of the attention these two authors paid to the educational needs of the weakest people (belonging to certain social classes) and their analysis of the reality in which the school tends to amplify their negative conditions.

Mathematics is a school subject that causes difficulties in a very high percentage of students. I believe that the task of research in mathematics education should be the investigation of situations that induce difficulties in students, trying to identify and analyse those that are usually not taken into consideration. I believe one method can be constituted by philosophical reflection and the use of qualitative research tools. That is the main method of this present article.

In the following, I want to analyse some specific initiatives of the school itself that induce the students to abandon the search for understanding and to choose other ways to be able to face tasks and to perform them. I am not referring to fraudulent ways but to strategies that the students adopt



during their own preparation, and which are aimed at passing a test, not at understanding. We will see how the school induces these ways, proposing shortcuts that exempt students from going through a preparation work that would require, maybe, greater investment of time. I believe that these choices of the school are also relevant from an ethical point of view because they compromise students' possibility of acting when they are called to face new situations.

I want to focus on the case of Aura (pseudonym), a female secondary school student. She is carrying out a reinforcement exercise under the guidance of the teacher who addresses the whole class. She must establish the sign of

$$\frac{1+x}{1-x}$$

and asks: "Do I always have to write the equation  $1+x=0$ ?" (similarly to what was done in other previous exercises). She knows to find the value for which the numerator is equal to zero, she has already understood that it is -1, but she doesn't realize that writing and solving that equation really means finding the value for which the numerator equals zero. In fact, in another situation, having to find the zero of the polynomial  $P(x) = 3x - 5$ , she does not know how to operate (she cannot identify it quickly and does not think she can set the equation). Aura can solve the exercise:

"Study the sign of the function  $f(x) = (3x - 5)/(1 - x)$ " following the steps:

- 1) I write the equations  $3x - 5 = 0$ ,  $1 - x = 0$ ;
- 2) I draw a number line on which I place the first solution, one on which I place the second;
- 3) on the first line I write + to the right of  $5/3$  and on the second line + to the left of 1 following the rule: "if  $x$  is positive [the coefficient is positive], the sign + goes to the right but if  $x$  is negative + goes to the left", learned as a further mnemonic rule;
- 4) "- with + gives - ; - with - gives + ; + with - gives -"

Note how there is no reference to the fact that we are talking about a quotient. Aura achieves the purpose of solving the exercise and is able to face verification tests with the same request. Would she be able to use the procedure as part of a task that, for example, asks her to sketch the graph of the function, preliminarily identifying the parts of the Cartesian plane in which the graph may be located and those from which it is excluded? Aura is unlikely able to use the procedure in the same exercises in which a student who knows the motivations of the different phases of the same procedure uses it. In such a context, she would have to reconstruct a further procedure that would become a new formal act for her, the substance of which would escape her. In other words, not knowing how to use the procedure above, she would not have it as a tool to give continuity to her education. See [2] for more about the case of Aura.

The steps followed by Aura, even some of those that appear formally correct, show various aspects that I consider unsatisfactory and lead me to speak of “lack of understanding”. The question mark in the title of this section, however, is a prelude to a reflection on what can be the meaning of “mathematical understanding”.<sup>1</sup>

In mathematics, methods that are not tied exclusively to the expression of formal inferences are recognized. This is discussed for example in Steen’s [14]. There, the author also discusses various aspects of the mathematical understanding and speaks of “deep” and “superficial” understanding. In this way he introduces an evident problematization of what can be intended by “understanding”. In [3], *inter alia*, I focused on how philosophical hermeneutics, particularly in the work of Hans-Georg Gadamer, deals with the same problem of understanding. What I consider important to recall here is that the process of interpretation, aimed at understanding, is actually a process that occurs progressively and is in itself inconclusive: this justifies my agreement with Steen’s observations, underlining however that I believe that even a “deep understanding” is not both definitive and exhaustive.

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<sup>1</sup> Karaali [10] poses three related questions 1. What does it mean to know something mathematical? 2. How do we come to know a mathematical truth? 3. What does it mean to understand mathematics? She opens up the three questions so that all readers intrigued by these questions will find something worth agreeing with and arguing against.

The teacher could for instance view the aspects of Aura's work that I considered unsatisfactory as transitory acts, examples of "superficial understanding," to be taken into account as a starting point for planning future activities. This would be another case of how the I-Other relation, concerning teacher and student that we had already seen above, can be concretized.

