

## A Mini-seminar: Teaching Ethics in Mathematics in an Hour a Week

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### Cover Page Footnote

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# A Mini-seminar: Teaching Ethics in Mathematics in an Hour a Week

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## Abstract

This article describes a one-credit class on ethics in mathematics, which met once a week for an hour and discussed topics in cryptography, algorithms, and big data, and the role of mathematics in public life. I offer anecdotal evidence that even a course with relatively little time commitment for both students and faculty can have a significant impact on students' understanding of and interest in ethics in mathematics. I also collect recommendations and resources in the hopes of encouraging others to consider teaching such a course.

**Keywords:** ethics in mathematics, teaching

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## Introduction

The teaching of ethics in mathematics is relatively underdeveloped in comparison to sibling disciplines like computer science and engineering. There are research articles analyzing syllabi from over a hundred technology ethics courses [22] and describing a range of approaches to engineering ethics education over decades [27], while no such proliferation of resources exists in math. But while the literature on teaching ethics in mathematics is limited, it is by no means nonexistent. For example, Axtell and Westphal [4] discuss incorporation of an ethics module into a summer mathematical research experience for undergraduates; Elliott, Stokes, and Cao [17] and Lesser and Nordenhaug [42] describe two very different ethics in statistics courses; Shulman [62] gives examples of 'methods to help students become more alert to ethical issues in mathematics'; and Franklin [23] and Webber [72] describe general courses in mathematical ethics. More recently, the

Cambridge University Ethics in Mathematics Project has developed a series of eight lectures for a course on ethics in mathematics [11], written a collection of discussion papers [13, 14, 15, 48], and hosted several conferences. Moreover, a broader collection of work analyzes the role of ethics in mathematics [10, 18, 19, 20, 28, 32, 63, 64] and gives examples of mathematicians articulating and debating their professional ethical obligations [1, 2, 30, 16, 21, 26, 37, 44, 49, 59, 60, 67, 73]. Additional pieces on the teaching of mathematics [3, 6, 7, 8, 9, 19, 25, 50, 56, 66], the cultural practices of mathematical research [12, 24, 41, 57, 65, 68], and on mathematics and social justice [39, 33] can also serve as complementary resources.

In Spring 2021, I taught an Ethics in Mathematics class that met once a week for 50 minutes. In this article I describe the institutional context and pedagogical approach of the course; summarize the course content; outline the assignments; and conclude by reflecting on student feedback and offering recommendations for other instructors. The appendices include sample readings, assignments, and weekly schedule together with complete written course evaluations.

### **Institutional context and pedagogical approach**

This class was offered as a one-credit course in the mathematics department of Rice University, a private research university with about 4,000 undergraduate students. The mathematics department had already established a course number for special and one-time courses, which allowed for a significantly shorter approval process than would be required for a permanent course. Although this course was offered within the pure mathematics department, topics ranged across pure and applied mathematics. (To my knowledge, the computational and applied mathematics department did not offer any ethics courses at this time, and so there was little concern about overlap.)

The formal prerequisite was set as any one of three offered courses in linear algebra, though it was waived for some interested students. Of course, this requirement influenced the level of technicality when we discussed the mathematics of public-key cryptosystems and machine learning. Even more importantly, setting this prerequisite was part of a decision to aim the class at students who viewed themselves as people who used and did mathematics.

As a result, while student backgrounds and declared majors varied, in designing this course I assumed that the students had already been extensively exposed to the idea that mathematics could be used effectively in a range of settings.

The primary goal of this course was to make students aware of ethical issues that arise when doing and using mathematics and to give them the opportunity to identify and refine their own professional values. More specifically, the primary learning objectives were that students should:

- Identify ethical questions that arise in a variety of professional contexts (as a math teacher, a journalist, a “pure” mathematician, a developer of “big data” algorithms).
- Articulate the aspects of human choice and judgment involved in doing mathematics even at a very basic level.
- Describe the implicit assumptions and ideologies underlying the role of data and quantitative arguments in public life.
- Reflect on their own values and analyze situations in which their principles conflicted.