However, an examination of the actual relation between Aura and the teacher might lead to an objection regarding what I maintain in the present article, namely, that it is in the ethical relation that understanding occurs. After all, Aura is involved and, from the point of view of Levinas, she appears willing to be guided, to receive a teaching, so that she wants to faithfully follow the teacher's instructions. Why then is her understanding of the task so lacking?

I think that the relation between the teacher and Aura should not be considered "ethical". In fact, during the lessons, the teacher used to guide the class and to address the students in turn with questions regarding the mathematical content she was presenting, and to give them the opportunity to ask clarification questions. No interventions had been used to address Aura's specific difficulties, for instance those on the meaning of an equation. The focus of the teacher, and consequently that of Aura, was on carrying out different examples of the same task, that is, on the exterior of the work in mathematics. In their relation there does not seem to have been the conversation in which all planning is abandoned: it seems that the exposition of this specific mathematical content was done according to a pre-established program.

#### **4. Infinity and acts**

The origin of language is linked to the relation with the Other through the face, and therefore derives from an ethical relation. This is linked to the willingness to receive a teaching. Learning, with respect to a mathematical content, cannot be limited to the slavish repetition of a verbal message, because mathematical knowledge is realized in the development of skills and competences that involves the application of concepts. The relation between mathematics teacher and student is characterized, perhaps more than in other disciplines, precisely by the 'celerity' with which it often transitions to situations that I would like to call "operative." These situations regard

exercises, problems, proofs etc. in which the students use what they learned. In mathematics education, the encounter between teacher and students leads to a coincidence of the respective actions in doing mathematics: solving an exercise or a problem or proving a theorem. Both inhabit the class, in different ways imposed by their different institutional roles. A teacher can choose an exercise, which I see as an instance of mathematical language, for her students. She can consider their needs, for example: to recall some concepts, to tackle certain calculation difficulties, to reinforce certain skills. It is a way to maintain the ethical relation. The teacher accepts implementing an action that is conditioned by the needs of the Other, considered in a position of superiority because, in fact, it is capable of influencing her acts. Within the ethical relation, an opportunity arises for students to accept the proposal that comes from a competent and authoritative person who guides them, but who will eventually face the resolution of that exercise by putting herself in the game, or who can simply listen the solution that the students propose. In this situation — in which I suppose the students accept the teacher's proposal, and the teacher considers how the students behave in operating with mathematics — they work together on an instance of mathematical language, in an ethical relation in which they recognize the Other in a prevailing position but, on the part of each, without the need to subject this relation to conscious reflection. In the words of Levinas, it is “the essential of language: the coinciding of the revealer and the revealed in the face, which is accomplished in being situated in height with respect to us — in teaching” [11, page 67].

When the students accomplish a task – one that is formally correct but that does not express their knowledge – they suffer the violence linked to the fact that they are conditioned to make a distorted use of language, which betrays its very essence. When studying for that task — knowing from previous experiences that the repetition of rules will be required — they do not need to put themselves in the attitude of receiving a teaching about a mathematical content. It is precisely the assignment and assessment of that task that determines all this, and this calls into question the teacher. I believe that here violence consists in distracting the students from the ethical relation on which language is based, only within which their knowledge can grow. I argue that their relation with the teacher who required that task, while being one of obedience, cannot be deemed ethical because it consists in carrying

out an action aimed at obtaining the approval of the teacher as guarantor of the institution, or to have benefits related to recognition of merit. I believe that, on the part of the students, it is a way to “grasp” the Other — the teacher.

Let’s think about a specific case in which (typically in a test) the teacher asks the students to write the following definition of the power of a rational number with negative exponent (taken from an Italian textbook and translated here): “The power of a rational number, other than 0, with a negative integer exponent is a power that has the reciprocal of the given number as its base and the opposite of the given number as its exponent”. Suppose the students have written the definition correctly. To make sure that they have really understood it, the teacher can assign additional work through questions such as: Why is “other than 0” specified? What is the meaning of the terms “reciprocal” and “opposite”? Write examples with whole numbers only; write examples with non-integer numbers, etc. The teacher thus requires a new task, further actions. Depending on the reaction of the students, first of all she ascertains whether they agree to take care of those actions, that is to turn their thoughts to them despite a surplus of work.

Even if the students do not wonder why those questions and not others, for what purposes, why a specialist in mathematics considers them important etc., thanks to their previous experience, I think they can prevent some of those questions posed by the teacher (not necessarily by making them explicit) during their personal study. They may assign themselves the task of explaining the terms that appear in the definition, may feel the need to understand why it is necessary for the definition to include all those different statements, in particular why it is necessary to specify “other than 0” etc. It is a possible sequence of questions and tasks that the students assign to themselves, which can be concretized in actions through which they will proceed towards a greater mastery of specific concepts and, more generally, of the way of dealing with mathematics. This process undertaken by the students is a manifestation of a critical and also self-critical attitude (in a positive, constructive sense) that concerns the willingness to put themselves to the test. It is a way to develop their mastery of procedures and concepts, that is their mathematical knowledge. The students access theoretical thinking through concrete actions. This is a potentially infinite pro-

cess that, at school, can stop with reference to those tasks that the teacher — as a specialist of mathematics — requires them. The students have to accept that the teacher examines — through those requests, according to an interpretation of Levinas, *per se* violent — their personal preparation. Those requests are precisely an initiative of the teacher but in many aspects reflect the way of working that unites the members of the mathematical community. Just as an aside, I would like to note that knowing how to respond to those requests helps the student to achieve the goals that the educational institution imposes, namely passing verification tests and exams.

On the part of the students, solving an exercise has the ultimate and substantial result of going beyond a mathematical concept. Speaking of possessing a mathematical concept seems to mean, in some sense, the fact that the intent has been achieved to enclose entirely — in a totality — a portion of mathematical theory. It is an illusion that does not take into account the possibility of “going beyond” — always — in the ethical relation: with Levinas, we can say that it is the idea of infinity that animates students’ progress in mathematics (in this case, evidently, we are not referring to the infinity that mathematicians use in their discipline, which here can however have a metaphorical value). Therefore, the idea of infinity resides in the pretension to possess, through the acquisition of a concept — in transcendence — all the possible manifestations of that concept. The infinite, on the other hand, also concerns the process that takes place through acts in which students put themselves to the test. This is the way to internalize — “incarnate” - that concept through a surplus of being: in itself, an always inconclusive process.

The next excerpt from TI reports the points that inspired what we have said about the role of the exercises. We can read it with reference to some keywords already used above: infinity, concept-theory-transcendence, exercise-activity-incarnation.

The notion of act involves a violence essentially: the violence of transitivity, lacking in the transcendence of thought [...] What, in action, breaks forth as essential violence is the surplus of being over the thought that claims to contain it, the marvel of idea of infinity. The incarnation of consciousness is therefore comprehensible only if [...] the idea of infinity moves consciousness. The idea of infinity [...] sustains activity itself. Theoretical thought,

knowledge, and critique, to which activity has been opposed, have the same foundation. The idea of infinity, which is not in its turn a representation of infinity, is the common source of activity and theory. [11, page 27]

The teacher may live what Levinas describes, in the case of an experience in which her ideas about the students (foreknowledge, prejudices...) come to consciousness through the same exercises she has chosen for them, through the observation of how they solve them. All this occurs as a manifestation of the teacher-student ethical relation, but certainly does not exhaust it.

The idea of violence as it appears in this last quotation does not concern the negative aspects that can, for example, summarised as “destruction of every possibility for action” to which Levinas refers in [11, page 21], quite the opposite. It is a violence that produces a relationship between the infinity of thought and theory, between the infinity of thought and activity, between theory and activity. In mathematics teaching and learning, it is therefore an indication of the ways in which the students accept the teaching of an expert in the discipline, and they proceed in mathematical knowledge.

## **5. Rhetoric**

I believe that the teaching of mathematics, more than all other disciplines, contributes to building the rational thinking of students. However, there are classroom situations that do not seem to contribute to this goal. Here, I want to focus on an example in which a procedure quickly leads to the solution of a problem but induces the students to bypass their personal investigation, with the associated difficulties and risks of failure. This procedure demeans their interiority, not stimulating them to recover their previous knowledge and not activating their abilities. It is a type of action projected onto the exterior that does not produce understanding. It may seduce the students by facilitating them to achieve what the school requires. In the words of Levinas: “[...] teaching leads to the logical discourse without rhetoric, without flattery or seduction and hence without violence, and maintaining the interiority of him who welcomes” [11, page 180].

Rhetoric is “speech or writing intended to be effective and influence people” or “clever language that sounds good but is not sincere or has no real meaning.”<sup>2</sup> “Influencing people” can suggest a hidden purpose that is, to make them believe — in a “not sincere” way — something that “has no real meaning”.