My priority was introducing students to examples of compelling ethical problems arising in and from mathematics and to how various practitioners approached such questions, rather than focusing on moral philosophy. However, we did briefly discuss some ethical frameworks as a way to clarify our conversation on cryptography, following [52].

In terms of pedagogy, my guiding principle was that given the small amount of class time — eleven 50-minute Zoom periods — students would benefit most from actively engaging in explorations and discussion. In preparation for each week’s class, students read news and academic articles, book chapters, and podcast transcripts and submitted short pre-class reflections. Class time was split roughly evenly over the course of the semester between mini-lectures on new mathematical content; small group discussion of the readings; small group collaboration on in-class activities; and whole class discussion.

In addition to pre-class reflections, students completed four assignments and a final project over the course of the semester. Sample readings, assignments, and course schedule are available in Appendix A and complete written course evaluations in Appendix B.

### Course content

The course began with a module on cryptography, which was chosen to engage students with new mathematical content (modular arithmetic and the Diffie-Hellman key exchange) while providing a context in which ethical issues of privacy, security, and transparency were naturally foregrounded. This part of the course was heavily influenced by [34], including a pre-class questionnaire that was remarkably effective: when the vast majority of the class agrees both that “Companies have a duty to protect the privacy of their customers” and that “Private companies have a duty to report criminal behavior”, students are pushed to move beyond stating broad principles to grappling with how to balance conflicting ideals in practice.

One of our early case studies was the battle over whether Apple would build a backdoor to aid the FBI in extracting data from the phone of a suspect in a mass shooting. Both Apple and the FBI made explicitly moral arguments for why such a backdoor should or should not exist [31, 40] and our conversation began with students analyzing the values and emotions appealed to in these arguments. Students quickly recognized that both parties also had less lofty interests at stake, which led the class to a richer analysis of how to evaluate these arguments. This case-study also provided an opportunity to introduce at a very basic level some ethical frameworks, such as the utilitarian, rights-based, fairness, common good, and virtue approaches. The Markkula Center for Applied Ethics website [43] was a useful resource for providing short introductions to these frameworks, as well as additional case studies. Rogaway’s piece “The moral character of cryptographic work” [60] provided the opportunity to deepen our classroom conversation. First, much of the discussion of the Apple vs. FBI case relied on the idea that there are inherent trade-offs between personal privacy and collective security, while Rogaway urges the reader to move beyond this framing to consider a different, surveillance-studies based approach.

More broadly, this piece challenged students to move beyond “some applications of math have ethical implications” towards “ethical implications are unavoidable” [13]. Class time activities included small group work on modular arithmetic problems, mini-presentations on ethical frameworks, and discussion of the implicit values and emotions underpinning various arguments.

In the beginning of the algorithms module we relied heavily on a single book: Cathy O’Neil’s *Weapons of Math Destruction* [51]. This very accessible read gives many examples of algorithms that play increasingly powerful roles in hiring, teacher evaluation, and prison sentencing, and yet are often better at shrouding bias in a technological veil than at actually reducing it. O’Neil defines ‘weapons of math destruction’ to be algorithms that are opaque to the people they most impact; have a broad enough scale to affect many aspects of a person’s life; and inflict significant damage, often via the creation of feedback loops. The “strictly mathematical” content of this part of the course was applied, focusing on gradient descent and the very basics of machine learning, with an emphasis on how training data and the designer’s choices impact the resulting algorithm. In class, we complemented O’Neil’s work by, among other activities, delving more deeply into the details of the *US News & World Report* college ranking algorithm and discussing what factors could be ethically used in determining car insurance rates.

While I originally planned to keep the algorithms and mathematics in public life modules distinct, student feedback together with weather-induced class cancellations led me to combine these modules a few weeks in. I believe this somewhat ad hoc restructuring improved the course experience significantly: by broadening our scope from big data to the uses of mathematics in court cases, the news, and more, we were able to situate O’Neil’s examples in a broader context. Discussion of the short piece “Mathematics as propaganda” [36] encouraged students to expand their focus from specific and egregious examples of the (mis)use of algorithms to the roles that mathematical arguments play in broader society, for good and bad. Additionally, we balanced O’Neil’s many instances of “math destruction” with more nuanced examples of people working to use data for the greater good. For example, “Governing an Algorithm in the Wild” [58] outlined the “highly transparent and participatory” process through which the algorithm that allocates kidneys for

transplants was revised. Finally, the talk “Abolish Big Data” [45] led to a discussion of the assumptions and ideologies implicit in much of our society’s current approach to data, and of what alternate beliefs might lead to a healthier data ecosystem.