In mathematics education, I wish to focus on two roles of textbooks and teachers: that of introducing mathematical content and that of guiding young students. It happens that this second role is betrayed, sometimes in a hidden way, “obliquely [de biais]” [11, page 70], when students are instructed how to write a correct series of formal steps to achieve the result, and yet it happens that these passages do not refer to authentic meanings. Therefore, students are persuaded that through those passages they are guided to understanding, while they get only a way to reach the solution — easy, in their eyes, and which they are persuaded to use. I do not wish to discuss here the reasons why this is done by the teacher or the author of a textbook. What I want to point out is that this must be considered as violence, thus sharing Levinas’s point of view on rhetoric.

As an example, let us consider the following problem:

“Convert a number from base 10 to base 2”.

In textbooks or websites, one can find a pattern like that in Figure 1 (which shows the case of the number 12), accompanied by an explanation describing the process.

As I have experienced in my classes, this is a process that most pupils are able to implement, just by following the process in the table on the left, without supplementary explanation. Other pupils read the steps of the process provided, sometimes partially, capturing some key words: division, quotient, remainder. In other cases, the teacher is asked to repeat the process, underlining the different phases. Pupils usually come to the answer easily and know how to apply the process to other numbers. The pattern is very orderly and allows them to operate without thinking about the meaning of what they are doing. Some fundamental questions remain and concern, for instance, why divide repeatedly by 2 and why consider the remainders to write the answer.

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<sup>2</sup> See Cambridge Dictionary, <https://dictionary.cambridge.org/>.



To convert from base 10 to base 2 use the remainder method. Write the place values (of the base you are converting to) in a table. Write down the decimal number to be converted with space below it.

2	12	0
2	6	0
2	3	1
1		

$$12_{10} = 1100_2$$

Figure 1: The Remainder Method for Base Conversion

The Process:

- (a) Divide the “desired” base (in this case base 2) INTO the number you are trying to convert
- (b) Write the quotient with a remainder
- (c) Repeat the division process using the whole number from the previous quotient
- (d) Continue repeating this division until the quotient is less than 2
- (e) Read the answer from the bottom up

The meaning of the process can be recovered through a reasoning shown in Figure 2 (please, note that, if described only graphically, this reasoning tends to become visually unclear, so it is better that pupils operate with coins or other material).

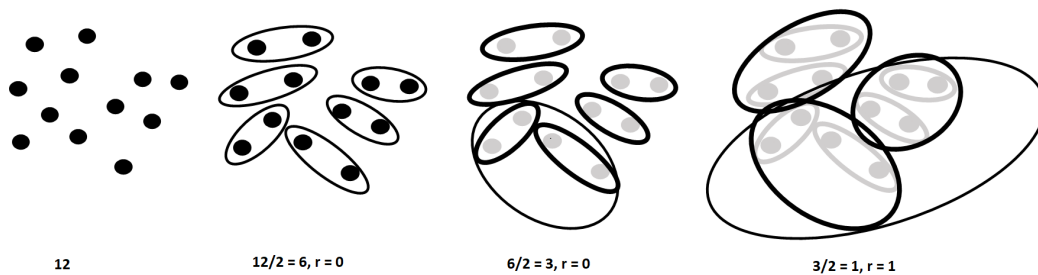


Figure 2: A Conceptual Demonstration of Converting Bases.

On the right, in the largest oval, there are 2 ovals (in each 4 coins, 8 in all) and another oval (still containing 4 coins) is at the top: we see that the division 3:2 has a remainder of 1. Going to the left, none of the divisions

(6:2 and 12:2) have remainder. The numbers 8, 4, 2 are powers of 2 and it is relevant to keep them in mind in order to understand the binary numbering system. The four parts of the figure thus recall the concepts concerning division, powers, numbering. The activity aims at pupils' understanding.

The reasoning shown in Figure 2 is more complex than the previous process. Pupils must master various concepts that come into play in the graphical representations and corresponding calculations. They are put to the test more in this reasoning than in the use of the process and the risk of failure is greater. This reasoning wants to highlight the reference to specific meanings that I consider are available to the pupils. As I mentioned above, I believe that it can give rise to an activity in which pupils are called to work through the use of concrete material. In the classroom, this allows for occasions in which the teacher can intervene considering the needs of some pupils, addressing them with appropriate considerations and expecting responses from them that inform her of how they are facing the task.

Let's consider the case in which the process, with a pattern, is proposed to pupils as the only way to solve the problem of the conversion from base 10 to base 2. In my opinion, this is a manner to approach them obliquely, hiding from them the complexity of the reasoning and offering them in exchange a procedure that easily leads to the searched solution. In all this, there is an aspect that we also find in the use of rhetoric. Rhetoric ensures that an interaction with the interlocutor is maintained but, being the art of persuading using adequate expressive devices, it tends to undermine the freedom of the Other. The previous process is a sophisticated device that reassures the pupils by convincing them that it is a quick way to provide the correct answer. But in this way, they are deluded into thinking that the relevant concepts are contained in the process, namely: learning a scheme, performing divisions, copying figures in the correct order. They are not called upon to refer to the concepts that really justify the process, which thus becomes simply an artifice. Their freedom to understand is undermined, obtaining their compliance through the possibility of completing a task within a short time. In the words of Levinas:

[Rhetoric] approaches the other not to face him, but obliquely — not, to be sure, as a thing, since rhetoric remains conversation, and across all its artifices goes unto the Other, solicits his yes.

But the specific nature of rhetoric (of propaganda, flattery, diplomacy, etc.) consists in corrupting this freedom. It is for this that it is preeminently violence, that is, injustice. [11, page 70]

I would like to point out that I used the term “process” when referring to how textbooks or sites illustrate the conversion from base ten to base two. I used the term “reasoning” when I wanted to illustrate the meaning of that process. Thus, I have implicitly referred to what conditions an instance of “mathematical reasoning” must satisfy. I do not pretend to arrive at a definition from the point of view of the epistemology of mathematics. I intend to refer to the field of education and I primarily consider an instance of mathematical reasoning as a teacher’s proposal to students. I don’t even intend to discuss how the teacher introduces it — for example, with a constructivist approach in mind. From my point of view, an instance of mathematical reasoning is not a mere exposition of formal inferences; it must contain the reference to the meanings and must be addressed to the student, that is to another person with whom the teacher has an ethical relation (in the terms discussed above).

## 6. Language, text, ethical relation

The community of mathematical insiders is “a group including mathematicians, mathematics educators, mathematics teachers, mathematics-related professionals and others who have specialised in mathematics. For insiders [...] mathematics is a love object: endlessly fascinating, attractive and beautiful. We see it as meaningful, useful, powerful and also as an avenue to personal success. As insiders we are deeply invested in mathematics, we see the world through the lens of mathematics; so not surprisingly we are strong advocates for the high valuation and prestige of mathematics in education and society” [7]. I argue that due to the fact that she belongs to this community, the teacher runs the risk of not really taking the student — who is in an initial approach to mathematics — into account. This may happen, for instance, when using mathematical symbolism to introduce a new concept without considering that the students’ difficulties may concern precisely the use of symbols. This produces what Levinas describes as “[a] curious result;

language would consist in suppressing the other — to whom it is addressed, whom it calls upon or invokes” [11, page 73].

The language of algebra allows us to express propositions in general terms. However, the ability of students to generalize must be built over the years, through numerous activities and exposure to relevant mathematical content. Students must first understand that expressing in general terms is an enrichment of the discourse. This might seem paradoxical to them since, in order to give meaning to a proposition expressed in general terms, they should look for suitable examples. If they were not able to do so, they would run the risk of falling into rote learning without understanding (consider, for example, the repetition of propositions after a mnemonic study seen as the way to fulfil the teacher’s requests). At that point the teacher would have failed in her intent to introduce students to such an advanced form of the language of mathematics. I am considering the case in which the teacher has set among her educational objectives precisely the use by the student of the algebraic language for mathematical discourse, but we can consider the hypothesis that the teacher may not have even considered that those difficulties may arise in students.

Therefore, the language of algebra risks excluding students from the possibility of dealing with mathematical discourse, not only to understand what the teacher proposes to them but also to express propositions that derive from their mathematical knowledge — their own reasoning. When students are not able to share with others the mathematical aspects they have internalized, their freedom to act according to their being is compromised.

What we have discussed now can relate to what Levinas says about language which is “far from presupposing universality and generality” [11, page 73]. For us, this underlines how introducing content through general propositions risks being inadequate from the point of view of mathematics education. Language, however, makes universality and generality possible through discourses inside the ethical relation.