## Assignments

**Pre-class reflections:** Students turned in short reflections the night before each class period. The instructions were to spend no more than twenty minutes on each reflection, and lengths varied (both between students and week-to-week) from a few sentences to multiple length paragraphs. I tracked completion of these reflections and occasionally left comments, but the primary goals of this assignment were to ensure that students were prepared for in-class discussion and to give me information about what ideas were particularly interesting or confusing for students in order to shift the focus of class discussion. Based on student feedback and my own observations, both of these aims were met, and I strongly recommend incorporating some version of this in similar courses.

**Cryptography assignments:** Due to time constraints, we were not able to spend enough time delving into the technical details of public key cryptography to allow students to directly analyze the impacts of certain mathematical choices. Therefore, there were two separate assignments for this part of the course, one focusing on modular arithmetic and the other on ethical considerations surrounding privacy and surveillance. For the problem set, given the variety of students’ mathematical backgrounds, I provided six modular arithmetic problems that were labeled according to difficulty, and asked students to submit solutions to the four that best matched their experience level. Additionally, students wrote a two to three page reflection in which they were asked to take a stance on when a society should allow governmental surveillance of private communications, with commentary on the ethical considerations at stake. This was heavily inspired by [34].

**Data and judgment problem set:** These problems were organized to go beyond the idea that one can lie with statistics to force students to recognize that even very simple analyses of data require human judgment [5].



For example, students were given a table of tax data from Ford's presidency showing that over four years, tax rates on each income group decreased, but the average tax rate increased overall [71]. They were then asked to explain what phenomena in the data caused this example of "Simpson's paradox", to decide whether the aggregated or disaggregated rates were a better descriptor of the situation, and to support their decision with an argument. Another problem asked students to find their own example of quantitative data in the news and to critically evaluate the choices that were made in obtaining, analyzing, and presenting that data.

**Algorithms policy brief:** Inspired by the Markkula Center for Applied Ethics' Smart Lamppost case study [53], students were asked to imagine that they were a staffer for a city council considering installing so-called "smart lampposts", which could include capabilities ranging from collection of air quality data, license plate identification and recording, and automatic notification of the police when gunshots were registered. Students then created short briefings on what the council members should keep in mind as they made their decision, including questions to ask the designers and identification of constituent groups that should be consulted.

**Final project:** Students picked a mathematical topic where ethical considerations were important, and chose a target audience they wished to communicate with about this topic. They then created an artifact (video, poster, presentation,...) that effectively communicated the issues at stake to their chosen audience, and also wrote a short paper discussing the ethical and mathematical decisions they made in that process. This was heavily inspired by Federico Ardila-Mantilla's "CAT(0) geometry, robots, and society" [2], which interweaves a relatively conventional mathematical introduction to the geometry underlying certain models of robotic motion with discussion of the societal and ethical implications of such work. This article is a fantastic model for how format and structure have significant rhetorical impact, and encourages students to think about how their own writing choices do or do not support their argument. The results of this project included a podcast episode about the ethics of autotune; an Instagram story on a dog-shaped robot briefly used by the NYPD; and an infographic for high school students about the *US News & World Report* college ranking system.

## Student feedback

Feedback on this course was remarkably and uniformly positive (see Appendix B for a complete copy of students' comments from the university-wide course evaluation form). Students repeatedly commented that while they had not previously been asked to consider the ethical considerations of STEM fields, they found this work intellectually and personally fulfilling.

Several students mentioned that the class had been useful in thinking about their future careers, with one reflecting on a new internal conflict:

“In terms of me personally, I feel very confused, as these agencies [the US National Security Agency and other intelligence agencies] were things I was aspiring for in my career, seeing as they are very helpful in giving mathematicians jobs, but now it really makes me feel strange to work towards something I ultimately don't agree with.”