In Levinas’s perspective, discourse through language is a manifestation of the ethical relation between human beings. Recalling Plato, “discourse is therefore not the unfolding of a prefabricated internal logic, but the constitution of truth in a struggle between thinkers, with all the risks of freedom”

[11, page 73]. Therefore, it is language that determines the passage from the individual to the general through this relation (“struggle” in the Levinas’s excerpt) between human beings. Let’s recall that it is in the I-Other relation that the foundation of ethics resides, not in a list of precepts that individuals learn, or in the rules that they possess independently. Language and ethical relation are inseparable. Excluding the Other by using language to address only apparently the Other is to perpetrate violence. We can understand how this type of violence is relevant to the teaching of mathematics, considering how the latter is based on language — a specific language.

Dougan [5] refers to Emmanuel Levinas and Hans-Georg Gadamer on the ethics of reading and on the encounter with the Other in literature. Here, I consider the contribution of the two philosophers to analyse how the students realize, through interpretation of a written mathematical text, the ethical relation based, in general terms, on the language and specifically on that text. I am going to consider that the teacher does violence to the students even when she hinders their relation with another person who can help them, who guides them to new mathematics — for example by imposing on them tasks that distract them from this relation, as we will see later, and in part we have already seen. Through the written mathematical text, a relation is established between human beings: the author (but not only) and the student. I argue that the enhancement of this relation favours a student’s better understanding of mathematical content, inside the process of interpreting a text (which Gadamer [9, page 371] considers as a dialogue — another form of dialogue). Students are asked to accept the Other’s proposal, welcoming the opportunity to follow an instance of mathematical reasoning with, and, in the meanwhile, their propensity to revise their own preconceptions.

In a text, I would like to focus, on the one hand, the mathematical content, on the other hand, the educational choices. The author, if it is a text taken from a schoolbook, may have searched through the text for an answer to the question regarding how to convey that mathematical content to students with a certain preparation, of a certain age, etc. As a teacher, when I get to choose a text, I ask myself the question “What aspects of this topic do my students need to deepen?” Moreover, for me the text does not only represent the possibility for my students to receive further explanation but also to offer them an opportunity to test and consolidate, through using knowledge and

math skills in new contexts, the learning of what has already been treated in previous lessons. In this way, interpretation becomes a problem — for these characteristics similar to those of classical word problems.

I consider the student's relation with the author as ethical, a relation between human beings through the text. This relation can be established thanks to the choices that the author has made to plan the text, giving to himself obligations towards other people. The author of the text is the student's Other. It is thanks to this ethical relation that the understanding of the text can take place.

## 7. Termination of an ethical relation

The text is often assigned to the students by the teacher (but it is sometimes possible that the students choose it by themselves). In [1] I show that, in a text, traces of the author can be found. But a text can carry traces of many human beings, due to the fact that it has a story. The text is a place where various ethical relations with the student can be established. The complexity of these (possible) relations induced by the text – which cannot be reported in a list – leads me to use the expression “ethical relation with the text”.

It is trivial to recall the importance of written texts in the transmission of mathematical knowledge: in history, in current scientific research, in science magazines, etc. The study of a text is often essential for the student when there is no explanation from the teacher as in the case in which the student has not been able to participate in the lesson in the classroom, or in the case of online courses, during which the interaction with the teacher is reduced, such as in the Covid-19 pandemic. When, in certain situations, the requirements of the school hinder and tend to interrupt this virtuous relation, the students are induced into an unethical relation with the written text that put at risk their understanding. Here, I return to what I said in the previous pages, mainly in Sections 3 and 5.

The text has an author who presents the content and a reader who follows and reconstructs the author's reasoning that is presented via arguments, algebraic passages, proofs etc. When a relation is established, the reader is called to share ideas with the author. However, it would be naive to think of a sort of identification of the reader with the author — also in this sharing of ideas —

because their attributions of meaning to procedures and concepts will hardly coincide (let's think to the case of a student and the author of a textbook). Student's interpretation is personal. In any case, I believe that these observations regarding sharing of reasoning — particularly evident in reading a mathematical text — emphasize the relation between human beings. This may seem paradoxical when speaking of mathematics, if we consider how the author's characteristics tend to (although not always, see [1]) disappear from a mathematical text. To underline the importance of that relation for students, let us think of the value of mathematical textbook for them who through it (although not exclusively) are introduced to the study of mathematics. The various human beings who leave trace in the text assume the role of guide and give a message of exhortation.