Another wrote the following as part of their pre-class reflection a few weeks into the course, after reading Rogaway's piece [60].

“This article was eye-opening for me in a way that was both intellectually stimulating and deeply concerning. I am studying to be a physicist/mathematician and before this, I have never once considered politics regarding math or physics, nor have I even been in a morally ambiguous situation while writing proofs or experimenting. Personally, I thought that subjects so dependent on concrete facts, observations, and logic were inherently apolitical, I never stopped to consider what now seems obvious: science has ethical consequences and needs to be approached with the safety of the public and the world in mind.”

In an end of the semester open-ended reflection, one student summed up their experience as follows:

“This class really surprised me with how impactful it has been on my own opinions of myself and my future choices. Even though we talked about a lot of bad stuff, I feel like thinking over these

ethical dilemmas actually made me love math more because I feel that I now have the ability to be intentional with how I use it, and that I can choose to have a positive impact instead of a negative one makes me feel closer to the subject. I remember when I signed up for this class I was like “I don’t know what we will possibly talk about” and so clearly my mind has been opened to a lot of things that it just couldn’t even conceptualize before.”

### Closing recommendations

#### **Find examples where the ethics and mathematics are inseparable:**

In the beginning of the course, we were focused on either mathematical topics (that have ethical implications in its common applications) or ethical questions (that require mathematical understanding to fully address.) However, in our discussion of quantitative data and judgment, the issues of ethics and mathematics could not be so easily separated. Conversations about Simpson’s paradox, the class-size paradox, mean versus median versus mode, and conditional probability served to emphasize both that there is always an aspect of human choice in how information is collected, analyzed, and presented and that some choices are better than others. Several students commented that they found this to be one of the most valuable components of the course.

**Intentionally build class atmosphere:** Engaging in a class on ethics and mathematics requires students to do both ethics and mathematics. Both require a certain amount of bravery from students, especially if they believe that their background, whether personal or mathematical, is meaningfully different from that of their peers. It is important to consciously include opportunities for students to become more comfortable with each other and build a sense of community and interpersonal respect. First day activities such as Matt Salomone’s “Is it a sandwich” [61] can give students a chance to engage with each other in a low-stakes and frankly entertaining way while beginning to talk about the human aspects of mathematics.

**Aim for a range of ambiguity:** In a cultural context that often assumes all technological progress is good, it is important to include cases where students see an unambiguous right and wrong and recognize that mathematics

can be used to further that unequivocal wrong. However, also including examples of people trying hard to use mathematics to make the world better, and grappling with the inevitable nuances and complexities of that task, sparks much richer discussion than focusing only on the most egregious uses and misuses of mathematics.

**Expect the unexpected:** Keep an open mind about students' positions—it behooves the instructor to be ready for discussion to take many paths. For example, after reading a relevant chapter of O'Neil's book, I asked to what extent targeted advertisements could be ethical. I expected to get a range of answers—maybe choosing an author to advertise based on previous book purchases is okay while targeting low-income consumers is not— but the class was unanimous that targeted advertising was intrinsically unethical.

**Find close-to-home examples:** Some of the richest conversation came from examples that were very close to student life. What this looks like will vary according to your student population; for this class, our exploration of the *US News & World Report* college ranking system was highly personally relevant and led to very high student engagement (and some amount of personal disillusionment!). Near the end of the semester, the Just Mathematics Collective released a statement [30] urging mathematicians to pledge to avoid collaborating with the National Security Agency, refuse to assist in their recruitment efforts, and refuse to write recommendation letters for NSA job applicants. This sparked a rich and thoughtful discussion in class, in which some students visibly worked to reconcile beliefs that professors have the right to refuse to support certain institutions with concerns about having a professor refuse to write them a letter for a job they were excited about.

**Take advantage of your class format:** In the context of spring 2021, students (and instructors!) were generally both stressed and tired of Zoom. I believe that this seminar's limited nature — only an hour a week of class time and correspondingly few assignments — was actually quite valuable in giving the students space to remain engaged throughout the semester. This is not meant as an argument that a more extensive ethics in mathematics class would not be valuable, but as encouragement to consider the benefits as well as the drawbacks of non-traditional class structures.