At school, however, it may happen that the ethical relation with the text is not adequately valorised, and some educational initiatives even tend to interrupt it. Obviously, these are not initiatives aimed at this; there is no express will. This represents an act of violence that leads the students to abandon the reference to the human beings who, through the text, can help orient them towards mathematical knowledge. In this negative process, the teachers still play an ambiguous role: on the one hand, proposing the text, they are among the persons who encourage the ethical relation; on the other hand — for the activities they propose and the tasks they ask to the students — they become those who breaks this relation.

I think that what is produced with respect to the students can be referred to as “making them play roles in which they no longer recognize themselves” [11, page 21], already cited above. I read this statement as follows: Students find in that relation the orientation toward mathematical knowledge; what the institution, through the teacher, requires may regard tasks in which the students no longer recognize themselves precisely because they lose their reference to the Other with which they had shaped — and they are going to shape — their being as mathematicians. For example, one of those tasks might be to repeat a definition, without concern by the teacher for student's understanding during previous classroom activities devoted to that definition. Let's consider that when the student is summoned by an Other whose trace is in the text, the process of understanding is started, even if the Other is behind the short text of the definition introduced in Section 4

Some tasks require students to express a mathematical statement and the teacher recognizes it as correct even if it is not the result of a process of understanding. This sort of task tends to interrupt the students' ethical relation with the text. Recall the examples given in the previous sections concerning an understanding that I have suggested may be lacking. If the students produce a statement recognized as valid even if not understood, they have the indication that the school's requests do not include the need to understand. If the school does not encourage students' search of understanding, they receive the teaching that the ethical relation with the text is not relevant and they are induced to make an instrumental use of the text, aimed at scholastic success. The violence that tends to interrupt the ethical relationship between them and the text results in the students' violence on the text — and in turn, on the human beings who have left their trace in it.

## 8. Concluding remarks

The concept of violence appears in TI under various aspects; I have not tried to define it in a concise form. Taking inspiration from TI — with reference to expressions like “making them carry out actions that will destroy every possibility for action” or “rhetoric” — I attributed a negative value to some situations of violence in mathematics class. Other situations are violent in themselves (“[t]he notion of act involves a violence essentially”) but help to illustrate the process of mathematical understanding, considered in a Levinasian perspective.

It should be noted that Levinas speaks of “The Asymmetry of the Interpersonal” with regard to the ethical relation determined by the face:

The being that presents himself in the face comes from a dimension of height, a dimension of transcendence whereby he can present himself as a stranger without opposing me as obstacle or enemy”. This happens even if I live “the inevitable orientation of being “starting from oneself” toward “the Other”. [11, page 215]

I would like to interpret this passage with reference to education: the students can see the teacher in a “dimension of height” and “transcendence” without considering this a premise for suffering violence. The same can be



said exchanging “students” for “teacher”; the asymmetry of the relation is maintained in this case, too.

Many authors have engaged with Levinas to tease out the contributions his thought can make to education. For instance, [15] — introducing a collection of papers — points out that it “is not a collection of congruent interpretations of Levinas but of disparate and diverse readings and renderings of Levinas’s thought to current contexts in education”. Perhaps his relevance also for mathematics education consists in being able to inspire reflections on class relations, in educators with the most diverse cultural backgrounds. However, Levinas’s thought has aspects regarding a radical subversion of relations between human beings, to which it is difficult to remain faithful. Personally, I feel the need to periodically go back to the same passages of his work, to reflect again, and to critically analyse what they had previously suggested to me.

As said above, a written mathematical text is a place where the student relies on the Other whose trace is in the text. In the ethical relation, the student must be grasped from the text, rather than grasping it for purposes related to academic achievement. With regard to this, the school plays a negative role through the teaching that does not value understanding and tends to interrupt that virtuous ethical relation, thus introducing a violent teaching.

Understanding is never total: neither an absolute nor a definitive act. The understanding of a mathematical text by an expert is different from that by a student. In the relation between teacher and student, criteria for understanding emerge through what the teacher — as an expert — suggests and asks to the student. I have shown that it is educationally counterproductive when students are aware that they have not understood parts of a text and, despite this, they receive an appreciation. From a Levinasian point of view, knowledge can be violence, if intended as domination and control. The awareness on the part of the teacher and the students that knowing mathematics is always a partial and incomplete process, I think, can help to create educational situations that allow avoiding that association of knowledge and violence.

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