## A. Course materials

### *Course schedule*

The following table outlines the main activities of each 50 minute class. Due to weather and pandemic induced cancellations., there were only eleven class meetings.

Week 1: Discussion of [55]; modular arithmetic mini-lecture and activity.

Week 2: Discussion of [31, 40]; Emails Exposed case study [70] activity.

Week 3: Discussion of [52]; modular multiplication and inverses activity.

Week 4: Discussion of [60]; mini-lecture on the Diffie-Hellman key exchange.

Week 5: Discussion of [51, Chapters 1-2], [36]; class-size paradox activity.

Week 6: Discussion of [51, Chapters 3-4], [35]); *US News & World Report* college ranking activity.

Week 7: Discussion of [51, Chapters 5-6], [54]; mini-lecture on neural networks.

Week 8: Discussion of [51, Chapter 7], [58]); mini-lecture on gradient descent.

Week 9: Discussion of [51, Chapter 8], [45]; ideology of big data activity.

Week 10: Discussion of reading [51, Chapters 9-10], [69]; car-insurance activity.

Week 11: Discussion of [51, Conclusion], [30], [32]; recap of the semester.

### *Sample readings and reflection prompts*

**Week 2:** Watch “Encryption: Last Week Tonight with John Oliver” [40] and read “The FBI Wanted a Back Door to the iPhone. Tim Cook Said No.” [31]. While you’re reading and watching, pay attention to the strategies the FBI, Tim Cook, and other participants use to support

their arguments. What values, ethical concerns, and emotions are they appealing to? In this particular case, which ethical argument, the FBI's or Apple's, do you find most convincing? What factors do you think contributed most to your evaluation?<sup>1</sup>

**Week 8:** Read “Sweating Bullets: On the Job” [51, Chapter 7] and “Governing an algorithm in the wild” [58], and then comment on the similarities and differences between the following two quotes:

“Managers assume that the scores are true enough to be useful, and the algorithms make tough decisions easy” [51, p. 133]

“I think, for the doctors involved, there was a temptation to say: ‘Oh, we’re going to use this medical fact to decide who gets the next organ.’ It sort of resolved for them a tension that is a very un-doctor-like thing they might otherwise have to do, which is make an explicit ethical judgment about who should come first. So there’s sort of this desire to invest, in some cases, these medical facts with a greater clinical significance than they may, in fact, possess.” [58]

**Week 9:** Read “Collateral Damage: Landing Credit” [51, Chapter 8] and “Abolish Big Data” [45].<sup>2</sup>

“I think about this a lot because you know, when we say making data a tool for social change instead of a weapon of political oppression, like that’s a cultural change.” [45]

Unpack this a little bit: what cultural assumptions and ideology about data would Milner say contribute to its role as a weapon of political oppression? What philosophy might an alternate data ecosystem be rooted in?

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<sup>1</sup> These readings were taken from and questions inspired by [34].

<sup>2</sup> In a future version of this course, I would likely replace this with [46] for accessibility reasons.

*Cryptography problem set*

Submit solutions to four of the following six problems. I encourage you to pick problems that are appropriate for your level of mathematical background. The problems are ordered chronologically within the course material; the stars denote my best guess at relative difficulty level, though opinions may vary.

- (a) (\*) Prove that ‘equivalence mod  $n$ ’ is indeed an equivalence relation, i.e. that the following properties are satisfied for all  $x, y, z \in \mathbb{Z}$ .
- (i) (Reflexivity)  $x \equiv x \pmod{n}$ .
  - (ii) (Symmetry) If  $x \equiv y \pmod{n}$ , then  $y \equiv x \pmod{n}$ .
  - (iii) (Transitivity) If  $x \equiv y \pmod{n}$  and  $y \equiv z \pmod{n}$ , then  $x \equiv z \pmod{n}$ .
- (b) (\*\*\*) Recall that for  $x \in \mathbb{Z}$  and  $n \in \mathbb{N}$  we defined  $[x]_n = \{a \in \mathbb{Z} : x - a \text{ is divisible by } n\}$ . For  $x, y \in \mathbb{Z}$  prove that the following are equivalent. (That is, (a) is true if and only if (b) is true if and only if (c) is true. One often proves such statements by proving (a) implies (b), (b) implies (c), and (c) implies (a).):
- (i)  $x \equiv y \pmod{n}$
  - (ii)  $[x]_n = [y]_n$
  - (iii)  $[x]_n \cap [y]_n \neq \emptyset$ .
- (c) (\*) Prove that addition on  $\mathbb{Z}/n\mathbb{Z}$  is well-defined, i.e. that if  $[a]_n = [c]_n$  and  $[b]_n = [d]_n$ , then  $[a + b]_n = [c + d]_n$ . Convince yourself (but do not submit a written proof) that multiplication is also well-defined.
- (d) (\*\*) Prove that  $[a]_n$  has a multiplicative inverse in  $\mathbb{Z}/n\mathbb{Z}$  if and only if  $a$  and  $n$  are relatively prime. You may use the following fact we discussed in class without proving it: *If the greatest common divisor of  $x$  and  $y$  is  $d$ , then there exist integers  $p, q$  such that  $xp + yq = d$ .*
- (e) (\*\*\*) Let  $p$  be a prime and  $[a]_p$  be a nonzero element of  $\mathbb{Z}/p\mathbb{Z}$ . Prove that  $[a]_p$  has finite order that is less than or equal to  $p - 1$ .

- (f) (\*\*) Given  $[a]_p \in \mathbb{Z}/p\mathbb{Z}$  and  $n \leq p - 1$ , give an efficient algorithm for computing (a minimal positive element of)  $[a]_p^n$ . (I don't mean anything particularly rigorous by 'efficient', but for large values of  $n$  your algorithm should be significantly faster than the basic "Compute  $[a]_p^2 = [a]_p * [a]_p$ . Compute  $[a]_p^3 = [a]_p * [a]_p^2$ . Continue on until you have computed  $[a]_p^n$ ." approach, which takes  $n - 1$  multiplications.)

### *Data and Judgment problems*

Some of these problems were incorporated into in-class activities; the rest were part of a take-home problem set.

- (a) University X needs to figure out its average course size. Here's some data modeled on our campus: there are 69 courses with 15 students each, 19 courses with 30 students each, 10 courses with 70 students each, and 2 courses with 120 students each.
- (i) What is the mean course size averaged across all classes?
  - (ii) What is the mean course size a student experiences? That is, if you asked all the students on campus to write down the sizes of their classes and averaged all of these numbers together, what would you get?
  - (iii) What's going on!? Can you explain why (a) was smaller than (b)? Will that always be the case? In which contexts is (a) a meaningful number? What about (b)?
  - (iv) If you have extra time, think about alternate summary statistics (e.g. median and mode). Do these also vary based on whether you take an average class versus an average student? What do you think the most meaningful of these values is?
  - (v) If you still have extra time, read [38] and discuss in your groups: can you identify other examples of when the difference between "individual and group level perspective" might have real impact on quantitative analysis?
- (b) This article by *US News & World Report* [47] goes into more depth in how they determine college rankings, as discussed in our readings [51].



Remember that a “proxy” variable is something that you measure when you can’t directly access the thing you really care about. For example, per capita GDP is sometimes used as a proxy for standard of living. In this example, all of “alumni giving rate”, “faculty salaries”, and “average class size” are probably proxies! Which do you think are “good” proxy variables and which “bad”? Why?

- (c) “The numbers speak for themselves,” said the police chief. “People from Group A commit crime C at a rate (2 percent) of TWICE group B (1 percent). Therefore, we need to police group A aggressively in order to stop crime C.”<sup>3</sup>

- (i) Assuming the numbers the chief stated are correct, fill in the table below.

	Commit crime C	Do not commit crime C	Total
Group A			1,000
Group B			9,000
Total			10,000

- (ii) What is the chance a randomly chosen person from group A committed crime C? What is the chance a randomly chosen person from group B committed crime C?
- (iii) What is the chance a randomly chosen person who committed crime C is from group A? From group B?
- (iv) Based on your calculations, how would you respond to the police chief?
- (d) The following is real data on tax rates before and after a change in tax policy during Gerald Ford’s presidency.

A copy of Table 2 from [71] was inserted here.

How can it be that tax rates on each income group went down, but the overall tax rate went up? (Don’t just say: “Simpson’s paradox”.

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<sup>3</sup> This problem was inspired by a worksheet at the Math Teacher Workshop on Social Justice issues led by Belin Tsinnajinnie and Aris Winger, hosted by the Alliance of Indigenous Math Circles in July 2020.

What is *causing* Simpson's paradox in the data? Can you give an intuitive explanation for what's going on?) Which do you think is more honest and accurate: did tax rates go up or down during Ford's presidency?

- (e) Find a news article that contains quantitative data. Include a link, and choose one piece of information to focus on. What choices have been made about how to obtain, compute, and display that information? How easy is it to figure out who made those choices and why?

### *Final project description*

One of the ongoing themes of this class has been that mathematicians have certain ethical obligations because of their specialized knowledge, and that one of the most important of these obligations is to communicate clearly the virtues and pitfalls of mathematical tools and algorithms. Pick an issue where you believe mathematics and ethics both have something important to say, and create the following:

- (a) An artifact (poster, video,...?) that effectively communicates what's at stake to a specific audience. Depending on your audience, you may have to simplify the mathematics in some ways—of course, how you do so is itself a decision with ethical implications!
- (b) An associated brief report (2-3 pages) that goes more in depth into the mathematical and ethical issues at stake, why you chose the audience and issue you did, and the decisions you made about what was most important to convey. This can be more technical.

### **B. Complete written comments from university-wide course feedback form**

All nine students in the course completed institutionally standardized course and instructor feedback forms. Six of the students chose to include comments, which are reproduced verbatim. At this institution, evaluations can be viewed by all students and faculty; as a result, comments are frequently written more for other students than for the course instructor.

Response 1: “This was a really excellent and fascinating course which explored a ton of topics of incredible importance about which I had previously thought very little. The content and all of the readings were extraordinarily engaging, and the in-class discussions raised numerous very difficult questions with no easy answers (the best and most meaningful type of questions, really). I feel (especially after having taken this course) that far too much STEM education is focused on the particular and technical details of “how?” and drastically underestimates the importance of the question “why?”. Before taking this course, I had really only ever been exposed to relentless technological optimism in my STEM classes. This was the first course I have ever taken which has forced me to question the ethics of applying technological and algorithmic “solutions” to every one of society’s problems and conflicts. It also reinforced the essential role of individuals in stemming the tide of unethical and inadvertently harmful algorithms and other math-based technologies. I considered the ethics of my own role in mathematics as a student at Rice and what this looked like. Overall, I think every math, computer science, or generally STEM student should take this class, and I’d say it even has much to offer those who concentrate in fields unrelated to mathematics.”

Response 2: “I really liked this course. This course challenged me to view the world in different ways and develop strong views of ethics in math.”

Response 3: “This class really changed the way I thought about the relationship between STEM and ethics. I highly recommend that all students take a class similar to this. ”

Response 4: “Really interesting and fun course. We’d spend about half our time discussing mathematical/ technical concepts, and the other half discussing ethical implications or case-studies. Topics included cryptography, algorithms, and math in public life. Readings were also interesting, and gave me a lot of food for thought about the ways in which I might pursue math & data-based careers after college.”

Response 5: “Really really insightful course. This class will have you actually apply mathematics to real life (who said math was useless irl?). It will teach you literally about the ethical issues involved with math like cryptography and delve into really cool case studies. I highly recommend this class!”

Response 6: “This was a really cool class to take. I recently read (on Wikipedia I believe) that very few universities offer courses on math ethics. I feel very lucky to have had access to such a course because it has really made me think critically about my role as a mathematician. I think this is a great class for non-Math majors to take, because many of the ethical issues discussed in this course occur at the intersection of math with other disciplines. Strongly recommend this course, especially if you are working on being more intentional with your career.”

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